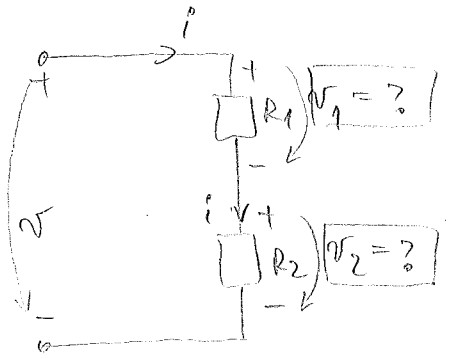
Voltage & current division

(1) Voltage division

- express voltage across a resistor in series with another resistor as a function of the voltage across both resistors.



KVL: $v = v_1 + v_2$

$v = iR_1 + iR_2 = i(R_1 + R_2)$

$$i = \frac{v}{R_1 + R_2}$$

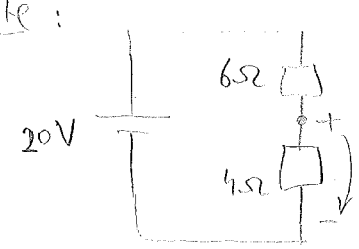
Ohm's law for R_1: $v_1 = R_1 \cdot i$

$$v_1 = \frac{R_1}{R_1 + R_2} v \quad (1)$$

Similarly:

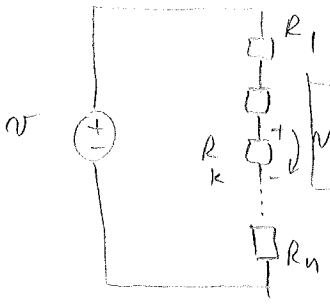
$$v_2 = \frac{R_2}{R_1 + R_2} v \quad (2)$$

Example:



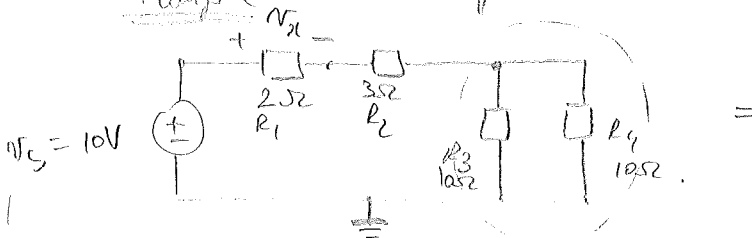
$$v_x = ? = \frac{4\Omega}{(4+6)\Omega} \cdot 20V = \frac{4}{10} \cdot 20V = 8V$$

Generalization:

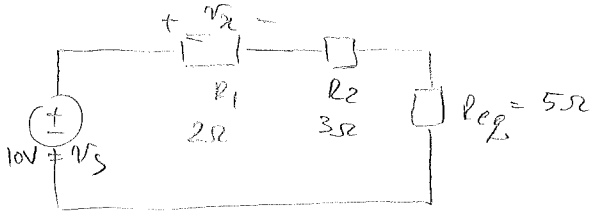


$$v_k = \frac{R_k}{R_1 + R_2 + \cdots + R_n} \cdot v \quad (3) \qquad = \frac{R_k}{\sum_{i=1}^{n} R_i} v = v_k$$

Example: use voltage division to determine $v_x = ?$



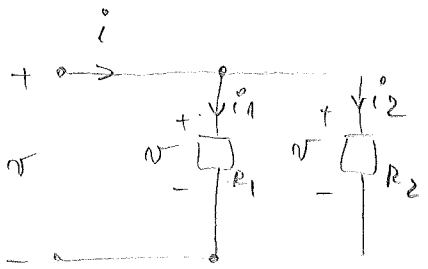
$$R_{eq} = R_3 \| R_4 = 5\Omega$$



$$v_x = \frac{R_1}{R_1 + R_2 + R_{eq}} \cdot v_s = \frac{2}{2+3+5} \cdot 10V = 2V$$

Note: Could have written $R_{eq} = R_2 + R_3 \| R_4$ and simplified to only 2 series resistors

(2) Current division



$i_1 = ? = i_1(i)$

$i_2 = ? = i_2(i)$

KCL: $i = i_1 + i_2 = \frac{v}{R_1} + \frac{v}{R_2} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right) \cdot v \Rightarrow \boxed{v = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} \cdot i}$

Rewrite by Ohm's law again: $i_1 = \frac{v}{R_1} = \frac{1}{R_1} \cdot \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} \, i = \frac{1}{R_1} \cdot \frac{R_1 R_2}{R_1 + R_2} \cdot i$

$$\boxed{i_1 = \frac{R_2}{R_1 + R_2} \cdot i} \quad (4)$$

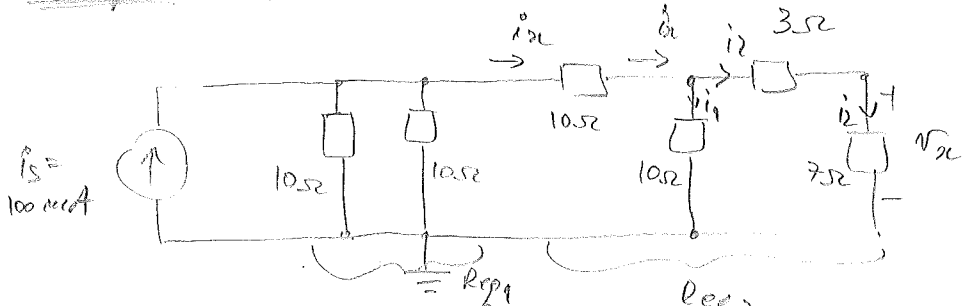
Similarly: $$\boxed{i_2 = \frac{R_1}{R_1 + R_2} \cdot i} \quad (5)$$

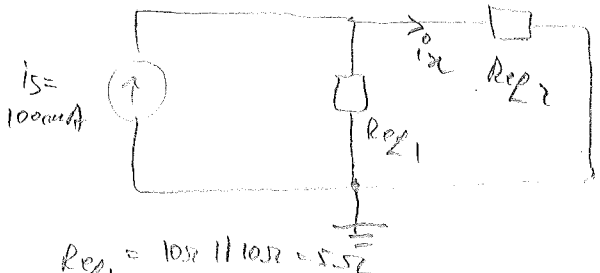
Note: ∃ differences from (1) & (2) equations!

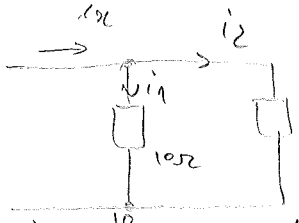
Generalization:

$$\boxed{i_k = \frac{\frac{1}{R_k}}{\frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}} \, i} \quad (6)$$

Example: Find i_x and v_x:







$R_{eq1} = 10\Omega \,||\, 10\Omega = 5\Omega$

$R_{eq2} = 10\Omega + \{10\Omega \,||\, (3\Omega + 7\Omega)\} = 15\Omega$.

(10Ω ; 5Ω)

$i_2 = \frac{10}{10+10} \cdot 25\,mA = 12.5\,mA$.

Ohm's law: $\boxed{v_x = 7\Omega \times 12.5\,mA}$

$i_x = \frac{R_{eq1}}{R_{eq1} + R_{eq2}} \cdot i = \frac{5}{5+15} \cdot 100\,mA = \boxed{25\,mA = i_x}$