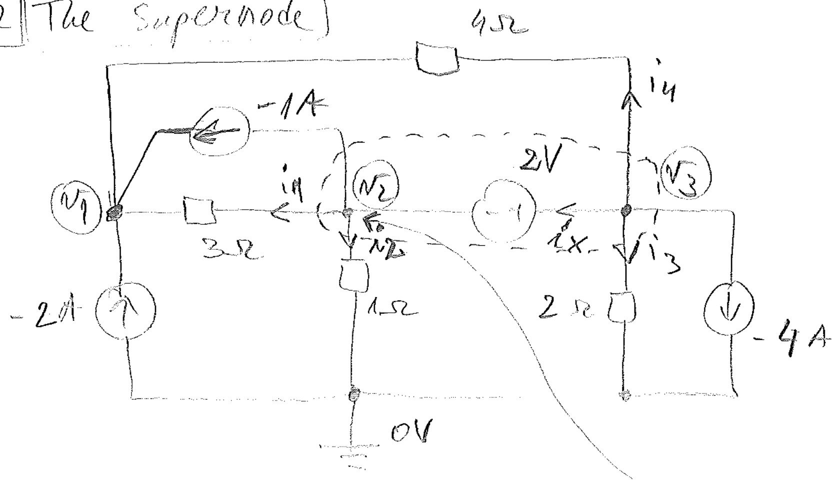
4.2 The supernode



$v_1, v_2, v_3 = ?$

NOTE: one way (but only now when we have one voltage source), is to select ground this node:

(a) The hard way:

Assign a current i_x thru 22V source.

$$\text{KCL}@v_1: \begin{cases} \dfrac{v_3-v_1}{4} + \dfrac{v_2-v_1}{3} + (-1) + (-2) = 0 \\ \dfrac{v_2-v_1}{3} + \dfrac{v_2}{1} = -1 - \boxed{i_x} \\ \dfrac{v_3-v_1}{4} + \dfrac{v_3}{2} + \boxed{i_x} + (-4) = 0 \end{cases}$$

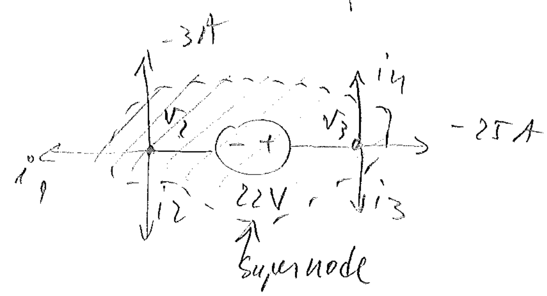
: nodal analysis eq's.

4 unknowns, 3 eq's

4 unknowns 4 equations!

Add also: $v_3 - v_2 = 2$

(b) using the supernode technique



$$\begin{cases} (v_1): & {}^{3}\dfrac{v_3-v_1}{4} + {}^{4}\dfrac{v_2-v_1}{3} = {}^{12}1+2 \\ (v_2+v_3): & {}^{4}\dfrac{v_2-v_1}{3} + {}^{12}\dfrac{v_2}{1} + {}^{12}(-1) + {}^{6}\dfrac{v_3}{2} + {}^{3}\dfrac{v_3-v_1}{4} + {}^{12}(-4) = 0 \\ \text{also}: & v_3 - v_2 = 2 \end{cases}$$

3 equations
3 unknowns.

(a) Using Cramer's rule:

$$v_1 = \frac{\begin{vmatrix} 36 & 4 & 3 \\ 60 & 16 & 9 \\ 2 & -1 & 1 \end{vmatrix}}{\underbrace{\begin{vmatrix} -7 & 4 & 3 \\ -7 & 16 & 9 \\ 0 & -1 & 1 \end{vmatrix}}_{=\Delta}} = \frac{36\times16\times1 + 2\times4\times9 + 3\times60\times(-1) - 2\times3\times16 - 60\times4\times1 - 9\times36\times(-1)}{-7\times16\times1 + 0\times4\times1 + 3\times(-1)\times(-7) - 0\times16\times3 - 4\times1\times(-7) - (-7)(4)(1)} \, [V]$$

$$v_2 = \frac{\begin{vmatrix} -7 & 36 & 3 \\ -7 & 60 & 9 \\ 0 & 2 & 1 \end{vmatrix}}{\Delta} = \ldots$$

$$v_3 = \frac{\begin{vmatrix} -7 & 4 & 36 \\ -7 & 16 & 60 \\ 0 & -1 & 2 \end{vmatrix}}{\Delta} = \ldots$$

(b) Using Gaussian elimination

$$\left[\begin{array}{ccc|c} -7 & 4 & 3 & 36 \\ -7 & 16 & 9 & 60 \\ 0 & -1 & 1 & 2 \end{array}\right] \otimes(-1) \oplus$$

$$\left[\begin{array}{cccc} -7 & 4 & 3 & 36 \\ 0 & 12 & 6 & 24 \\ 0 & -1 & 1 & 2 \end{array}\right] \otimes \frac{1}{12} \oplus$$

$$\left[\begin{array}{cccc} -7 & 4 & 3 & 36 \\ 0 & 12 & 6 & 24 \\ 0 & 0 & \frac{3}{2} & 4 \end{array}\right]$$

$$\Rightarrow \boxed{v_3 = \frac{4}{\frac{3}{2}} = \frac{8}{3} \, V}$$

$$12v_2 + 16 = 24 \Rightarrow v_2 = \frac{8}{12} \, V = \boxed{\frac{2}{3} \, V = v_2}$$

$$-7v_1 + 4\times\frac{2}{3} + \cancel{3}\times\frac{8}{\cancel{3}} = 36 \Rightarrow \boxed{-7v_1 = 28 - \frac{8}{3}}$$

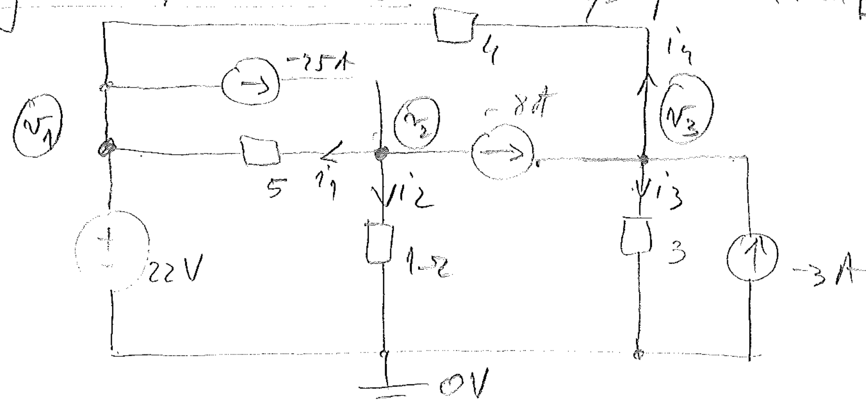
$$\begin{cases} 3v_3 - 3v_1 + 4v_2 - 4v_1 = 36 \\ 4v_2 - 4v_1 + 12v_2 + 6v_3 + 3v_3 - 3v_1 = 60 \\ -v_2 + v_3 = 2 \end{cases}$$

$$\Rightarrow \begin{cases} -7v_1 + 4v_2 + 3v_3 = 36 \\ -7v_1 + 16v_2 + 9v_3 = 60 \\ -v_2 + v_3 = 2 \end{cases} \Leftrightarrow \boxed{Ax = b}$$

$$A = \begin{bmatrix} -7 & 4 & 3 \\ -7 & 16 & 9 \\ 0 & -1 & 1 \end{bmatrix} \quad X = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \quad b = \begin{bmatrix} 36 \\ 60 \\ 2 \end{bmatrix}$$

← solved on verso of 1

(c) Re-selecting the reference node – not always possible and $\boxed{\text{only}}$ if we are allowed to change reference!



Note: you could have just selected reference, w/o redrawing!

Node v_2: Node v_3:

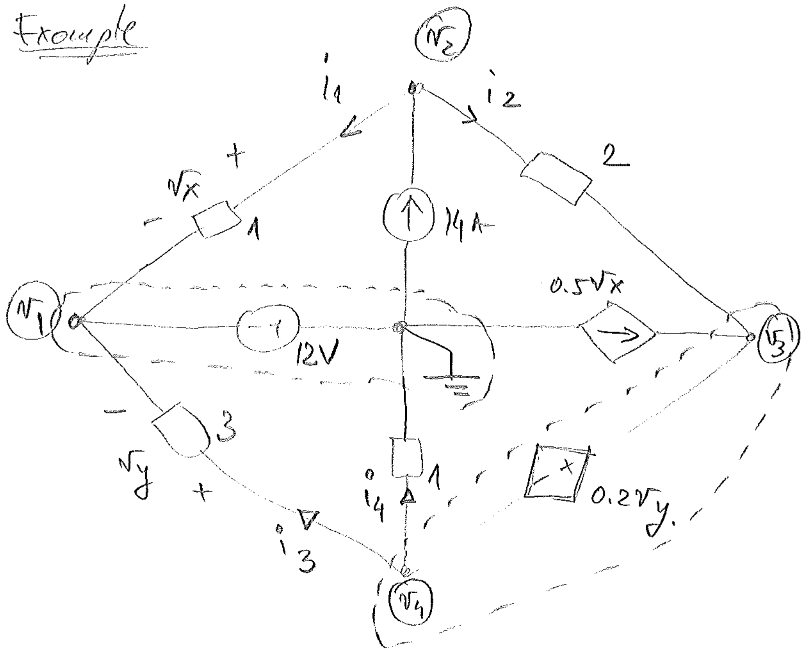
$$\begin{cases} \frac{v_2 - 22}{5} + \frac{v_2}{1} - 8 = -25 \\ \frac{v_3 - 22}{4} + \frac{v_3}{3} = -8 - 3 \end{cases} \Leftrightarrow \begin{cases} v_2 - 22 + 5v_2 = 40 - 125 \\ 3v_3 - 66 + 4v_3 = -11 \end{cases}$$

$$\begin{cases} 6v_2 = 66 - 125 = -59 \\ 7v_3 = 55 \end{cases}$$

$$\boxed{v_2 = -\frac{59}{6}} \, [V] \qquad \boxed{v_3 = \frac{55}{7}} \, [V]$$

Note: v_2, v_3 have values for the reference chosen! they are not the same v_2, v_3 as in the original circuit.

Example



$v_1, v_2, v_3, v_4 = ?$

$\boxed{v_1 = -12\ V}$

Node v_2: $14A = \frac{v_2 - (-12)}{1} + \frac{v_2 - v_3}{2}$

Supernode v_3, v_4: $0.5\underbrace{v_x}_{=(v_2+12)} + \frac{v_2 - v_3}{2} = \frac{v_4}{1} + \frac{v_4 - (-12)}{3}$

Add also: $v_3 - v_4 = 0.2 \times v_y = 0.2 \times [v_4 - (-12)]$

3 equations
3 unknowns: v_2, v_3, v_4.