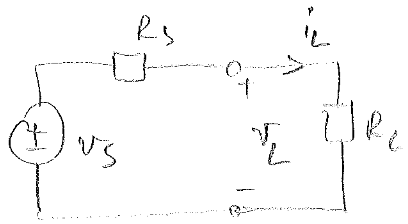
Maximum power transfer theorem



practical voltage source

Power delivered to the load R_L:

$$\boxed{P_L = i_L^2 \cdot R_L = \frac{v_S^2 \cdot R_L}{(R_L+R_S)^2}} \quad (1)$$

$$i_L = \frac{v_S}{R_S+R_L}$$

Differentiate wrt R_L to find the maximum value of R_L that absorbs maximum power from the practical voltage source.

$$(1) \Rightarrow \frac{dP_L}{dR_L} = \frac{(R_L+R_S)^2 \cdot v_S^2 - v_S^2 R_L \cdot 2 \cdot (R_L+R_S)}{(R_L+R_S)^4}$$

$$\frac{dP_L}{dR_L} = 0 \iff (R_L+R_S)^{\cancel{2}} \cdot \cancel{v_S^2} - \cancel{v_S^2} \cdot R_L \cdot 2 \cancel{(R_L+R_S)} = 0$$

$$R_L + R_S - 2R_L = 0 \iff \boxed{R_L = R_S} \quad (2)$$

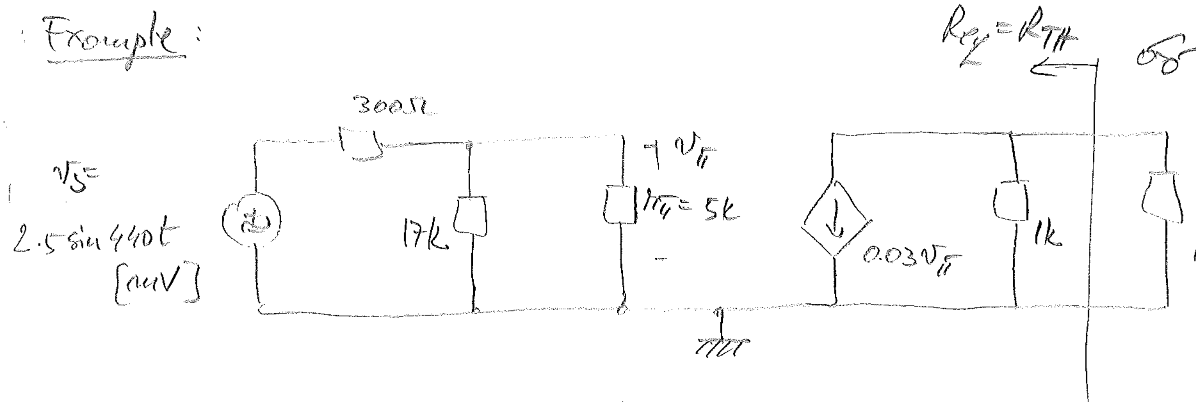
(i) Theorem: an independent voltage source in series with a resistance R_S (or an independent current source in parallel with a resistance R_S) delivers a maximum power to that load resistance R_L for which $R_L = R_S$!

(ii) A network (one-port) delivers the maximum power to a load resistance R_L when R_L is equal to the Thévenin equivalent resistance of the one-port network!

what is that power?

$$P_{L\,max} = \frac{v_S^2 \cdot R_S}{(R_S+R_S)^2} = \boxed{\frac{v_S^2}{4R_S} = P_{L\,max}} \quad (3)$$

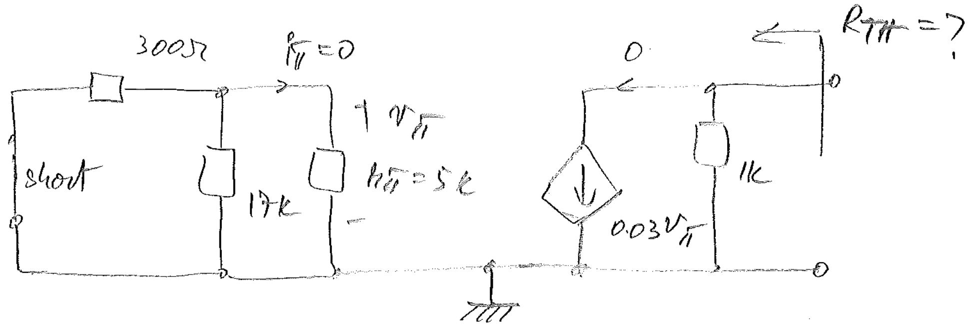
Example:



$R_L = ?$ to have max power transfer to it?

Thévenin's equivalent resistance?

(a)

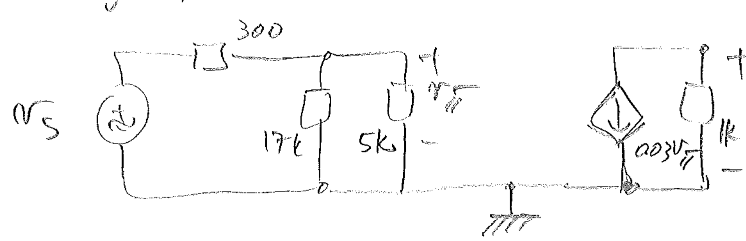


Method 1: $i_\pi = 0 \Rightarrow v_\pi = 0 \Rightarrow 0.03 v_\pi = 0 \Rightarrow \boxed{R_{TH} = 1k.}$

$\Rightarrow$ To get maximum power we need $\boxed{R_L = R_{TH} = 1k}$!

Method 2: $R_{eq} = \frac{v_{oc}}{i_{sc}}$

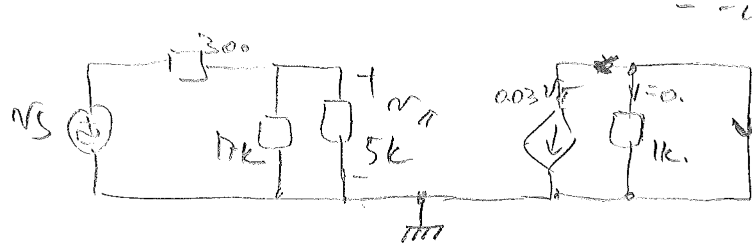
(Find v_{oc} anyway needed for the Thévenin's equivalent)



$\boxed{v_{oc} = -1k \times 0.03 v_\pi}$

$$v_\pi = \frac{5 /\!/ 17}{5 /\!/ 17 + 300} v_S \Rightarrow v_{oc} = -0.03k \cdot v_\pi$$

$$= -0.03k \times \frac{\underbrace{5 /\!/ 17}_{=3864}}{\underbrace{5 /\!/ 17}_{=3864} + 300} v_S$$



$\boxed{i_{sc} = -0.03 v_\pi}$

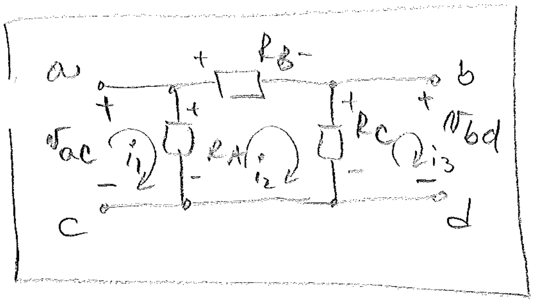
$$\Rightarrow R_{TH} = \frac{v_{oc}}{i_{sc}} = \frac{-1k \times \cancel{0.03 v_\pi}}{-\cancel{0.03 v_\pi}} = \boxed{1k = R_{TH}}$$

(b) Max power value?
We need $v_{oc} = v_{TH} = ?$

$$v_{oc} = -0.03k \times \frac{3864}{3864 + 300} \cdot v_S = -69.6 \cdot \sin 440t \text{ [mV]}$$

$$\Rightarrow P_{max} \overset{(3)}{=} \frac{v_{TH}^2}{4 R_{TH}} = 1.211 \sin^2 440t \ [\mu W]$$

Delta-Wye conversion $\quad \boxed{\Delta - Y}$ ← Homework!



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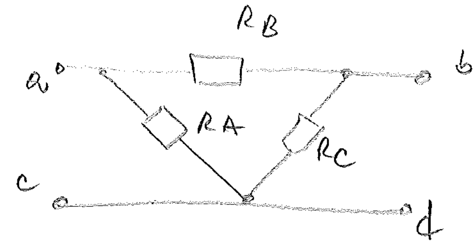
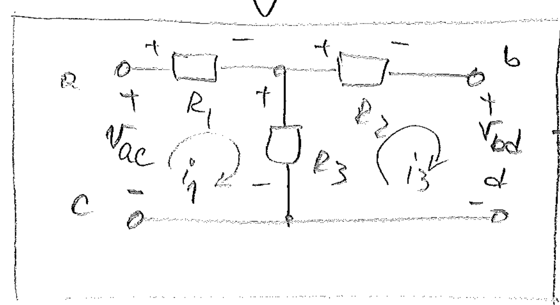


how? $\Downarrow$?



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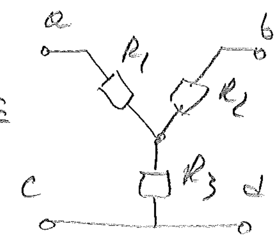


Networks are equivalent if terminal voltages and currents are equal!

Method 1: Do mesh analysis:

$$\begin{cases} v_{ac} = R_A(i_1 - i_2) \\ R_A(i_1 - i_2) = R_B i_2 + R_C(i_2 - i_3) \\ R_C(i_2 - i_3) = v_{bd} \end{cases}$$

$$\begin{cases} v_{ac} = R_1 i_1 + R_3(i_1 - i_3) = (R_1+R_3)i_1 - R_3 i_3 \\ R_3(i_1 - i_3) + R_2 i_3 = v_{bd} = -R_3 i_1 + (R_2+R_3) i_3 \end{cases}$$

remove $\boxed{i_2}$

$$v_{ac} = \left(R_A - \frac{R_A^2}{R_A+R_B+R_C}\right) i_1 - \frac{R_A R_C}{R_A+R_B+R_C} i_3$$

compare =>

$$v_{db} = -\frac{R_A R_C}{R_A+R_B+R_C} i_1 + \left(R_C - \frac{R_C^2}{R_A+R_B+R_C}\right) i_3$$

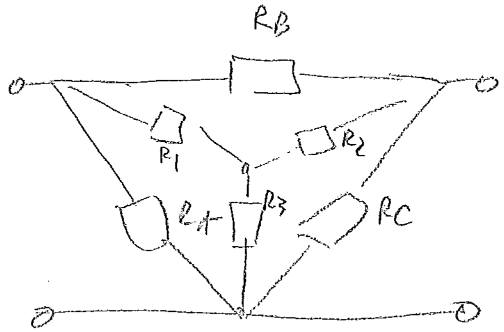
(h) => $\boxed{\Delta \Rightarrow Y}$

$$\boxed{R_3 = \frac{R_A R_C}{R_A+R_B+R_C}} \quad \boxed{R_2 = \frac{R_A R_C}{R_A+R_B+R_C}} \quad \boxed{R_1 = \frac{R_A R_B}{R_A+R_B+R_C}}$$

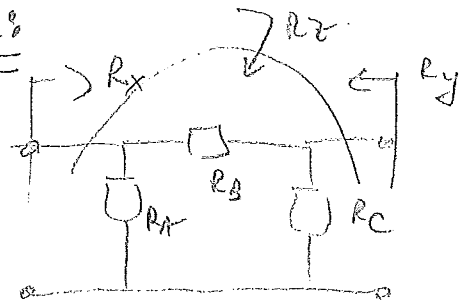
(b) => $\boxed{Y \Rightarrow \Delta}$

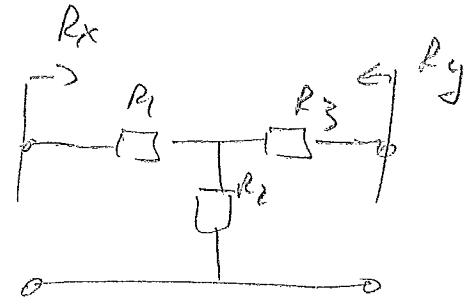
$$\boxed{R_A = \frac{R_1R_2+R_2R_3+R_1R_3}{R_2}} \quad \boxed{R_B = \frac{R_1R_2+R_2R_3+R_1R_3}{R_3}} \quad \boxed{R_C = \frac{R_1R_2+R_2R_3+R_1R_3}{R_1}}$$

see verso!



Method 2:





$$R_x = R_A \parallel (R_B + R_C) = \frac{R_A(R_B+R_C)}{R_A+R_B+R_C} \qquad R_x = R_1 + R_2$$

$$R_y = R_C \parallel (R_B + R_A) = \frac{R_C(R_B+R_A)}{R_A+R_B+R_C} \qquad R_y = R_2 + R_3$$

$$R_Z = R_B \parallel (R_A + R_C) = \frac{R_B(R_A+R_C)}{R_A+R_B+R_C} \qquad R_Z = R_1 + R_3.$$

$\Rightarrow$

$\boxed{\Delta \to Y}$ easier $\Rightarrow$ $R_1 = \dfrac{R_A R_B}{R_A+R_B+R_C}$ $\quad R_2 = \dfrac{R_A R_C}{R_A+R_B+R_C}$ $\quad R_3 = \dfrac{R_A R_C}{R_A+R_B+R_C}$

$\boxed{Y - \Delta}$ more difficult $\Rightarrow$ $R_A =$

$R_B =$

$R_C =$.