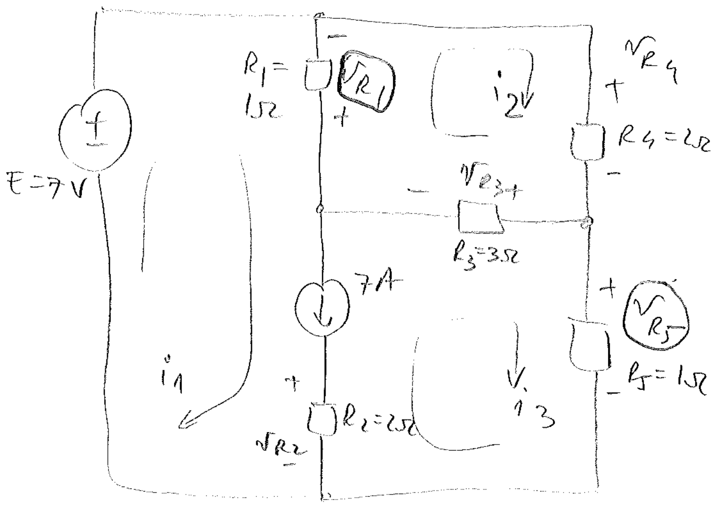
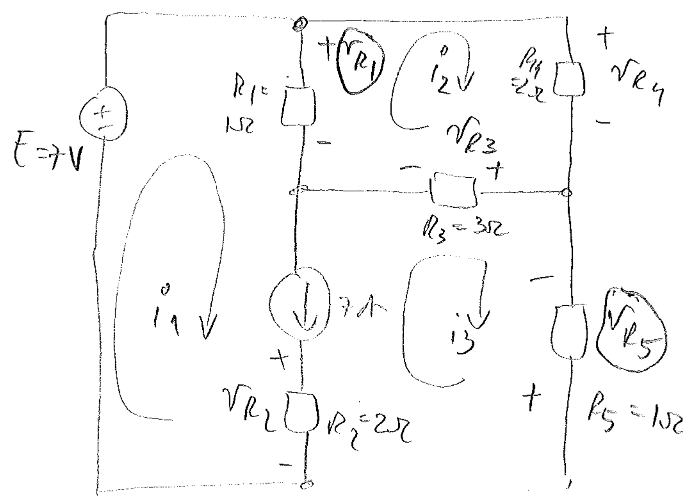
Example – from last time:

a) one selection of voltage polarities



b) another selection of voltage polarities.



(a)

loop i_2: $\begin{cases} v_{R_1} + v_{R_4} + v_{R_3} = 0 \\ E + v_{R_1} + v_{R_3} = v_{R_5} \end{cases} \Rightarrow$

loop super-mesh i_1, i_3

$$\Rightarrow \begin{cases} R_1(i_2 - i_1) + R_4 i_2 + R_3(i_2 - i_3) = 0 \\ E + R_1(i_2 - i_1) + R_3(i_2 - i_3) = R_5 i_3 \end{cases}$$

$$\Rightarrow \begin{cases} i_2 - i_1 + 2i_2 + 3(i_2 - i_3) = 0 \\ i_2 - i_1 + 3(i_2 - i_3) - i_3 = -7 \end{cases}$$

$$\Rightarrow \begin{cases} -i_1 + 6i_2 - 3i_3 = 0 \\ -i_1 + 4i_2 - 4i_3 = -7 \\ i_1 - i_3 = 7 \end{cases}$$

equ. for current source → $i_1 - i_3 = 7$

$$\Rightarrow \begin{cases} i_1 - 6i_2 + 3i_3 = 0 \\ i_1 - 4i_2 + 4i_3 = 7 \\ i_1 - i_3 = 7 \end{cases} \Rightarrow Ax = b$$

$$A = \begin{bmatrix} 1 & -6 & 3 \\ 1 & -4 & 4 \\ 1 & 0 & -1 \end{bmatrix} \quad x = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ 7 \\ 7 \end{bmatrix}$$

$\boxed{i_1 = 9A}$ $\boxed{i_2 = 2.5A}$ $\boxed{i_3 = 2A}$

(b)

loop i_2: $\begin{cases} v_{R_1} = v_{R_4} + v_{R_3} \\ E + v_{R_3} = v_{R_1} + v_{R_5} \end{cases}$

super mesh:

$$\Rightarrow \begin{cases} R_1(i_1 - i_2) = R_4 i_2 + R_3(i_2 - i_3) \\ E + R_3(i_2 - i_3) = R_1(i_1 - i_2) + R_5(-i_3) \end{cases}$$

$$\Rightarrow \begin{cases} i_1 - i_2 = 2i_2 + 3(i_2 - i_3) \\ 7 + 3(i_2 - i_3) = i_1 - i_2 - i_3 \end{cases}$$

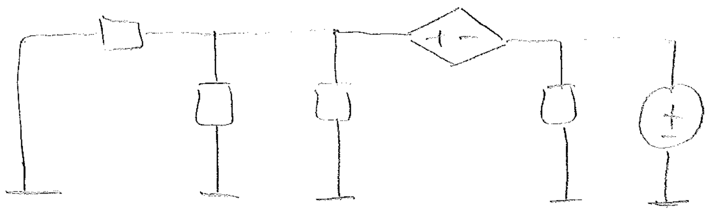
$$\Rightarrow \begin{cases} -i_1 + 6i_2 - 3i_3 = 0 \\ -i_1 + 4i_2 - 4i_3 = -7 \end{cases}$$

equ. for curr. source $i_1 - i_3 = 7$

Same equations!

$\Rightarrow$ $\boxed{i_1 = 9A}$ $\boxed{i_2 = 2.5A}$ $\boxed{i_3 = 2A}$!

Drawing schematic diagrams



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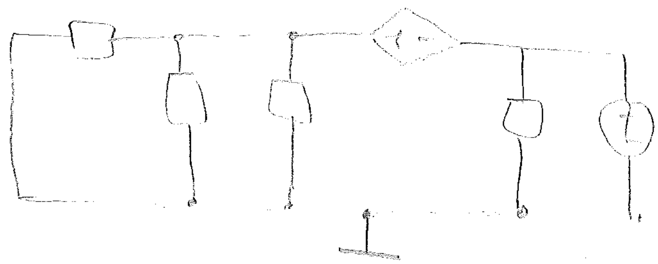
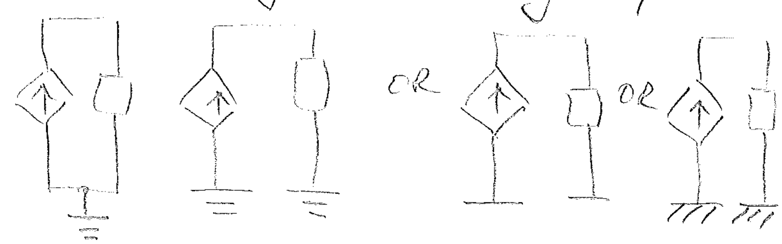
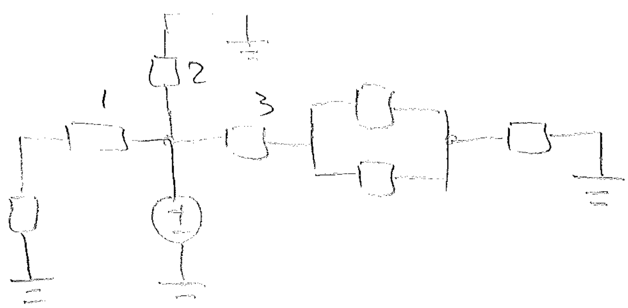


You'll see articles, textbooks etc. utilizing this drawing style:



Also, you may see:



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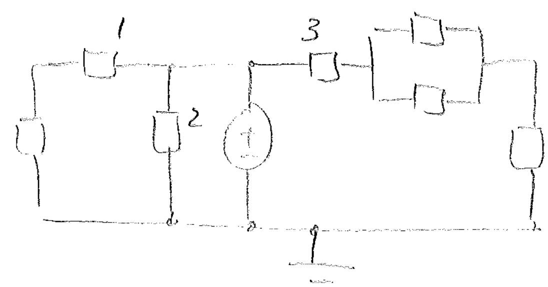


Linearity & superposition

- We analyze linear circuits.

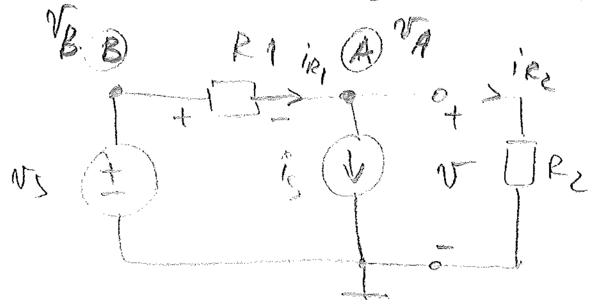
Def - A circuit is **linear** if it satisfies the following properties:

(1) - the **scaling** property. The branch currents and node voltages resulting from a single source are linearly proportional to the source. This indicates that multiplying the source by a constant, multiplies all currents and voltages by that constant.

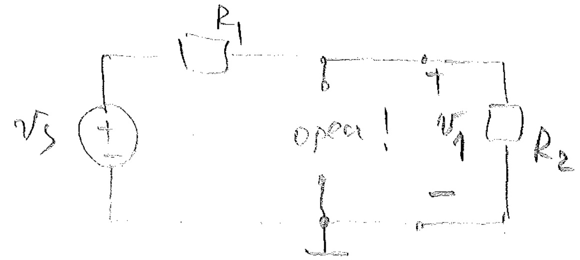
(2) - the **additive** property. In a circuit with multiple sources, each branch current and node voltage is the algebraic sum of the contributions from each source acting alone.

- A linear element: $v(t) = R\, i(t)$

Det2 - A linear circuit: composed of independent sources, linear dependent sources, and linear elements.

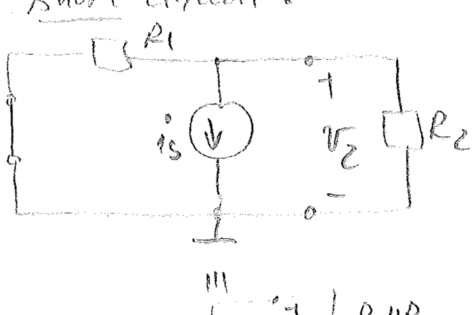


(1) Contribution from v_S with $i_S = 0$. $\boxed{i_S = 0}$ is equivalent to an open circuit.



$$v_1 = \frac{R_2}{R_1 + R_2} v_S$$ - contribution of v_S to v.

(2) Contribution from i_S with $v_S = 0$. $\boxed{v_S = 0}$ represents the i-v characteristic of a short circuit:



$$v_2 = -R_1 \| R_2 \cdot i_S = -\frac{R_1 R_2}{R_1 + R_2} i_S$$

|||

Now, we claim that $v = v_1 + v_2$

$$\boxed{v = \frac{R_2}{R_1+R_2} \cdot v_S - \frac{R_1 R_2}{R_1+R_2} i_S} \quad (1)$$

Verification of claim by <u>nodal analysis</u>:

Node B: $v_B = v_S$

Node A: $\underbrace{\frac{v_B - v_A}{R_1}}_{= i_{R_1}} = \underbrace{\frac{v_A - 0}{R_2}}_{= i_{R_2}} + i_S \quad \Rightarrow \quad \frac{v_S - v_A}{R_1} = \frac{v_A}{R_2} + i_S$

Also: $\boxed{v_A = v} \Rightarrow ?$

$$\Rightarrow R_2(v_S - v_A) = R_1 v_A + R_1 R_2 i_S$$

$$R_2 v_S - R_1 R_2 i_S = R_1 v_A + R_2 v_A$$

$$R_2 v_S - R_1 R_2 i_S = (R_1 + R_2) v_A$$

$$\Rightarrow \boxed{v =} \boxed{v_A = \frac{R_2}{R_1+R_2} v_S - \frac{R_1 R_2}{R_1+R_2} i_S} \quad (2)$$

The same!

The superposition principle's essence:

Calculate the contributions from the individual independent sources alone and then superimpose these contributions algebraically!

- Useful when only a <u>specific</u> voltage or current is sought, since it may relieve us from solving systems of equations!
- Also useful to compare the relative contributions from various sources!