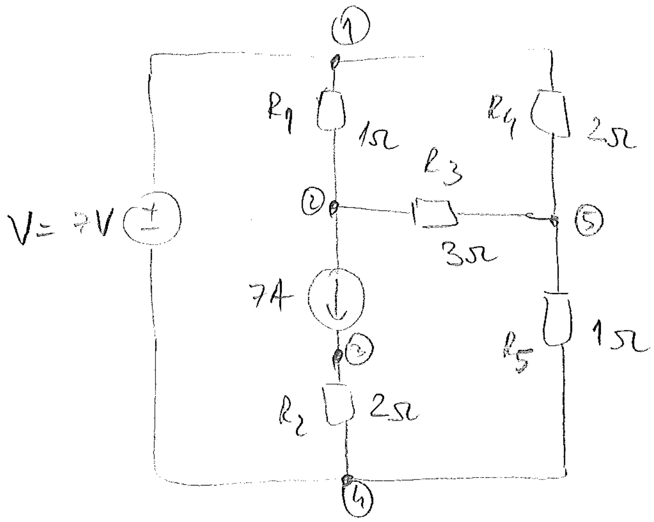
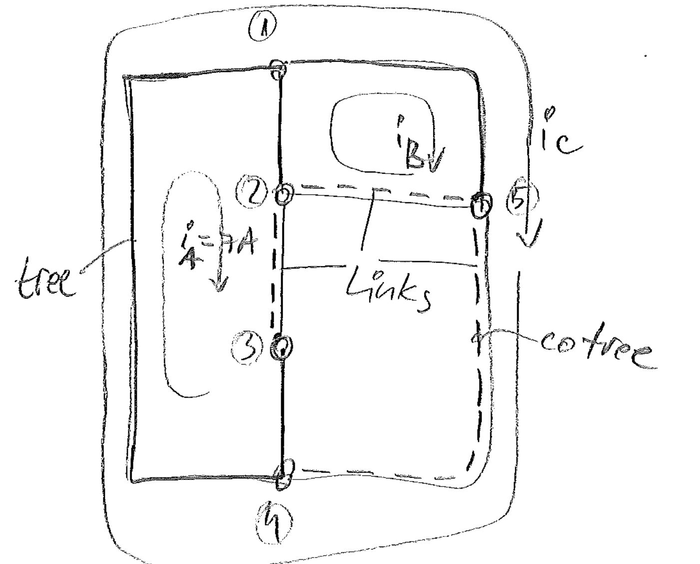
Links and general loop analysis

- The dual of the method of writing nodal equations.
- Similar steps as in the other method (the general nodal analysis)
- However now we assign current to each element in the cotree





> assign currents to branches in the cotree

> see these currents as loop currents thru loops constructed using the link (cotree branch) & the tree

Write KVL equations around these loops:

first: $i_A = 7A$

$$\left.\begin{array}{ll} i_B \text{ loop:} & R_1(i_B - i_A) + R_4(i_B + i_C) + R_3(i_B) = 0 \\ i_C \text{ loop:} & -7V + R_4(i_B + i_C) + R_5 i_C = 0 \end{array}\right\} \Rightarrow \left\{\begin{array}{l} i_B - 7 + 2i_B + 2i_C + 3i_B = 0 \\ -7 + 2i_B + 2i_C + i_C = 0 \end{array}\right.$$

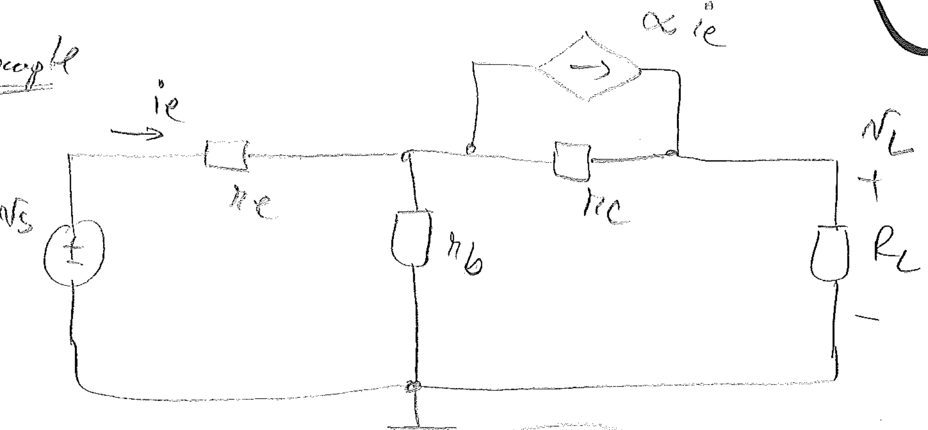
$$\Rightarrow \left\{\begin{array}{l} 6i_B + 2i_C = 7 \\ 2i_B + 3i_C = +7 \end{array}\right. \quad A = \begin{bmatrix} 6 & 2 \\ 2 & 3 \end{bmatrix} \quad b = \begin{bmatrix} 7 \\ -7 \end{bmatrix} \quad X = \begin{bmatrix} i_B \\ i_C \end{bmatrix} \Rightarrow$$

$$\Rightarrow \boxed{AX = b}$$

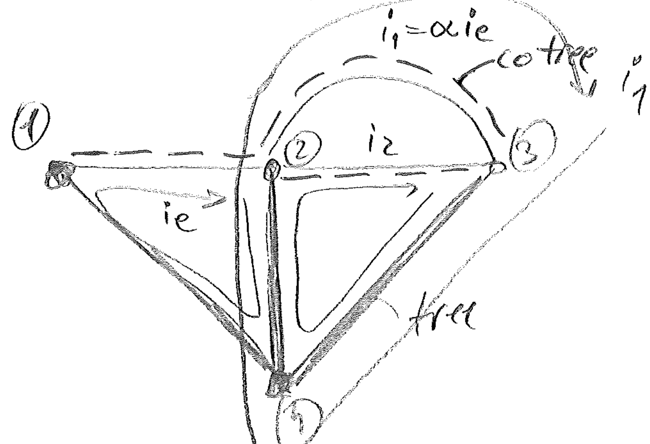
$$i_B = \frac{\begin{vmatrix} 7 & 2 \\ +7 & 3 \end{vmatrix}}{\begin{vmatrix} 6 & 2 \\ 2 & 3 \end{vmatrix}} = \frac{21 - 14}{18 - 4} = \frac{7}{14} = \frac{1}{2} = \boxed{0.5\ A = i_B} \qquad \boxed{i_A = 7\ A}$$

$$i_C = \frac{\begin{vmatrix} 6 & 7 \\ 2 & +7 \end{vmatrix}}{\begin{vmatrix} 6 & 2 \\ 2 & 3 \end{vmatrix}} = \frac{42 - 14}{14} = \frac{28}{14} = \boxed{2A = i_C}$$

Example



$i_e = ?$
$v_L = ?$



$G(V, E)$

N nodes, B branches

$\boxed{i_1 = \alpha i_e}$

Loop i_e: $-v_s + r_e i_e + r_b(i_e - i_2 - \alpha i_e) = 0$

Loop i_2: $r_b(i_2 + \alpha i_e - i_e) + r_c i_2 + R_L(i_2 + \alpha i_e) = 0$

$$\Rightarrow \begin{cases} [r_e + (1-\alpha) r_b]\, i_e - r_b i_2 = v_s \\ [r_b(\alpha - 1) + \alpha R_L]\, i_e + (r_b + r_c + R_L)\, i_2 = 0 \end{cases}$$

$$A = \begin{bmatrix} r_e + (1-\alpha) r_b & -r_b \\ \alpha R_L - (1-\alpha) r_b & r_b + r_c + R_L \end{bmatrix} \qquad b = \begin{bmatrix} v_s \\ 0 \end{bmatrix} \qquad x = \begin{bmatrix} i_e \\ i_2 \end{bmatrix}$$

$$\boxed{i_e = \frac{\begin{vmatrix} v_s & -r_b \\ 0 & r_b + r_c + R_L \end{vmatrix}}{\begin{vmatrix} r_e + (1-\alpha) r_b & -r_b \\ \alpha R_L - (1-\alpha) r_b & r_b + r_c + R_L \end{vmatrix}} = \frac{r_b + r_c + R_L}{(\quad)(\quad) + r_b(\quad)} \cdot v_s}$$

$$i_2 = \frac{\begin{vmatrix} r_e + (1-\alpha) r_b & v_s \\ \alpha R_L - (1-\alpha) r_b & 0 \end{vmatrix}}{\begin{vmatrix} r_e + (1-\alpha) r_b & -r_b \\ \alpha R_L - (1-\alpha) r_b & r_b + r_c + R_L \end{vmatrix}} = -\frac{\alpha R_L - (1-\alpha) r_b}{(\quad)(\quad) + r_b(\quad)} \cdot v_s;$$

$\boxed{v_L = R_L(i_1 + i_2)} = \dots$

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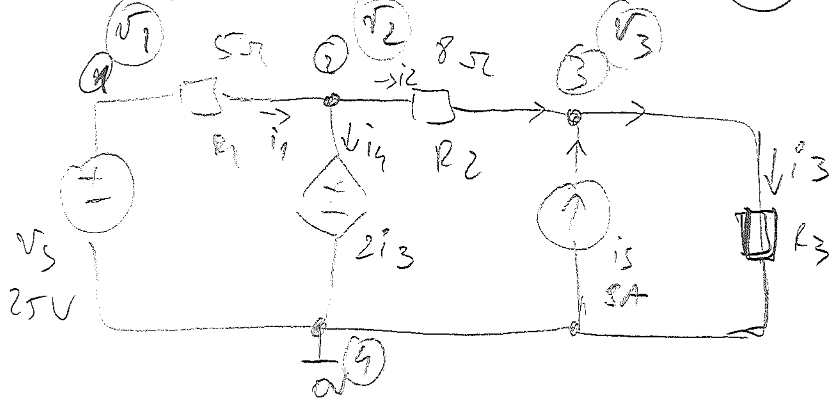


M_1 Regular nodal analysis

1

$\boxed{v_1 = V_s}$: we know.

$\boxed{V_2 = 2i_3}$

$\boxed{V_3 = ?} = R_3 i_3$

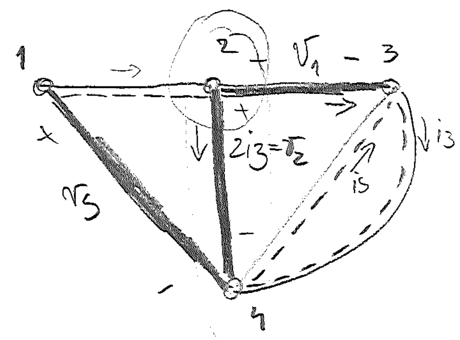
1 unknown only.

2. $\frac{v_1 - v_2}{R_1} + \frac{v_2 - v_3}{R_2} = i_4$

3. $\frac{v_2 - v_3}{R_2} + i_s = \frac{V_3}{R_3} = i_3$

$\Leftrightarrow \left\{ \begin{array}{l} \frac{v_1 - 2i_3}{R_1} + \frac{2i_3 - R_3 i_3}{R_2} = i_4 \\ \frac{2i_3 - R_3 i_3}{R_2} + i_s = i_3 \end{array} \right.$

M_2 General nodal analysis



Node 3: $\frac{v_1}{R_2} + i_s = \frac{v_2 - v_1}{R_3}$

Node 2: $\frac{v_s - v_2}{R_1} = \frac{v_1}{R_2} + i_4$

$\Rightarrow \left\{ \begin{array}{l} \frac{v_1^{?}}{R_2} + i_s = \frac{2i_3 - v_1^{?}}{R_3} \\ \frac{v_s - 2i_3}{R_1} = \frac{v_1^{?}}{R_2} + i_4 \end{array} \right. \Rightarrow$ 2 eqns 2 unknowns.

$v_1 + i_3 R_3 = 2i_3 \Rightarrow v_1 = 2i_3 - i_3 R_3$