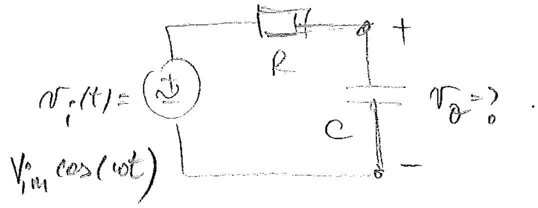
Seminar

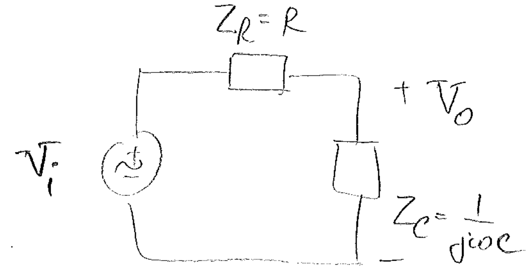
More examples of frequency-domain analysis (or phasor analysis)

AC divider



time-domain

get/draw $\longrightarrow$ frequency-domain representation of the circuit.



freq-domain

This is a voltage divider: $\boxed{V_o = \frac{Z_C}{Z_C + Z_R} V_i = \frac{1/j\omega C}{R + 1/j\omega C} \cdot V_i = \frac{V_i}{1 + j\omega RC}}$

$$V_o = |V_o| \cdot \angle\phi$$

where:
$$\begin{cases} |V_o| = \dfrac{|V_i|}{|1 + j\omega RC|} = \dfrac{V_{im}}{\sqrt{1 + (\omega RC)^2}} \\ \angle\phi = \angle 0^\circ - \tan^{-1}\dfrac{\omega RC}{1} = \angle -\tan^{-1}(\omega RC) \end{cases}$$

$$\boxed{V_o = \frac{V_{im}}{\sqrt{1 + (\omega RC)^2}} \angle -\tan^{-1}(\omega RC)}$$

OBS. 1: Note that this is a low-pass filter!

- Low frequencies: $\omega \to 0$, $|Z_C| \to \infty$
- High frequencies: $\omega \to \infty$, $|V_o| \to 0$
- Borderline frequency is the special frequency ω_0 that makes

$$|Z_C(\omega_0)| = |Z_R|$$

$$\frac{1}{\omega_0 C} = R \Rightarrow \boxed{\omega_0 = \frac{1}{RC}}$$

which is the $\boxed{\text{cutoff}}$ freq. of the filter!

OBS. 2

In the high-frequency limit, the RC circuit approximates an $\boxed{\text{integrator}}$!

$$V_o\big|_{\omega \to \infty} \to \frac{V_i}{j\omega RC} = \left(\frac{1}{j\omega}\right) \cdot \frac{V_i}{RC}$$

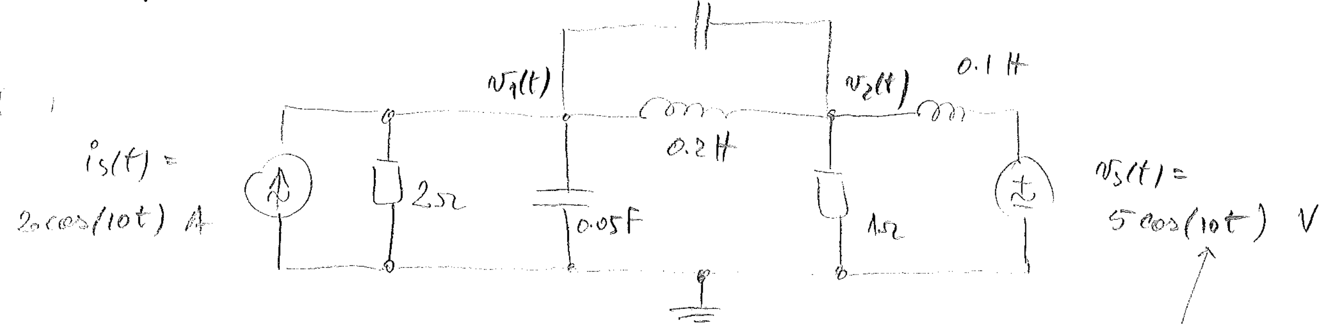
Tools: $x(t) = X_m\cos(\omega t + \theta) \longrightarrow X_m \angle\theta$

$$x_{der}(t) = \frac{d}{dt}\left(x(t)\right) = \omega X_m \cdot \sin(\omega t + \theta) = \omega X_m \cos(\omega t + \theta + 90^\circ) \to \omega X_m \angle \theta + 90^\circ$$

same as: $\to j\omega X_m \angle\theta$

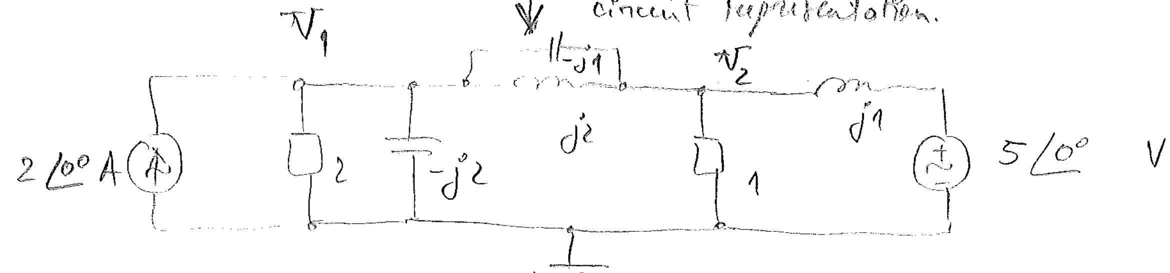
So, derivation is multiplication by $\boxed{j\omega}$ and integration is multiplication by $\boxed{\frac{1}{j\omega}}$!

Example: Find v_1, v_2.



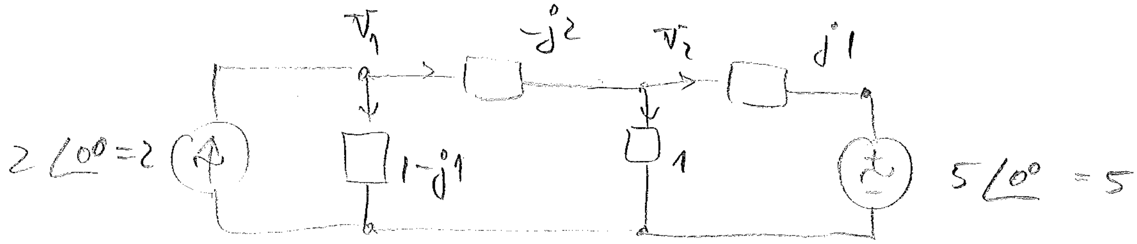
$\omega = 10$ rad/sec.

get the freq.-domain circuit representation.



simplify by grouping impedances.

$$\frac{2\times(-j2)}{2-j2} = \frac{-j4}{2-j2} = \frac{-j\cancel{4}(2+j2)}{\cancel{8}} = 1-1j$$



Use nodal analysis:

$$\begin{cases} \text{KCL node } V_1: & \dfrac{V_1}{1-j1} + \dfrac{V_1 - V_2}{-j2} = \overset{=2}{2\angle 0^\circ} = 2 \\[2ex] \text{KCL node } V_2: & \dfrac{V_1 - V_2}{-j2} = \dfrac{V_1}{1} + \dfrac{V_2 - \overset{=5}{5\angle 0^\circ}}{j1} \end{cases}$$

$$\Rightarrow \begin{cases} (1-j3)V_1 + (-1+j1)V_2 = -4-j4 \\ V_1 + (1+j2)V_2 = 10 \end{cases} \Leftrightarrow \boxed{Ax = b.}$$

$$A = \begin{bmatrix} 1-j3 & -1+j1 \\ 1 & 1+j2 \end{bmatrix} \quad x = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad b = \begin{bmatrix} -4-j4 \\ 10 \end{bmatrix}$$

$$\begin{cases} (1-j3)V_1 + (-1+j1)\,V_2 = -4-j4 \\ V_1 + (1+j2)\,V_2 = 10 \qquad / \times[-(1-j3)] \oplus \end{cases}$$

$$\Rightarrow (-1+j1)V_2 + \underbrace{(1+j2)(-1+j3)}_{\begin{array}{c}-1-6-j2+j3 \\ (-7+j1)\cdot V_2\end{array}} V_2 = -4-j4-10+j30$$

$$(-8+j2)\,V_2 = -14+j26$$

$$(-4+j1)\,V_2 = -7+j13 \quad \Rightarrow \quad \boxed{V_2 = \frac{7-j13}{4-j1} = 3.581\,\angle -47.66^\circ\ [V]}$$

$$\boxed{V_1 = 10-(1+j2)\,V_2 = \frac{7-j11}{4-j1} = \sqrt{10}\,\angle -43.49^\circ\ V}$$

Therefore, converting to time-domain:

$$\begin{cases} v_1(t) = \sqrt{10}\cdot\cos(10t - 43.49^\circ)\ \ V \\ v_2(t) = 3.581\cdot\cos(10t - 47.66^\circ)\ \ V \end{cases}$$

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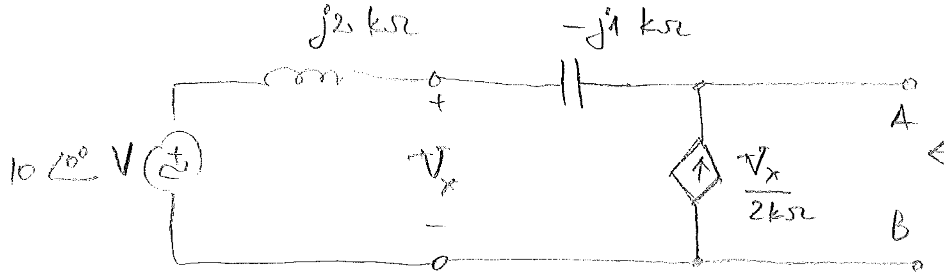
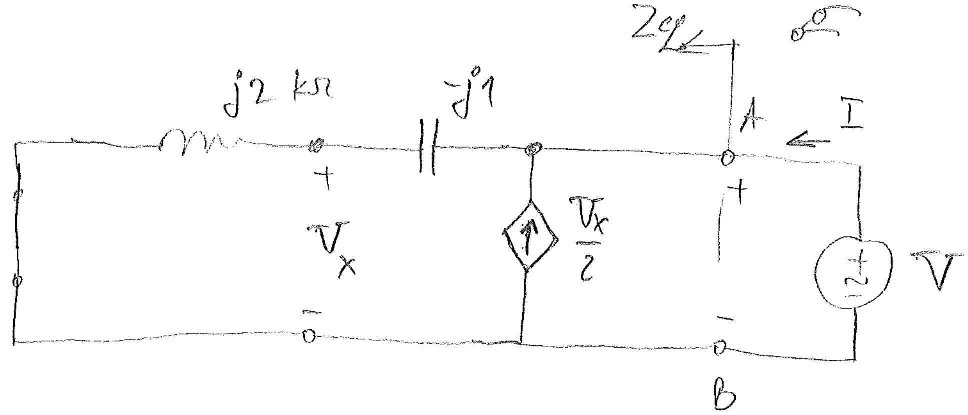
Equivalent impedance of an AC port



$Z_{eq} = ?$

Rule: Take the 2-port network, suppress all independent sources and apply V, "measure"/compute I, then: $\boxed{Z = \frac{V}{I}}$



KCL: $I + \frac{V_x}{2} = \frac{V - V_x}{-j1}$

Voltage division: $V_x = \frac{j2}{j2 - j1} V = 2 \cdot V$

$\Rightarrow$

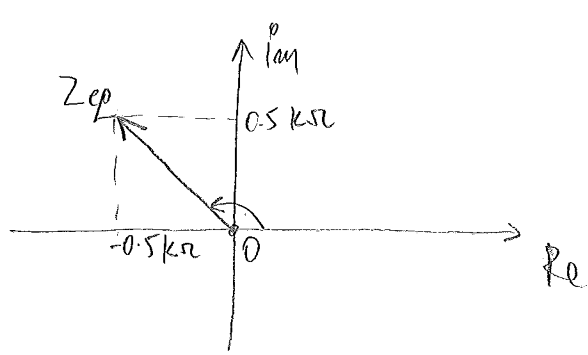
$$\Rightarrow \quad I + \frac{\cancel{2}V}{\cancel{2}} = \frac{V - 2V}{-j1}$$

$$I + V = \frac{1}{j} V$$

$$I = (-1 - j1) V \Rightarrow Z_{eq} = \frac{V}{I} = \frac{1}{-1-j1} = \frac{-1+j1}{1^2+1^2} = -0.5 + 0.5j \ [k\Omega]$$

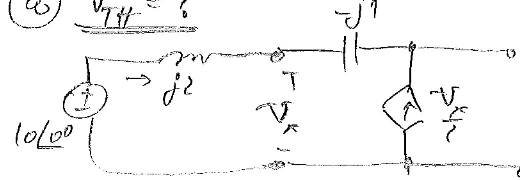
$$\boxed{Z_{eq} = -0.5 + 0.5j \ [k\Omega]}$$

phasor-diagram



Thevenin equivalent :

(a) $V_{TH} = ?$



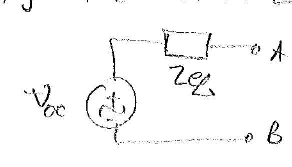
$V_{TH} = V_{oc}$.

$$\left.\begin{array}{l} V_{oc} = V_x + (-j1) \frac{V_x}{2} = (1 - j0.5) \cdot V_x \\ V_x = 10\angle 0° + (j2) \frac{V_x}{2} \Rightarrow V_x = \frac{10}{1-j1} = 5 + j5 \ V \end{array}\right\} \Rightarrow$$

$$\Rightarrow V_{oc} = (1 - j0.5)(5 + j5) = 7.906 \angle 18.43° \ [V]$$

(b) $Z_{eq} = -0.5 + j0.5 \ k\Omega = 0.707 \angle 135° \ k\Omega$

see above problem



Thevenin equivalent