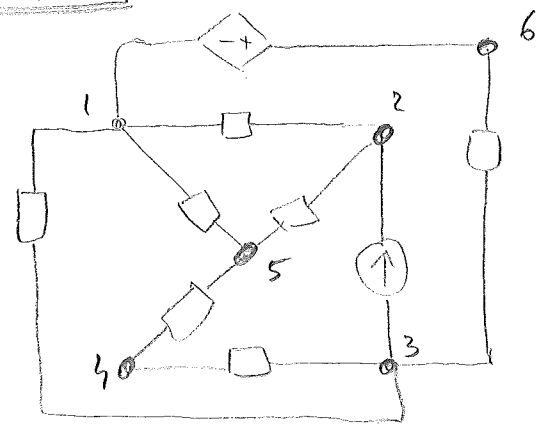
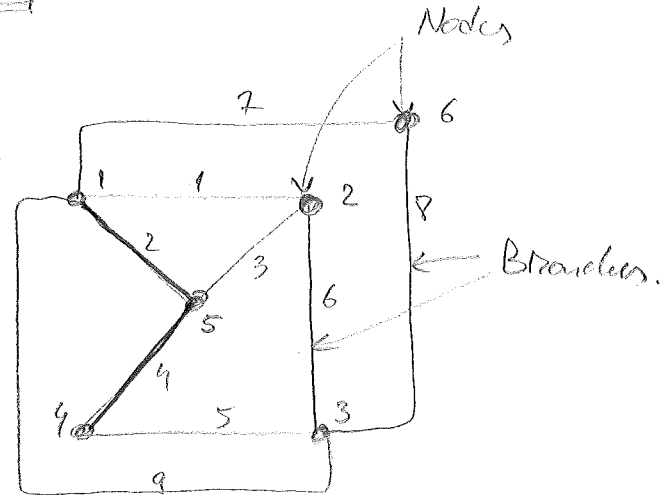
Trees and general nodal analysis

Definitions:



circuit:

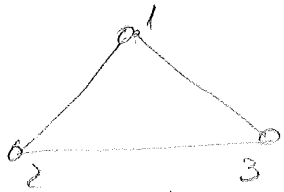
=>



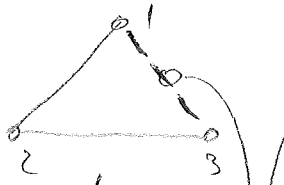
(linear) graph = (Nodes, Branches) = G(V, E)
corresponding to the circuit.
(vertices), (arcs) (edges)
|V| = 6
|E| = 9

Path from 1 to 4: {1, 5, 4}
Another path from 1→4: {1, 6, 3, 4}
Loop ≜ closed path.

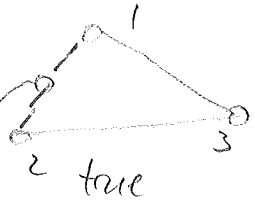
Tree = a set of branches that connect all nodes with no loop!



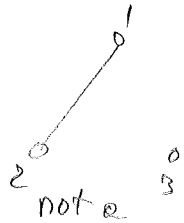
graph



tree



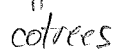
true



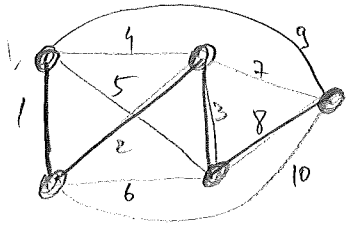
not a tree.

How many trees are there?

Cotree = the complementary set of edges of a tree



cotrees



red is the tree = {1, 2, 3, 8}

cotree = {4, 5, 6, 7, 8, 9, 10}

link is a branch ∈ cotree.

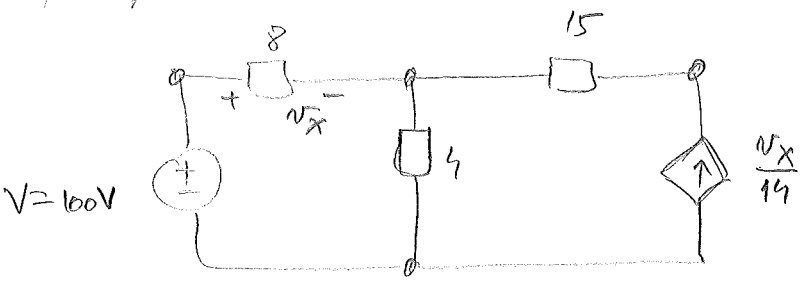
Links!

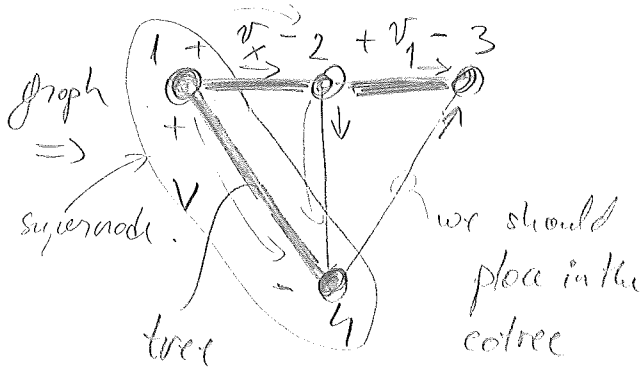
General Method to write a set of nodal equations

0 – Given circuit
1 – Draw graph, identify a tree
2 – Place all voltage sources in the tree (dependent & independent)
3 – Place all current sources in the cotree (– " – & – " –)
4 – Place all control-voltage branches for voltage-controlled dependent sources in the tree, if possible.
5 – Place all control-current – " – current-controlled – " – – " – cotree, if possible.

6 – Assign voltage variables.
7 – Write a set of equations using KCL for all but one nodes and supernodes!

Example:





The only 2 unknowns are v_x and v_1.

KCL Node 3: $\frac{v_1}{15} + \frac{v_x}{14} = 0$

KCL Node 2: $\frac{v_x}{8} = \frac{v_1}{15} + \frac{V - v_x}{4}$ ← found along the path between nodes 2 & 4 in the tree

$$\begin{cases} \frac{v_1}{15} + \frac{v_x}{14} = 0 \\ \frac{v_x}{8} = \frac{v_1}{15} + \frac{V - v_x}{4} \end{cases}$$

$$\begin{cases} \frac{14v_1}{15} + \frac{15v_x}{14} = 0 \\ \frac{4v_1}{15} + \frac{15 \cdot 3v_x}{4} = \frac{15 \cdot 100}{4} \end{cases} \Leftrightarrow \begin{cases} 14v_1 + 15v_x = 0 \\ -4v_1 + 45v_x = 1500 \end{cases} \Rightarrow \boxed{Ax = b}$$

$$A = \begin{bmatrix} 14 & 15 \\ -4 & 45 \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ 1500 \end{bmatrix} \quad x = \begin{bmatrix} v_1 \\ v_x \end{bmatrix}$$

$$\Downarrow$$

$$\boxed{v_1 = -60\ V}$$
$$\boxed{v_x = 56\ V}$$