Ch.7 Capacitors

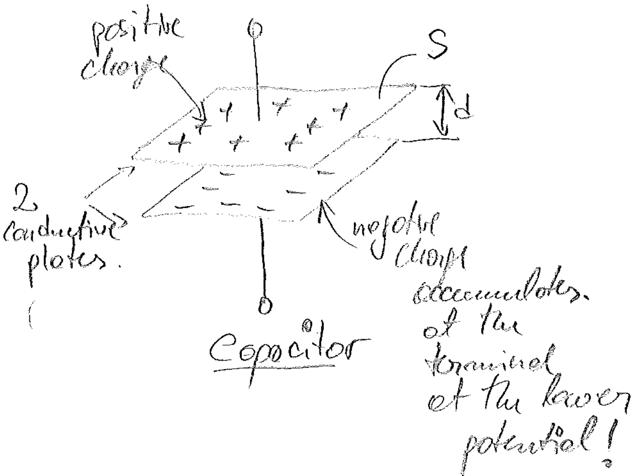
- Another passive circuit element. It is a energy storage element.

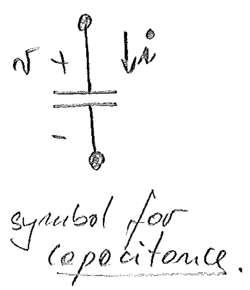
Reminder: a passive circuit element cannot supply power greater than zero over an infinite period of time.
V. important

(a) Capacitance: represents the ability of a circuit element to store charge in response to voltage.

Such circuits elements are called capacitors or condensers

Note: people use capacitance and capacitors interchangeably.





The Rate at which the accumulated charge varies with the applied voltage is denoted as capacitance.

$$\boxed{C \triangleq \frac{dq}{dv}} \quad (1)$$

[F] Farad is the SI unit

$$1F = \frac{1C}{1V}$$

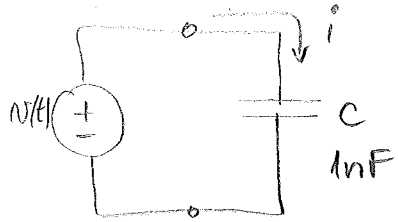
Capacitance C depends on
- the area of the plates S
- the distance between plates
- the insulating material — " — "

$$\boxed{C = \frac{\varepsilon \cdot S}{d}} \quad (2)$$

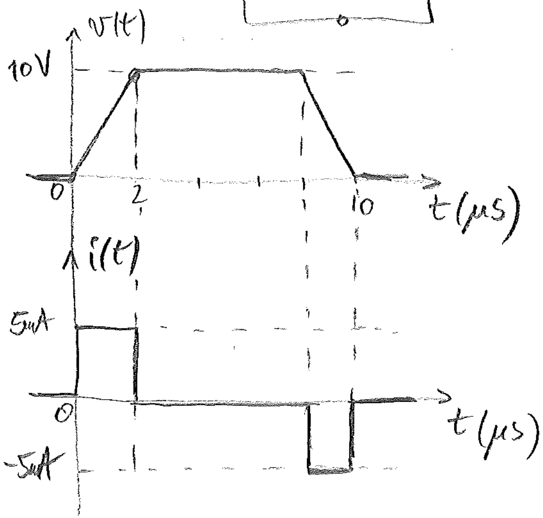
ε is permittivity which is $\varepsilon_0 = 10^{-9}/(36\pi)$ F/m for vacuum.

(b) The i-v characteristic

· Imagine $\boxed{v}$ is increased by dv => charge will increase by $dq = C \cdot dv$ on upper plate and charge $-q$ decrease by $dq = C\,dv$ on lower plate.



indicates => current flows like in the figure! even though no charges actually cross the insulator between plates!



$$i = \frac{dq}{dt} = \frac{C\,dv}{dt}$$

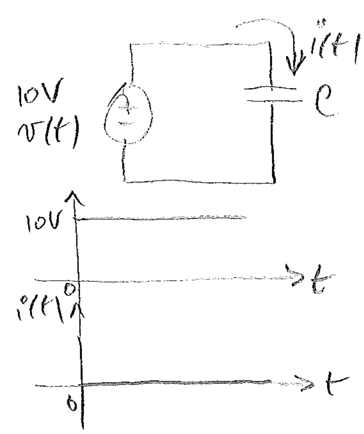
also: $\boxed{dq = C\,dv}$

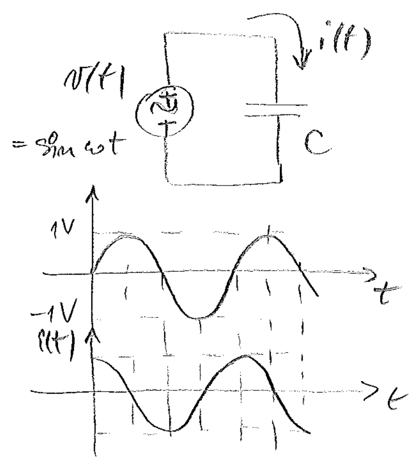
$$\boxed{i = C\frac{dv}{dt}} \quad (2) \qquad \boxed{i(t) = C\frac{dv(t)}{dt}} \quad (2)$$

The capacitance performs $\boxed{\text{voltage}}$ $\boxed{\text{differentiation}}$!

- Current depends on rate of change of the voltage!
- To sustain current, voltage must change. Otherwise current is zero! (if constant)
- The more rapidly v changes the larger i!
- One of the most import applications of C: DRAM's.

Example:





$$i(t) = C \cdot \frac{dv}{dt} = C\omega \cos \omega t$$

c) The v–i characteristic

Turn around equation (2): to find the voltage developed by a capacitor in response to an applied varying current.

pronounced "sai"

$$\boxed{v(t)=\frac{1}{C}\int_0^t i(\xi)\cdot d\xi + v(0)} \quad (3)$$

voltage across the capacitor at time $t=0$.

$$\boxed{v(0)=\frac{q(0)}{C}}$$

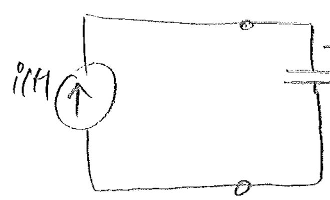
we deal only with linear capacitances!

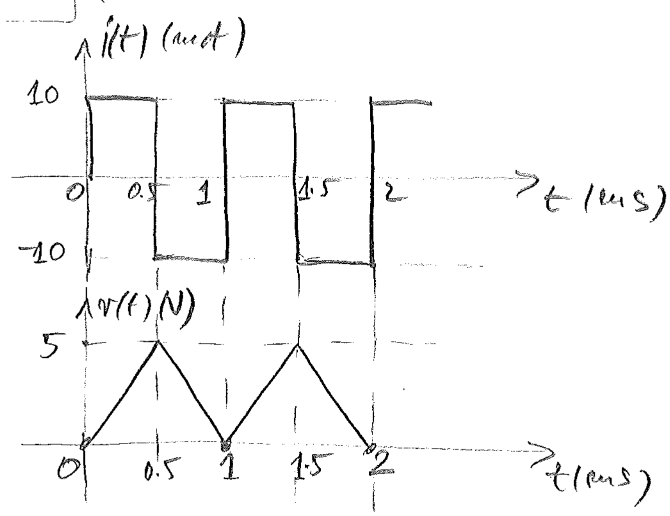
$$\boxed{C=\frac{Q}{V}}$$

Seen like this, the capacitor performs the operation of $\boxed{\text{current}}$ $\boxed{\text{integration}}$!

Particular case: if current is constant: $i(t)=I$

$$\Rightarrow \boxed{v(t)=\frac{I}{C}\cdot t+v(0)} \quad (3')$$





d) Capacitance Energy

The process of charging a capacitor requires energy expenditure.

Remember: $\boxed{p=\frac{dw}{dt}}$,

$$\Rightarrow \boxed{w(t)=\int_0^t \underbrace{p(\xi)}_{=v\cdot i = vC\cdot\frac{dv}{d\xi}}d\xi = \int_0^t v\cdot C\cdot\left(\frac{dv}{d\xi}\right)\cdot d\xi = \int_0^{v(t)} C\,u\,du} \quad (4)$$

$\underbrace{v'(\xi)\cdot d\xi}_{=du}$

variable change (see prima formulă de schimbare de variabilă (vezi pe spate)) $\boxed{u=v(\xi)}$

For a linear capacitance for which $\boxed{C=\frac{q}{v}}$; we get $w(t)=C\int_0^{v(t)} v\,dv$
$\boxed{q}$ linearly proportional to $\boxed{v}$.
$\boxed{q=Cv}$, therefore

constant, does not depend on v

$$= C\,\frac{v^2}{2}\Big|_0^{v(t)} \overset{(4')}{=} \boxed{C\,\frac{v^2(t)}{2}=w(t)}$$

This is what we'll consider most of the time!

(see verso)

Example

$C = 1\,\mu F$

$V = 10V$

$w = 0.5 \cdot 10^{-6} \times 10^{2} = 50\,\mu J$

$\boxed{\text{Capacitive energy}}$ is stored in the form of potential energy in the electric field between plates.

$\boxed{\text{Reminders}}$

① formula de schimbare de variabilă:

Fie $\underset{\varphi(t)}{\varphi} : [a,b] \to J$; $f : J \to \mathbb{R}$; $J \subset \mathbb{R}$

(a) f is continuous on J

(b) φ is derivabilă (admits derivative) and its derivata este continuă or is differentiable!

$$\int_a^b f(\varphi(t)) \cdot \varphi'(t)\,dt \underset{\varphi(t)=x}{=} \int_{\varphi(a)}^{\varphi(b)} f(x)\,dx \qquad \boxed{1}$$

② Fie $\varphi : [a,b] \to [c,d]$; $f : [c,d] \to R$

(a) f is continua pe $[c,d]$

(b) φ is bijectivă, φ, φ^{-1} derivabile cu derivatele continue

$$\int_a^b f(\varphi(t))\,dt = \int_{\varphi(a)}^{\varphi(b)} f(x) \cdot (\varphi^{-1})'(x)\,dx \qquad \boxed{2}$$