(2) Average power

Obtained for a time interval t_1 to t_2 by integrating the instantaneous power $p(t)$ and dividing by the length of the interval:

$$\boxed{P \triangleq \frac{1}{t_2-t_1}\int_{t_1}^{t_2} p(t)\,dt} \quad (1)$$

- In the particular case that $p(t)$ is periodic with period T, the average power can be computed as:

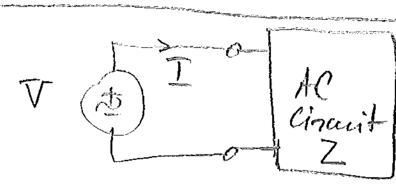
$$\boxed{P = \frac{1}{T}\int_{t_x}^{t_x+T} p(t)\,dt} \quad (2)$$

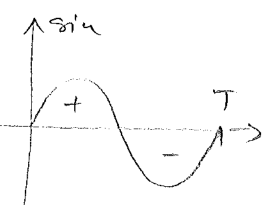
t_x arbitrary.

Sinusoidal steady-state (forcing function is a sinusoidal) power (ac power)

$v(t) = V_m \cos(\omega t + \theta)$

$i(t) = I_m \cos(\omega t + \phi)$





$p(t) = v(t)\,i(t) = V_m I_m \cos(\omega t + \theta)\cdot\cos(\omega t + \phi)$

$$p(t) = \frac{1}{2} V_m I_m \cos(\theta - \phi) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta + \phi)$$

Average ac power: $P = \frac{1}{T}\int_{t_x}^{t_x+T} \frac{1}{2} V_m I_m \cos(\theta-\phi)\,dt + 0 = \frac{1}{T}(t_x + T - t_x)\cdot\frac{1}{2} V_m I_m \cos(\theta - \phi) =$

$$\boxed{P = \frac{1}{2} V_m I_m \cos(\theta - \phi)} \quad (3) \quad [W]$$

Average power absorbed by a resistor:

$$\boxed{P = \frac{1}{2} V_m I_m \cos(0^\circ) = \frac{1}{2} V_m I_m}$$

OBS: R represents the ability of an impedance to dissipate power! in response to ac current!

Average power absorbed by a C, L (ideal elements):

$$\boxed{P = \frac{1}{2} V_m I_m \cos(\pm 90^\circ) = 0}$$

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Generally, we identify 3 cases

instantaneous power:

$$p(t) = \frac{1}{2} V_m I_m \cos(\theta - \phi) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta + \phi)$$

case 1 $\theta - \phi = 0$; purely resistive load.

$$p(t) = \frac{1}{2} V_m I_m [1 + \cos 2\omega t]$$

whose maximum avg. value is denoted S and is called the apparent power

$$\boxed{S = \frac{1}{2} V_m I_m} = \underbrace{V_{rms} \cdot I_{rms}}_{\text{studied later.}} \quad [VA]$$

case 2 $\phi = \pm 90°$; purely reactive load.

$$p(t) = \frac{1}{2} V_m I_m \cdot \cos 2(\omega t \pm 90°)$$

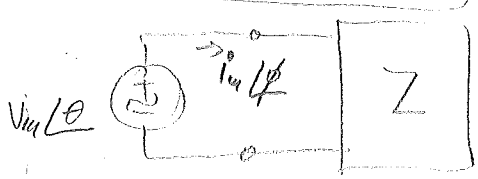
whose average is zero. (because the energy absorbed during a positive alternation is returned to the source during the subsequent negative alternation)

case 3 $0 < \cos(\theta - \phi) < 1$

$$\boxed{0 < P < S}$$

$\cos(\theta - \phi)$ is called the power factor (p.f.) and is frequency dependent because so is $\theta - \phi$! Hence the average ac power is frequency dependent!

Power and impedance



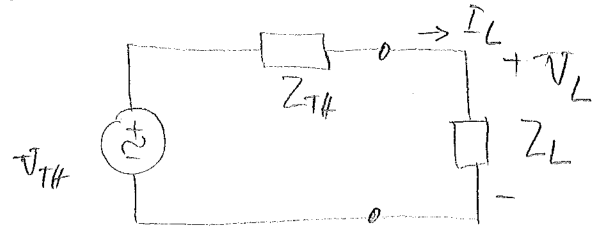
$$P = \frac{1}{2} V_m I_m \cos(\measuredangle Z) =$$

$$= V_{rms} \cdot I_{rms} \cdot \cos(\measuredangle Z)$$

$$= |Z| \cdot I_{rms}^2 \cdot \cos(\measuredangle Z) = R(\omega) \cdot I_{rms}^2$$

$$\boxed{P = R(\omega) \cdot I_{rms}^2}$$

(3) Maxim power transfer



Max. Power transfer takes place when

$$\underbrace{Z_{TH}^*}_{R_{TH}-jX_{TH}} = \underbrace{Z_L}_{R_L+jX_L} \Leftrightarrow \boxed{\begin{matrix} R_{TH} = R_L \\ -X_{TH} = -X_L \end{matrix}}$$

- Power transfer from an active ac port to a load is maximized when the load impedance is made equal to the complex conjugate of the equivalent impedance of the port!

- The load power is: $P_L = R_L \cdot i_{rms}^2 = \frac{1}{2} R_L i_m^2$

where: $i_m = \left| \frac{V_{TH}}{Z_{TH}+Z_L} \right|$

$$P_L = \frac{1}{2} R_L \cdot \frac{|V_{TH}|^2}{(R_{TH}+R_L)^2 + \underbrace{(X_{TH}+X_L)^2}}$$

any non zero value would reduce P_L. So, P_L is maximized when $-X_{TH} = X_L$!

Then, we're left with:

$$P_L = \frac{1}{2} R_L \frac{|V_{TH}|^2}{(R_{TH}+R_L)^2}$$

which we already solved when we looked at circuits with resistances!

$R_L = R_{TH}$

Finally the maximum average power:

$$\boxed{P_{L(max)} = \frac{1}{8} \frac{|V_{TH}|^2}{R_{TH}}}$$