Definitions (more)

- Admittance = is the ratio of phasor current to phasor voltage. It is denoted as $\underline{Y}$. It is a complex quantity (similar to impedance).

$$\boxed{\underline{Y} = \frac{I}{V}} \quad \boxed{= \frac{1}{Z}} \qquad [S] \text{ siemens.}$$

$$\underline{Y} = G + jB = \frac{1}{Z} = \frac{1}{R + jX}$$

admittance, conductance, susceptance, impedance, resistance, reactance.

Example:

$$Z = 1 - j2\ \Omega$$

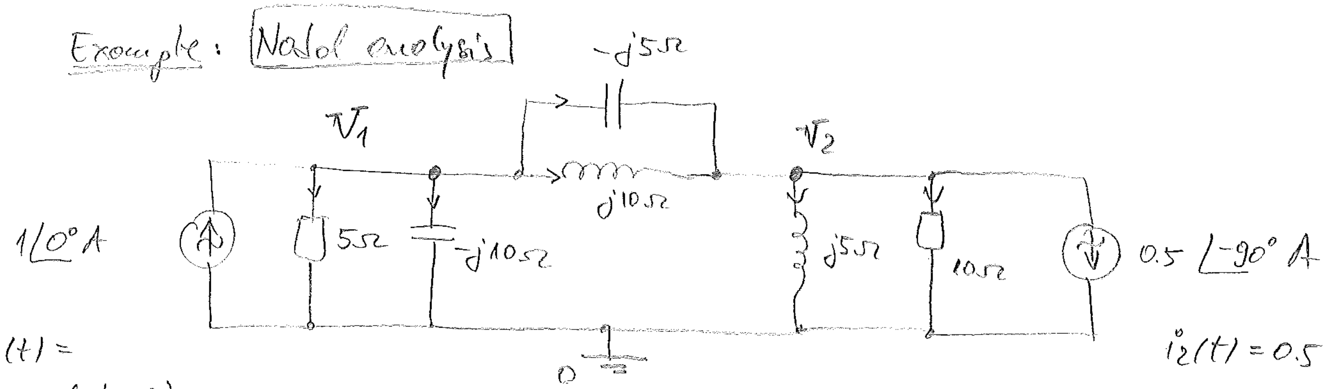
$$\underline{Y} = \frac{1}{Z} = \frac{1}{1-j2} = \frac{1+j2}{(1+j2)(1-j2)} = \frac{1+j2}{5} = \frac{1}{5} + j\frac{2}{5}\ S$$

multiply/divide by conjugate

- Connection
  - in parallel => summation of admittances
  - in series => similar to resistances in parallel.

$\boxed{\text{Nodal and mesh analysis}}$ using phasor to find the steady-state component of the complete-response; when the forcing function is a sinusoidal!

Example: $\boxed{\text{Nodal analysis}}$



$i_1(t) = 1\cdot\cos(\omega t + 0°) = \cos\omega t$

$i_2(t) = 0.5\cos(\omega t - 90°)$

OBS: { – In this figure we work w/ impedances and phasors! – Current sources have the same frequency!!

$$\begin{cases} \text{KCL node } V_1: & \dfrac{V_1}{5} + \dfrac{V_1}{-j10} + \dfrac{V_1 - V_2}{j10} + \dfrac{V_1 - V_2}{-j5} = 1\angle 0° = 1 + j0 = 1 \\ \text{KCL node } V_2: & \dfrac{V_2}{j5} + \dfrac{V_2}{10} - \dfrac{V_1 - V_2}{j10} - \dfrac{V_1 - V_2}{-j5} = -(0.5\angle -90°) = j0.5 \end{cases}$$

2 eq. 2 unknowns

$$\Rightarrow \begin{cases} (0.2 + j0.2)V_1 - j0.1\,V_2 = 1 \\ -j0.1\cdot V_1 + (0.1 - j0.1)V_2 = j0.5 \end{cases}$$

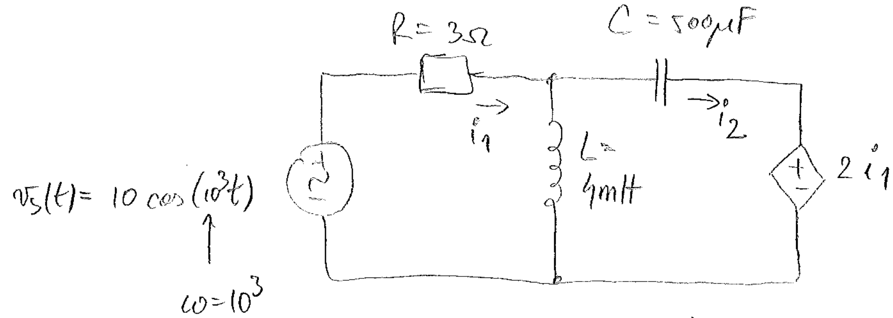
$$\Rightarrow \begin{cases} V_1 = 1 - j2 \ [V] = 2.24\angle -63.4° \\ V_2 = -2 + j4 \ [V] = 4.47\angle 116.6° \end{cases}$$

↑ in polar form

Hence, converting to the time-domain:

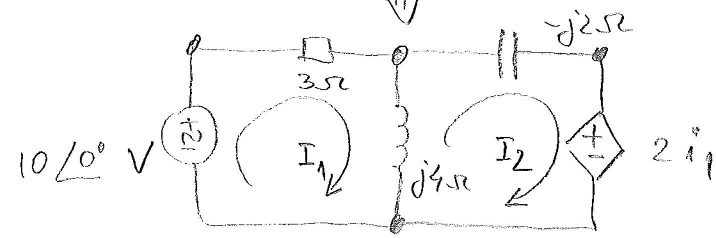
$$\begin{cases} v_1(t) = 2.24\cdot\cos(\omega t - 63.4°) \ [V] \\ v_2(t) = 4.47\cdot\cos(\omega t + 116.6°) \ [V] \end{cases}$$

Example : Mesh analysis



$i_1, i_2 = ?$

⇓ convert into impedances and phasors.



$$\frac{1}{j\omega C} = -j\frac{1}{\omega C} = -j\frac{1}{10^3 \cdot 500 \cdot 10^{-6}} = -j2\,\Omega$$

$$j\omega L = j \cdot 10^3 \cdot 4 \cdot 10^{-3}\,\Omega = j4\,\Omega$$

Loop I_1: $3I_1 + j4(I_1 - I_2) = 10\angle 0° = 10 + j0 = 10$

Loop I_2: $j4(I_2 - I_1) - j2 I_2 + 2 I_1 = 0$

2 eq. 2 unknowns.

$$\begin{cases} (3 + j4) I_1 - j4 I_2 = 10 \\ (2 - j4) I_1 + j2 I_2 = 0 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} I_1 = \dfrac{14 + j8}{13} = 1.24 \angle 29.7° \ [A] \\ I_2 = \dfrac{20 + j30}{13} = 2.77 \angle 56.3° \ [A] \end{cases}$$

in polar form

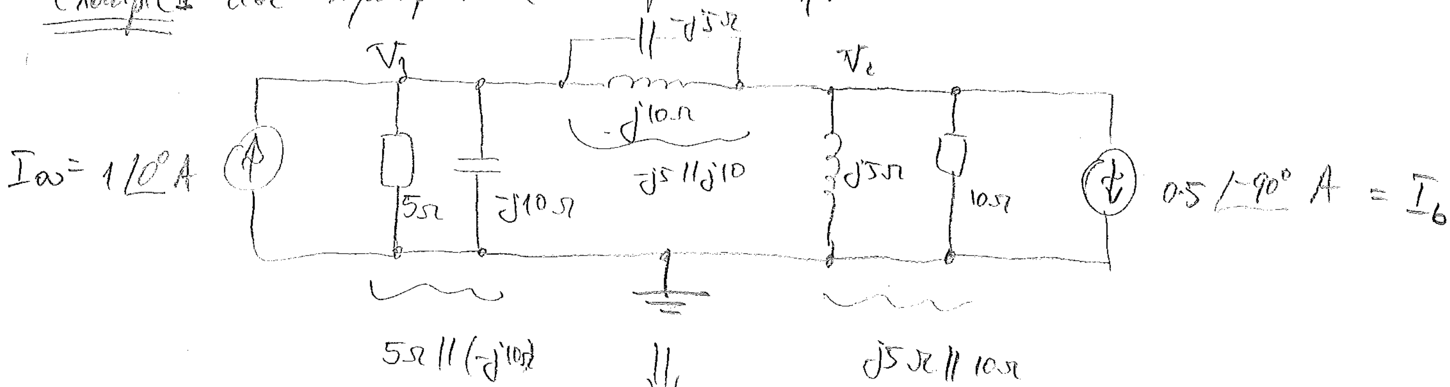
Converting to time domain!

$$\begin{cases} i_1(t) = 1.24 \cdot \cos(\omega t + 29.7°) \ [A] \\ i_2(t) = 2.77 \cdot \cos(\omega t + 56.3°) \ [A] \end{cases}$$

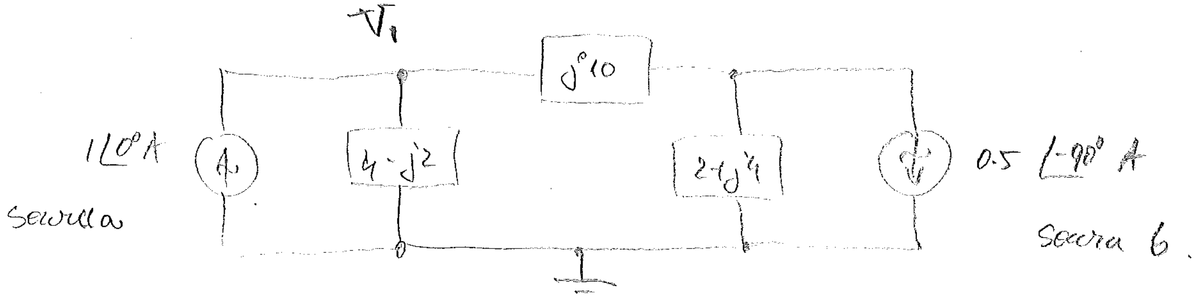
Superposition

✓ Done.

Example: Use superposition to find V_1.

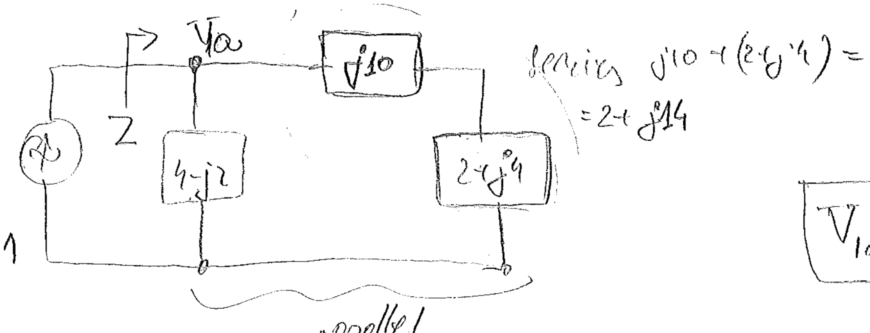


⇓



$$V_1 = \underbrace{V_{1a}}_{\text{due to source a}} + \underbrace{V_{1b}}_{\text{due to source b.}}$$

(a) contribution due to source a:



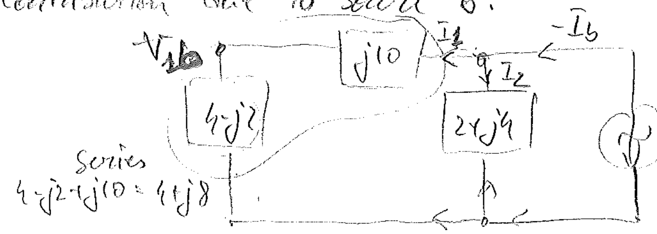
$I_a = 1\angle 0° A$
$= 1 + j0 = 1$

$$Z_{eq} = (4-j2) \| (2+j14) = \frac{(4-j2)\times(2+j14)}{4-j2+2+j14}$$

$$\boxed{V_{1a} = \underbrace{Z}_{} \cdot \underbrace{I_a}_{=1} = 2-j2 \; [V]}$$

$$\boxed{V_1 = V_{1a} + V_{1b} = = 2-j2-1 = 1-j2} \; [V]$$

(b) Contribution due to source b:



$I_b = 0.5\angle -90° A$
$= -j0.5$

$$I_1 = \frac{2+j4}{(2+j4)+(4+j8)} \times (-I_b) = -\frac{2+j4}{6+j12} I_b$$

$$\boxed{V_{1b} = (4-j2)\cdot I_1 = -\frac{(4-j2)(2+j4)}{6+j12} \times (\underbrace{0.5\angle -90°}_{=-j0.5}) = -1 [V]}$$