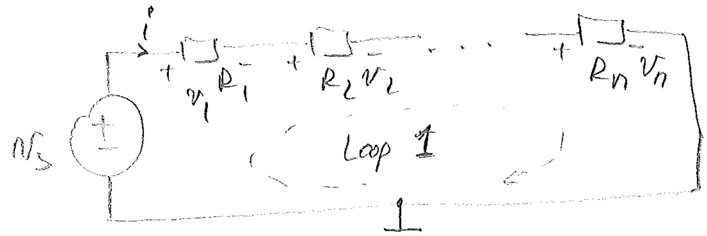
Resistor combinations

(1) Series



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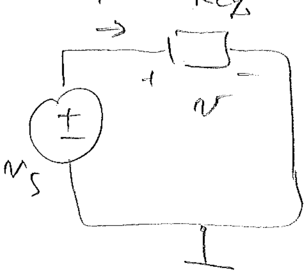


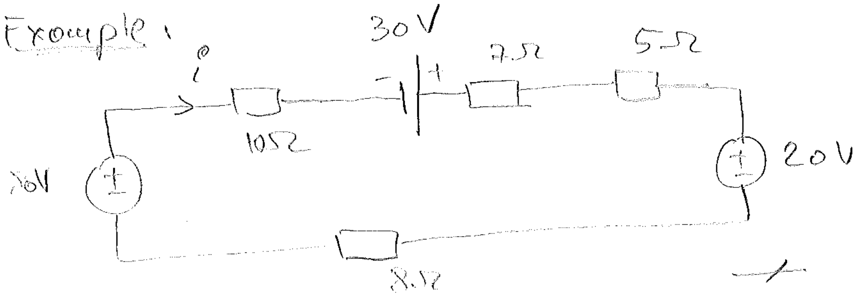
KVL for loop 1: $v_s = v_1 + v_2 + \ldots + v_n$; $v_s = v$

$v_s = i(R_1 + R_2 + \ldots + R_n)$; $v_s = i R_{eq}$.

$\Rightarrow R_{eq} = R_1 + R_2 + \ldots + R_n$

$$R_{eq} = \sum_{i=1}^{n} R_i \quad (1)$$

Example: $i = ?$



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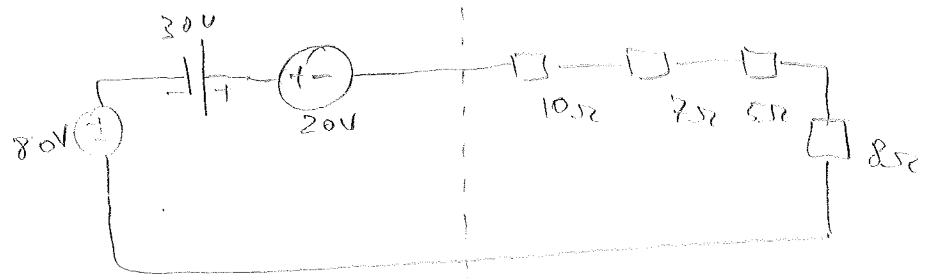
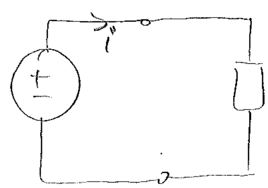


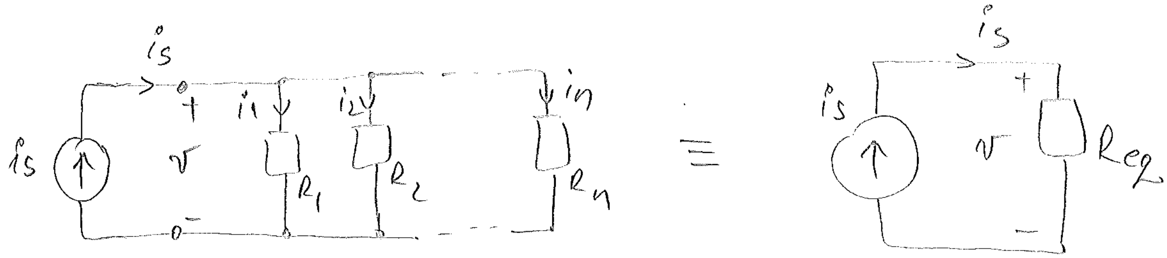
$v_{eq} =$
$80V + 30V - 20V =$
$= 90V$.



$R_{eq} = (10 + 7 + 5 + 8)\,\Omega = 30\Omega$.

$$i = \frac{v_{eq}}{R_{eq}} = \frac{90V}{30\Omega} = 3A.$$

(2) Parallel connection of resistors



KCL for top node:

$$i_S = i_1 + i_2 + \dots + i_n \qquad i_S = \frac{v}{R_{eq}}$$

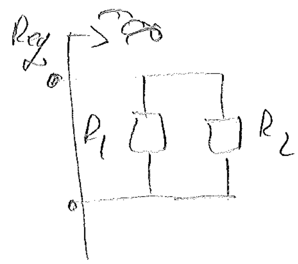
$$i_S = \frac{v}{R_1} + \frac{v}{R_2} + \dots + \frac{v}{R_n} \qquad i_S = \frac{v}{R_{eq}}$$

$$\Rightarrow \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n} = \frac{1}{R_{eq}}$$

$$\Rightarrow \boxed{R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}} = \frac{1}{\sum_{i=1}^{n} \frac{1}{R_n}}} \quad (2)$$

In terms of conductances: $\boxed{G_{eq} = G_1 + G_2 + \dots + G_n.}$ (3)

Example:



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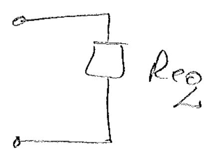
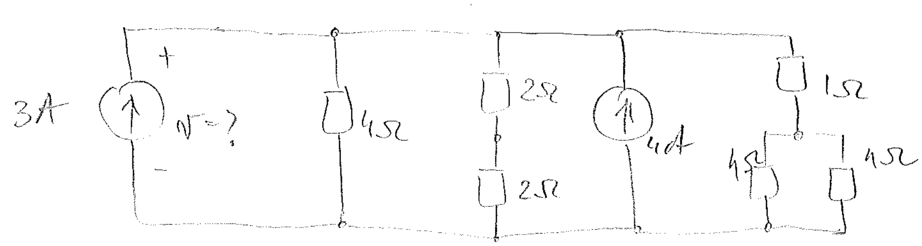
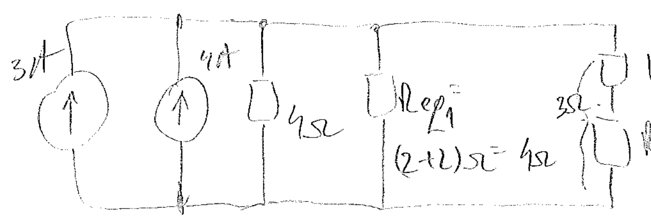


$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \boxed{\frac{R_1 R_2}{R_1 + R_2} = R_1 \| R_2} \quad (4)$$

good to memorize!

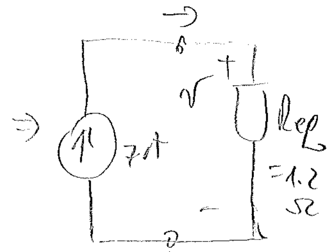
Example:





$$R_{eq_2} = \frac{4 \cdot 4}{4+4}\,\Omega = 2\,\Omega$$

$\Rightarrow$



$i = 7A$, $R_{eq} = 1.2\,\Omega$

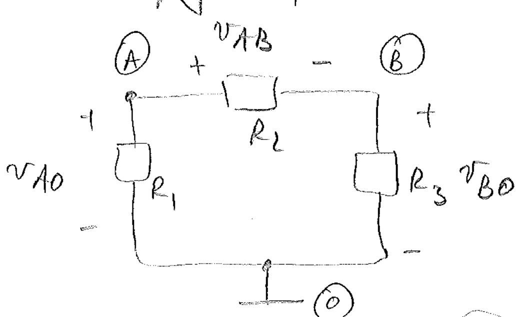
$R_{eq} = 4\Omega \| 4\Omega \| 3\Omega$

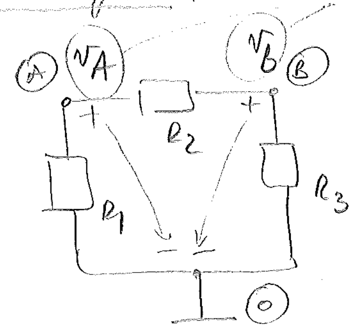
$$R_{eq} = \frac{1}{\frac{1}{4}+\frac{1}{4}+\frac{1}{3}}\,\Omega = \frac{12}{10}\,\Omega = 1.2\,\Omega.$$

$$v = R_{eq}\cdot i = 1.2\,\Omega \times 7A = \boxed{8.4\,V = v}$$

Reference Node

- because only potential differences (or voltages) have practical meaning, it is convenient to reference all node potentials in a circuit to the potential of a common node called the **reference node**.
- This node is identified by the symbol ⏚ ⊥ and its potential is zero by definition!
- Therefore, when referenced to the reference node, node potentials are simply referred as node voltages.





node potentials ≡ node voltages wrt reference node.

$$v_{AB} = v_A - v_B$$
$$v_{AO} = v_A - \underbrace{v_O}_{=0} = v_A$$
$$v_{BO} = v_B - \underbrace{v_O}_{=0} = v_B$$

branch voltage drops (left column) — node potentials = node voltages.

Note: while v_{AO}, v_{BO} happen to be node voltages, v_{AB} is not! v_{AB} is a branch voltage!

- Reference node is not necessarily the bottom-most node in a circuit. It is rather the node with the largest number of connections!

Show node voltages for the two different cases of node reference!