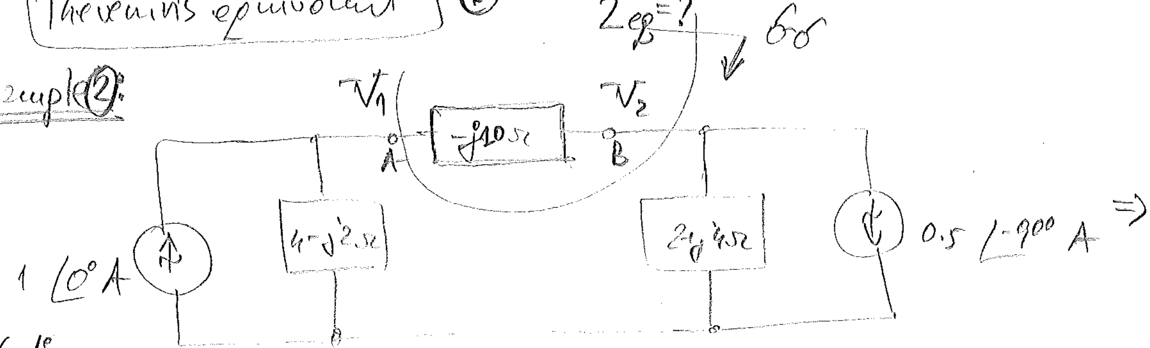
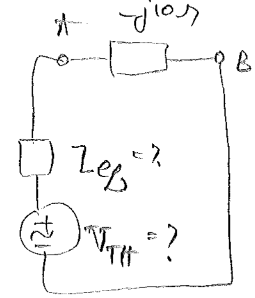
## Thévenin's equivalent ①

Example ②:




$\Rightarrow$

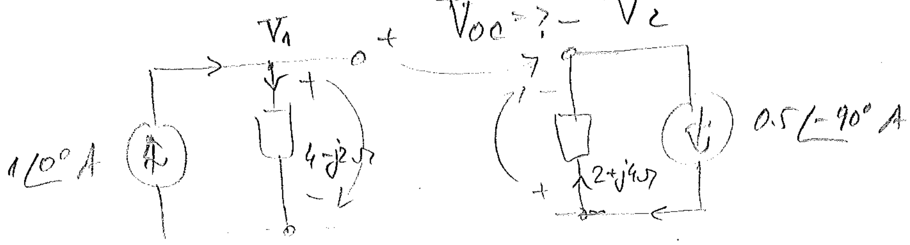



$Z_{eq}=6+j2\ [\Omega]$

$V_{TH}=6-j3\ [V]$

Solution

(i) $\boxed{V_{TH}=V_{OC}=?}$



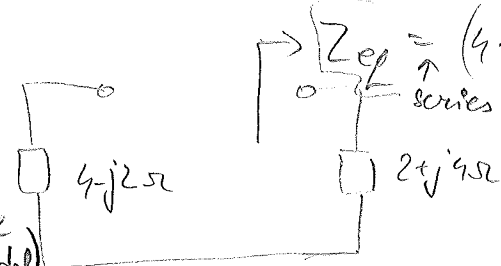

KVL:

$$\boxed{V_{OC}} = (4-j2)\cdot(1\angle 0^\circ) + (2+j4)(0.5\angle -90^\circ)$$

$$= (4-j2)\cdot(+1) + (2+j4)\cdot(-j0.5)$$

$$= 4-j2 + 2-1j = \boxed{6-j3}\ [V]$$

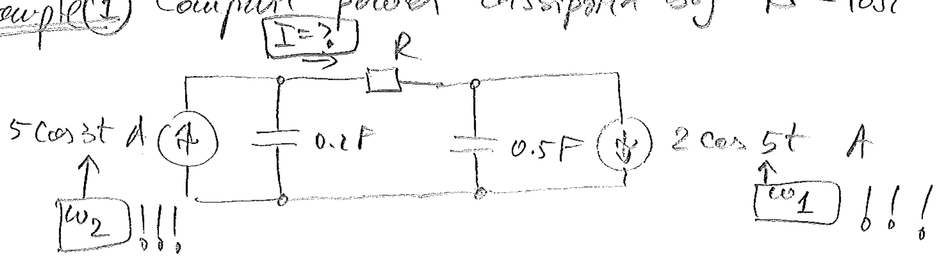
(ii) $\boxed{Z_{eq}=?}$




$\Rightarrow \boxed{Z_{eq} \underset{\text{series}}{=} (4-j2)+(2+j4) = 6+j2\ \Omega}$

## Superposition ② Done (reloaded)

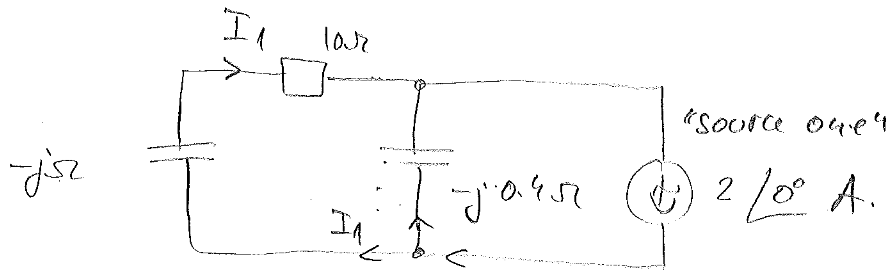
Example ① Compute power dissipated by $R=10\,\Omega$ (in steady-state!)




Note: we have 2 different frequencies!

Apply superposition to find the contribution of each of the current sources!

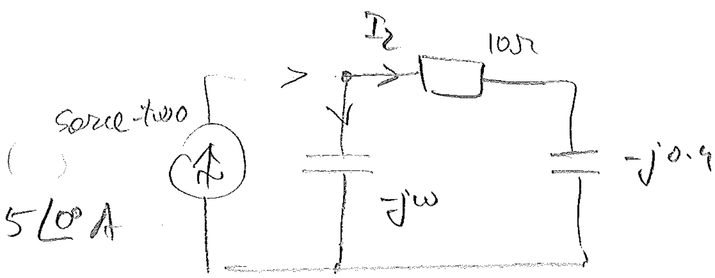
(a) Contribution due to source one: $I_1$ (phasor current!)




Use current division:

$$\boxed{I_1 = \frac{j0.4}{j0.4 + (10 - j)} \times (2\angle 0^\circ) = 79.23\angle -82.03^\circ \text{ mA}}$$

(b) Contribution due to source two: $I_2$




Also by current division:

$$\boxed{I_2 = \frac{-j\omega}{-j\omega + (10 - j0.4)} \times (5\angle 0^\circ) = 811.7\angle -76.86^\circ \text{ mA}}$$

Finally: $I = I_1 + I_2$ thru impedance of $10\Omega$.
Converting to the time domain:

$$i(t) = \underbrace{79.23\cos(5t - 82.03^\circ) + 811.7\cos(3t - 76.86^\circ)}\ [\text{mA}]$$

Power:
$$P_{10\Omega}(t) = (i(t))^2 \times 10 = 10 \times [\quad]^2\ [\mu W]$$

Give $\boxed{\text{HW \# 20}}$ the "Phasor diagrams", pp. 404–406. !

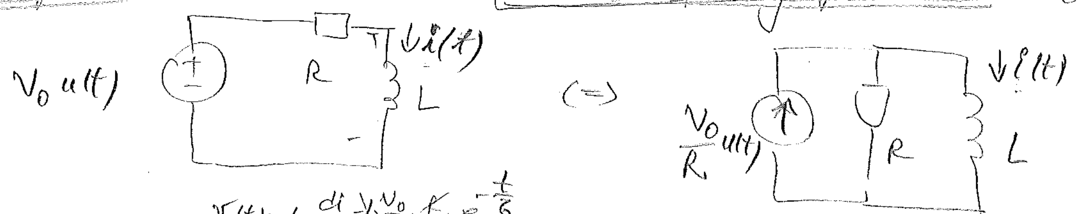
## Chapter 11 : AC Circuit power analysis

### 1) Instantaneous power

= the product of the instantaneous voltage and current:

$$\boxed{p(t) = v(t)\cdot i(t)}$$

ⓐ Example 1: RL circuit with $\boxed{\text{constant forcing-function}}$ $V_0$; $t \geq 0$.




$$v_L(t) = L\frac{di}{dt} = L\cdot\frac{V_0}{R}\cdot\frac{1}{\tau}\cdot e^{-\frac{t}{\tau}} = V_0\cdot e^{-\frac{t}{\tau}};\ t \geq 0$$

$$i(t) = \frac{V_0}{R}\left(1-e^{-\frac{t}{\tau}}\right)\cdot u(t)$$

or: $i(t) = \begin{cases} \frac{V_0}{R}\left(1-e^{-\frac{t}{\tau}}\right); & t \geq 0 \\ 0; & t < 0 \end{cases}$ where: $\boxed{\tau = \frac{L}{R}}$

Power delivered by source (equals the power absorbed by RL network):

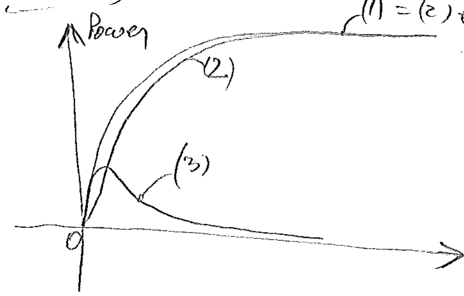
$$\boxed{p(t) = v(t)\cdot i(t) = \left(V_0\cdot u(t)\right)\times\left(\frac{V_0}{R}\left(1-e^{-\frac{t}{\tau}}\right)u(t)\right) = \frac{V_0^2}{R}\left(1-e^{-\frac{t}{\tau}}\right)\cdot u(t)} \quad (1)$$

Power delivered to R:

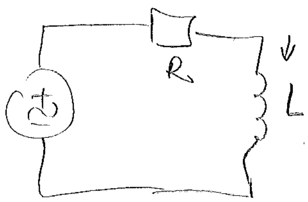
$$P_R(t) = R\,i^2(t) = \frac{V_0^2}{R}\left(1-e^{-\frac{t}{\tau}}\right)^2\cdot u(t) \quad (2)$$

Power delivered to L:

$$P_L(t) = \underbrace{\left(V_0\,e^{-\frac{t}{\tau}}\cdot u(t)\right)}_{=v_L(t)} \times \underbrace{\left(\frac{V_0}{R}\left(1-e^{-\frac{t}{\tau}}\right)u(t)\right)}_{=i(t)} = \frac{V_0^2}{R}e^{-\frac{t}{\tau}}\left(1-e^{-\frac{t}{\tau}}\right)\cdot u(t) \quad (3)$$



(b) Sinusoidal forcing-function



$v_s(t) = V_m \cdot \cos(\omega t)$

$\phi_v = 0$

$i(t) = I_m \cdot \cos(\omega t + \phi)$, $\phi = \phi_i$

$$= \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \cdot \cos\left(\omega t - \tan^{-1}\frac{\omega L}{R}\right)$$

← OBS: this is the steady-state only component. (ac analysis)

Instantaneous Power delivered to the network: $p(t) = v(t) \cdot i(t)$

$$p(t) = V_m \cdot I_m \cdot \cos(\omega t) \cdot \cos(\omega t + \phi)$$

$$= \frac{V_m I_m}{2} \cdot \left[\cos(2\omega t + \phi) + \cos\phi\right]$$

$$\boxed{p(t) = \underbrace{\frac{V_m I_m}{2} \cdot \cos\phi}_{\text{constant}} + \boxed{\underbrace{\frac{V_m I_m}{2} \cdot \cos(2\omega t + \phi)}_{\text{depends on } 2\omega \text{ and } t}}}$$

twice the applied frequency.