Mesh/loop analysis

Note: # textbooks use different terminology!

Applicable to planar circuits (can be drawn on a plane with no crossing branches), involves the steps:

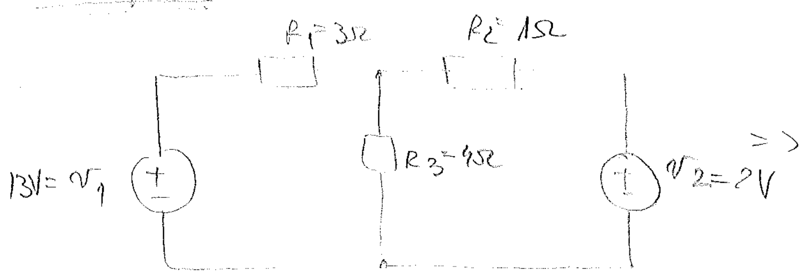
(1) Select a set of meshes such that at least one mesh passes thru each branch. Label each mesh with the corresponding mesh current and assign each current an arbitrarily chosen direction, say clockwise.

(2) Apply KVL around each labeled mesh, but with each branch voltage expressed in terms of the corresponding $\boxed{\text{mesh currents}}$ via Ohm's law. Make sure that you relate current directions and voltage polarities according to the "passive sign convention."

(3) Solve the resulting set of simultaneous equations for the unknown mesh currents.

Def: mesh $\triangleq$ a loop that does not contain other loops (→ a matter of selection.) within it.

Example 1:



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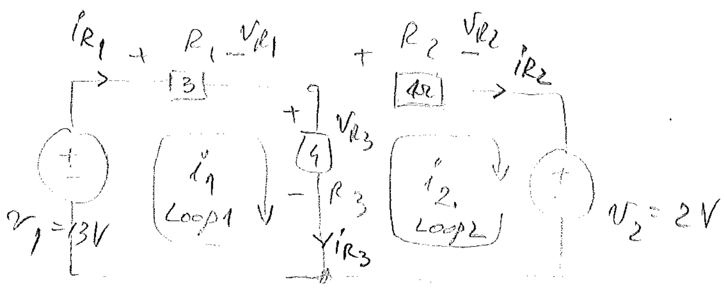


find $i_{R_1}, i_{R_2}, i_{R_3} = ?$ //

$i_1, i_2 \equiv$ mesh current; may be different from branch currents!

"Picture"/"visualize" mesh currents as circulating around the meshes! (around their perimeter)

$$\begin{cases} i_{R_1} = i_1 \\ i_{R_2} = i_2 \\ i_{R_3} = i_1 - i_2 \end{cases}$$

← mesh/currents.

Branch currents

SKIP THIS

KVL Loop 1: $\begin{cases} v_1 = v_{R_1} + v_{R_3} \\ v_{R_3} + v_{R_2} = v_2 \end{cases}$ " Loop 2:

$$\Rightarrow \begin{cases} v_1 = R_1 i_1 + R_3(i_1 - i_2) \\ R_3(i_1 - i_2) = R_2 i_2 + v_2 \end{cases} \Rightarrow \begin{cases} (R_1 + R_3) i_1 - R_3 i_2 = v_1 \\ R_3 i_1 - (R_2 + R_3) i_2 = v_2 \end{cases}$$

$$\Rightarrow \begin{cases} 7 i_1 - 4 i_2 = 13 \\ 4 i_1 - 5 i_2 = 2 \end{cases}$$

→ Solving by Gaussian elimination or Cramer's rule.

(a) Gaussian:

$$\left[\begin{array}{cc|c} 7 & -4 & 13 \\ 4 & -5 & 2 \end{array}\right] \otimes -\frac{4}{7} \oplus$$

$$\left[\begin{array}{cc|c} 7 & -4 & 13 \\ 0 & \underbrace{\frac{16}{7}-5}_{=\frac{16-35}{7}=-\frac{19}{7}} & \underbrace{2-\frac{52}{7}}_{\frac{14-52}{7}=-\frac{38}{7}} \end{array}\right] \Leftrightarrow \left[\begin{array}{cc|c} 7 & -4 & 13 \\ 0 & -\frac{19}{7} & -\frac{38}{7} \end{array}\right] \begin{array}{l} \Rightarrow 7i_1 - 4i_2 = 13 \\ \Rightarrow -\frac{19}{7} i_2 = -\frac{38}{7} \end{array}$$

$$\boxed{i_2 = 2\ [A]}$$

$$\Rightarrow 7i_1 - 8 = 13 \Rightarrow \boxed{i_1 = 3}\ [A]$$

(b) Cramer's rule. (do it at home)

$$A = \begin{bmatrix} 7 & -4 \\ 4 & -5 \end{bmatrix} \quad X = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \quad b = \begin{bmatrix} 13 \\ 2 \end{bmatrix} \quad \boxed{A \cdot X = b}$$

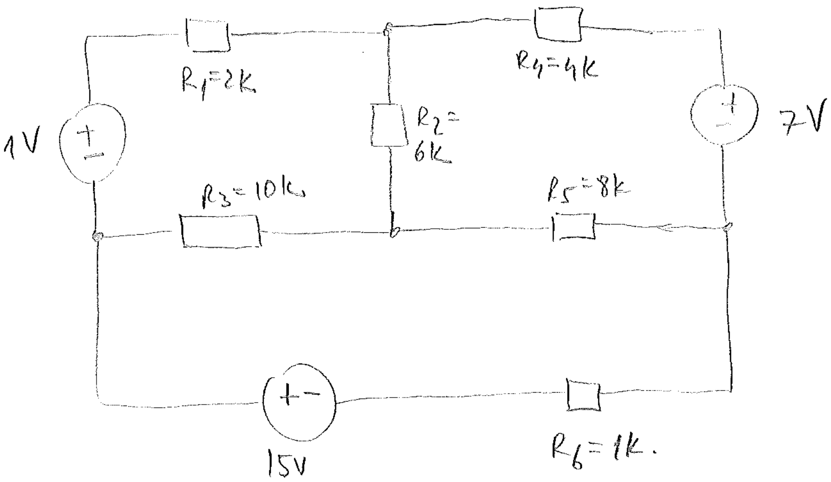
$$i_1 = \frac{\begin{vmatrix} 13 & -4 \\ 2 & -5 \end{vmatrix}}{\underbrace{\begin{vmatrix} 7 & -4 \\ 4 & -5 \end{vmatrix}}_{\triangleq \Delta}} = \frac{13\times(-5) - 2\times(-4)}{7\times(-5) - 4\times(-4)} = \frac{-65+8}{-35+16} = \frac{-57}{-19} = 3\ A$$

$$\boxed{i_1 = 3\ [A]}$$

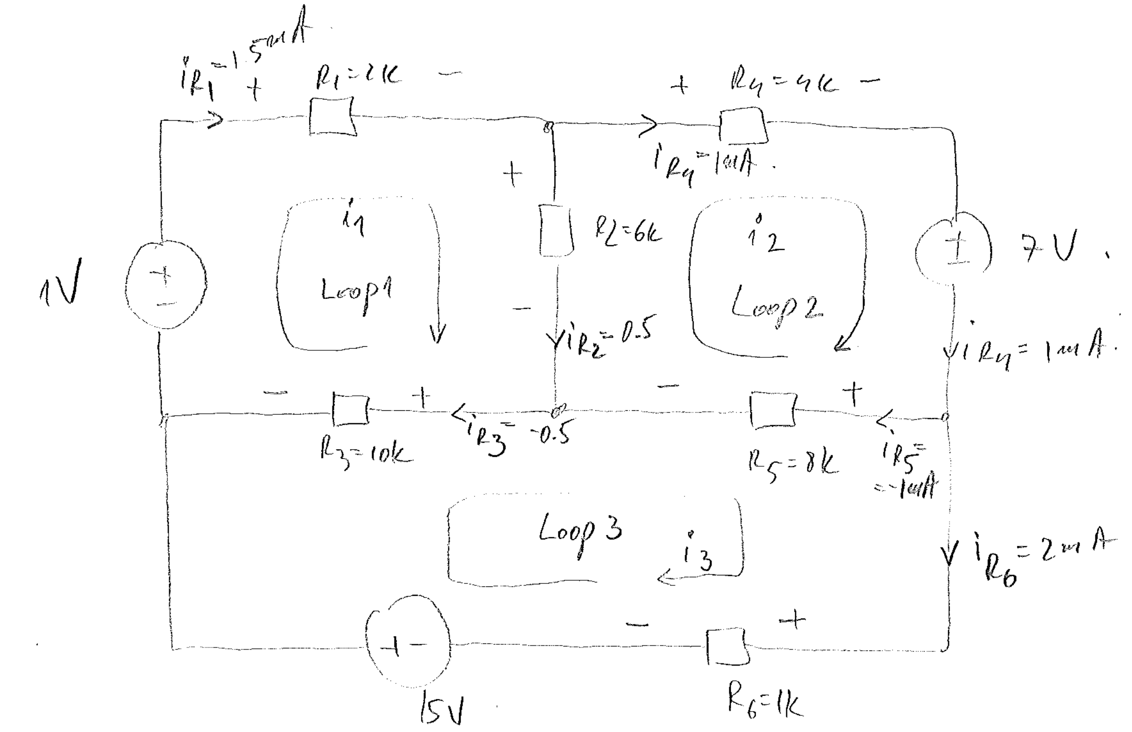
$$i_2 = \frac{\begin{vmatrix} 7 & 13 \\ 4 & 2 \end{vmatrix}}{\Delta} = \frac{7\times 2 - 4\times 13}{-19} = \frac{14-52}{-19} = \frac{-38}{-19} = 2\ A$$

$$\boxed{i_2 = 2\ [A]}$$

Example 2: Apply loop/mesh analysis to the given circuit:



Find out $i_{R_1}, i_{R_2}, i_{R_3}, i_{R_4}, i_{R_5}, i_{R_6}$.
by finding out the mesh currents!



KVL Loop 1: $1V = R_1 i_1 + R_2(i_1 - i_2) + R_3(i_1 - i_3)$

KVL Loop 2: $R_2(i_1 - i_2) = R_4 \cdot i_2 + 7V + R_5(i_2 - i_3)$ $\Rightarrow$

KVL Loop 3: $R_3(i_1 - i_3) + R_5(i_2 - i_3) + 15V = R_6 i_3$

$$\Rightarrow \begin{cases} 2i_1 + 6(i_1 - i_2) + 10(i_1 - i_3) = 1 \\ 2(i_1 - i_2) - 4i_2 - 8(i_2 - i_3) = 7 \\ 10(i_1 - i_3) + 8(i_2 - i_3) - 1 \cdot i_3 = -15 \end{cases}$$

$$\Rightarrow \begin{cases} 18 i_1 - 6 i_2 + 10 i_3 = 1 \\ 2 i_1 - 14 i_2 + 8 i_3 = 7 \\ 10 i_1 + 8 i_2 - 19 i_3 = -15 \end{cases} \Longrightarrow \boxed{A \cdot X = b}$$

matrix form.

$$A = \begin{bmatrix} 18 & -6 & 10 \\ 2 & -14 & 8 \\ 10 & 8 & -19 \end{bmatrix}, \quad X = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 7 \\ -15 \end{bmatrix}$$

(do it at home!)
Use Gaussian elimination or Cramer's rule to arrive to: $\boxed{i_1 = 1.5\ mA}$ $\boxed{i_2 = 1\ mA}$ $\boxed{i_3 = 2\ mA}$

Therefore: $\begin{cases} i_{R_1} = i_1 = 1.5mA;\ i_{R_2} = i_1 - i_2 = 0.5mA;\ i_{R_3} = i_1 - i_3 = -0.5mA \\ i_{R_4} = i_2 = 1mA;\ i_{R_5} = i_2 - i_3 = -1mA;\ i_{R_6} = i_3 = 2mA. \end{cases}$