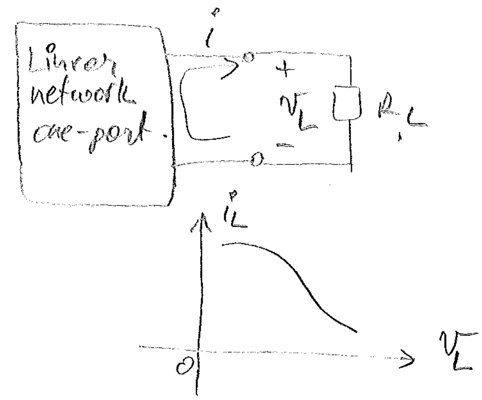
Circuit theorems: Thévenin and Norton equivalent circuits.

Definitions:

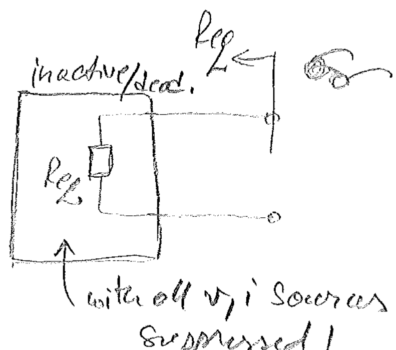
One-port = a network with just one pair of terminals emerging from it is called a one-port network, or a one-port for short.

i-v characteristic:

- open-circuit voltage $\boxed{v_{oc}}$
- short-circuit current $\boxed{i_{sc}}$



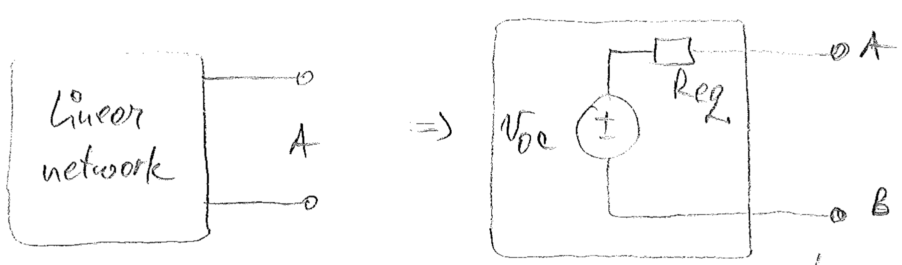
- equivalent resistance of the one-port: $\boxed{R_{eq}}$
  with all voltage and current sources in the network suppressed!
  (i.e. ...)
  it is given by: $\boxed{R_{eq} = \frac{v_{oc}}{i_{sc}}}$
  (see last page for proof).



- The most general technique $\boxed{R_{eq} = \frac{v_{test}}{i_{test}}}$ because i_{sc} or v_{oc} may be zero!?! in pathological examples

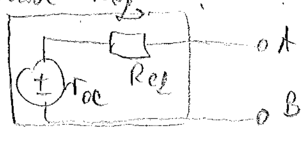
Thévenin's theorem

Any linear resistive one-port is equivalent to an ideal voltage source v_{oc} in series with a resistance R_{eq}, where v_{oc} and R_{eq} are: the open-circuit voltage and the equivalent resistance of the one-port!

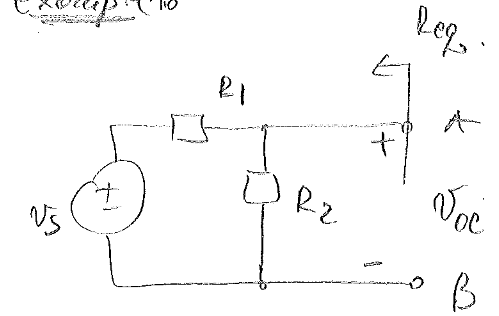


If the linear network has dependent sources and the controlling variables are within the network, then R_{eq} is the network as seen at terminals A, B with sources suppressed! (i.e. the dead/inactive network)

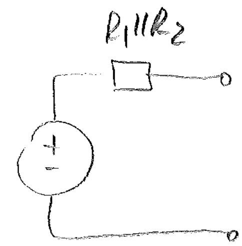
Only the independent ones!



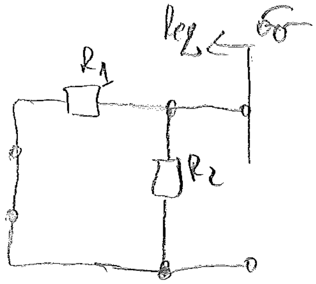
Example 1:



=>



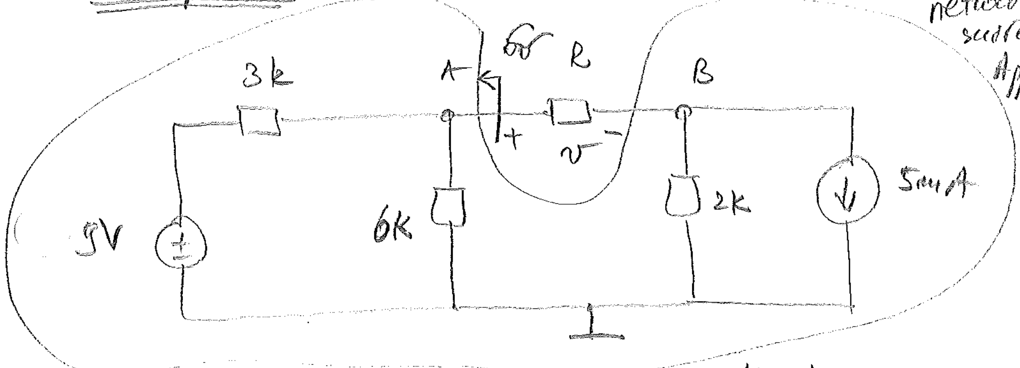
$$v_{oc} = \frac{R_2}{R_1+R_2} v_s$$



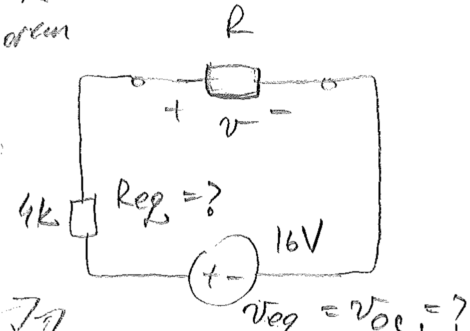
$$R_{eq} = R_1 \| R_2 = \frac{R_1 R_2}{R_1+R_2}$$

Example 2: (skip)

network surrounding R
Apply Thevenin's theorem



=

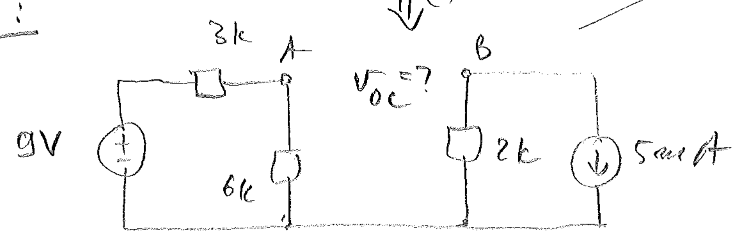


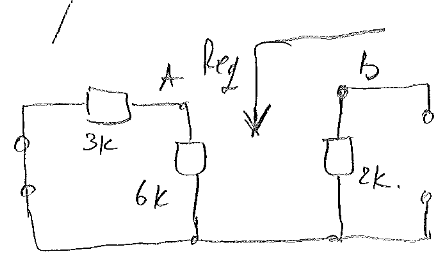
$v_{eq} = v_{oc} = ?$

$v(R) = v = ?$ for $R = 1k, 4k, 6k, 12k, 16k$

Method 1:

(1) (2) (3)





$$v_{oc} = v_A - v_B = \frac{6k\Omega}{(3+6)k\Omega} \times 9V - (2k \times 5mA)$$

$$= (6+10)V = 16V$$

$$R_{eq} = 2k + (3\|6)k$$
$$= \left(2 + \frac{3\cdot 6}{3+6}\right)k = 4k.$$

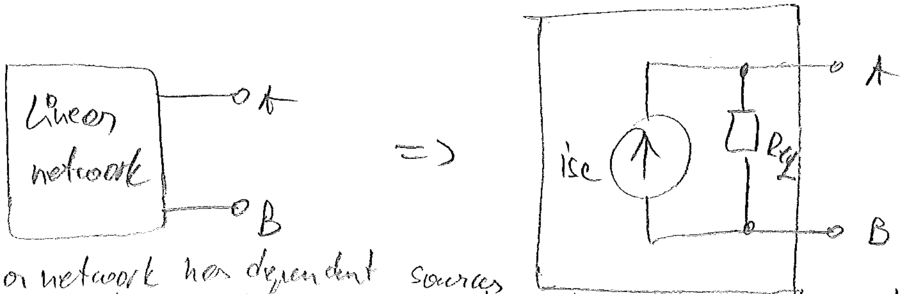
$$v = \frac{R}{R_{eq}+R} \cdot v_{eq} = \boxed{\frac{R}{R+4k} \cdot 16V = v(R)}$$

$$\begin{cases} R = 1k \Rightarrow v = \frac{16}{5} V \\ R = 4k \Rightarrow v = 8V \\ R = 6k \quad \ldots \end{cases}$$

Method 2: Use nodal analysis.

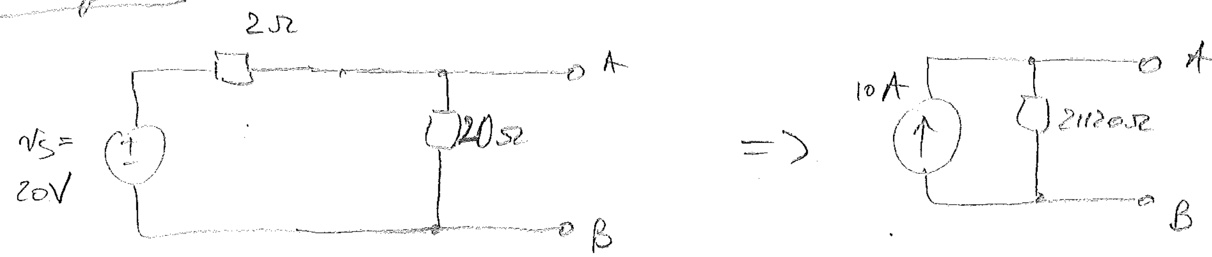
Norton's Theorem

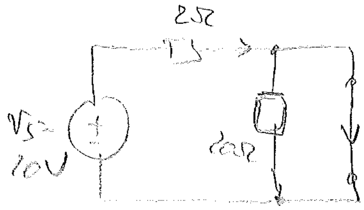
- Any linear resistive one-port is equivalent to an ideal current source i_{sc} in parallel with a resistance R_{eq}, where i_{sc} and R_{eq} are: the short-circuit current and the equivalent resistance of the one-port network.



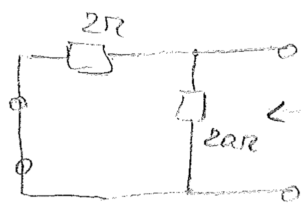
if the linear network has dependent sources and the controlling variables are within the network R_{eq} is the dead/inactive network!

Example 1:



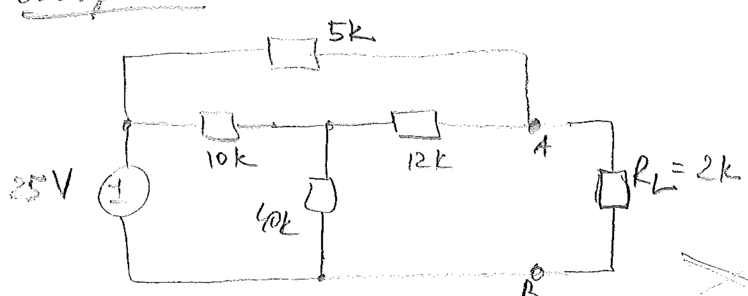


$i_{sc} = \frac{v_s}{2\Omega} = 10A$



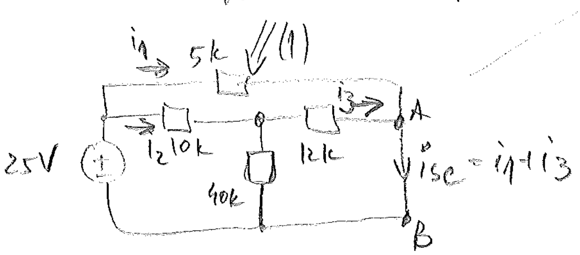
$R_{eq} = (2\|20)\Omega = \frac{40}{22}\Omega$

Example 2:



Use Norton's equivalent to find the power in R_L!

(1)



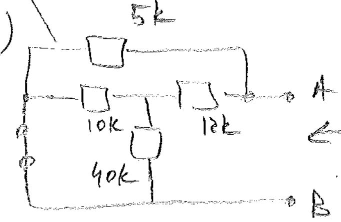
$i_{sc} = i_1 + i_3$

$i_1 = \frac{25V}{5k} = 5mA$

$i_3 = i_2 \times \frac{40}{40+12} = \frac{40}{52} i_2 \; ; \; i_2 = \frac{25V}{10 + 40\|12} = 1.3mA$

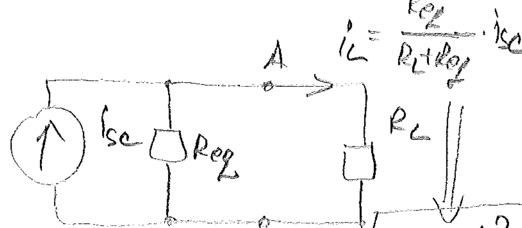
$\Rightarrow i_{sc} = 5mA + 1.3mA = 6.3mA$.

(2)



$R_{eq} = 5\|[(12+10)\|40] = 4k$

(3)



$i_L = \frac{R_{eq}}{R_L + R_{eq}} \cdot i_{sc}$

$\boxed{P = R_L \, i_L^2}$

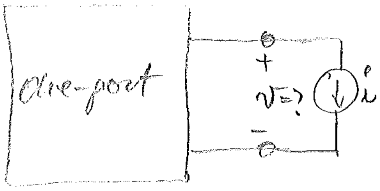
Extra material (4)

Proof of $\boxed{R_{eq} = \frac{v_{oc}}{i_{sc}}}$

OBS: not covered in class normally → no time.

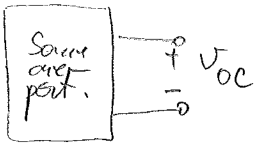
Method 1: i-v characteristics

a)

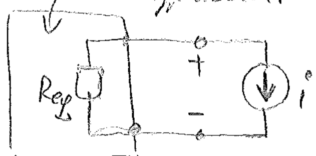


$v = v_1 + v_2$ — contribution of port (v_1), contribution of i (v_2)

Use superposition:



with all sources suppressed.

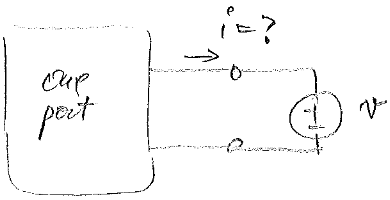


$v_1 = v_{oc}$ $\qquad$ $v_2 = -R_{eq} \cdot i$ $\qquad \Rightarrow \boxed{v = v_{oc} - R_{eq} \cdot i}$ (1)

$$\Downarrow$$

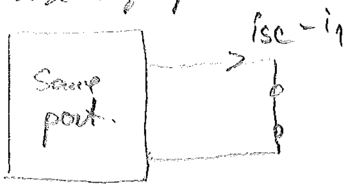
$$\boxed{i = \frac{v_{oc}}{R_{eq}} - \frac{v}{R_{eq}}}$$

b)

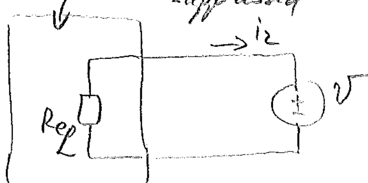


$i = i_1 + i_2$

Use superposition.



with all sources suppressed



$i_1 = i_{sc}$ $\qquad$ $i_2 = -\frac{v}{R_{eq}}$ $\qquad \Rightarrow \boxed{i = i_{sc} - \frac{v}{R_{eq}}}$ (2)

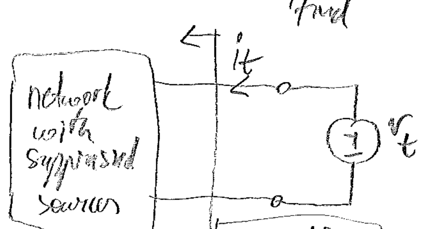
They are equivalent if i-v characteristics are the same! $\Rightarrow \frac{v_{oc}}{R_{eq}} = i_{sc} \Rightarrow \boxed{R_{eq} = \frac{v_{oc}}{i_{sc}}}$ nice!

Method 2 of finding $\boxed{R_{eq} = ?}$

Steps: (1) suppress all v, i sources of the one-port network

(2) apply a test voltage v_t and measure/find current i_t.

(3) $\boxed{R_{eq} = \frac{v_t}{i_t}}$



$\boxed{R_{eq} = \frac{v_t}{i_t}}$

or apply i_t and find v_t!