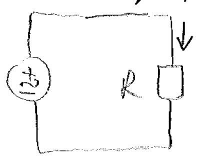
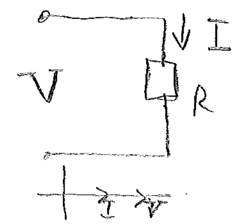
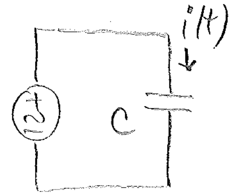
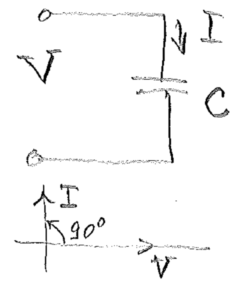
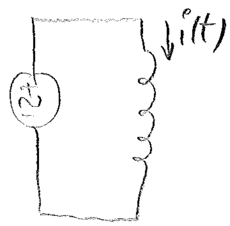
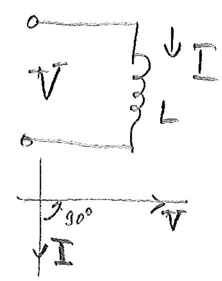
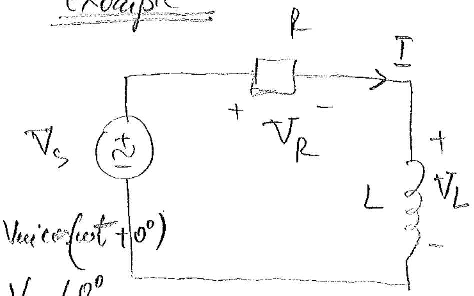
Summary on phasors (so far)

	Time-domain	Frequency-domain	
$v(t) = V_m \cdot \cos(\omega t + \phi_v)$, $i(t) = I_m \cdot \cos(\omega t + \phi_i)$ 	$v = Ri$; $V_m = R \cdot I_m$, $\phi_v = \phi_i$	$\boxed{V = R\,I}$	
$v(t)$, $i(t)$ 	$i = C\frac{dv}{dt}$ $\left(v = \frac{1}{C}\int i\,dt\right)$; $V_m = \frac{1}{\omega C}\cdot I_m$, $\phi_i = \phi_v + 90^\circ$	$\boxed{V = \frac{1}{j\omega C}\cdot I}$	
$v(t)$, $i(t)$ 	$v = L\frac{di}{dt}$; $V_m = \omega L \cdot I_m$, $\phi_i = \phi_v - 90^\circ$	$\boxed{V = j\omega L \cdot I}$	

The power of these phasor relationships is that we can do/apply "phasor-based analysis" techniques which will involve only algebraic equations, similar to how we did it for circuits with only R, indep. & dep. sources!

Example



$v(t) = V_m \cos(\omega t + 0^\circ)$

$\bar{V}_s = V_m \angle 0^\circ$

$$\bar{V}_s = \bar{V}_R + \bar{V}_L$$

$$\boxed{\bar{V}_s = R\,\bar{I} + j\omega L\,\bar{I}}$$ ← algebraic equation.

$$\bar{I} = \frac{\bar{V}_s}{R + j\omega L}$$

$$\boxed{V_s \text{ is given} \qquad \bar{V}_s = V_m \angle 0^\circ \qquad [v_s(t) = V_m \cos(\omega t + 0^\circ) = V_m \cos \omega t]}$$

- So, $\bar{I} = \dfrac{V_m \angle 0^\circ}{R + j\omega L}$

- To get the time-domain expression of the current $i(t)$, we first use the polar notation:

$$\bar{I} = \frac{V_m}{\sqrt{R^2 + (\omega L)^2}} \angle -\tan^{-1}\frac{\omega L}{R}$$

- Therefore: $$i(t) = \underbrace{\frac{V_m}{\sqrt{R^2 + (\omega L)^2}}}_{= I_m} \cdot \cos\Big(\omega t \underbrace{- \tan^{-1}\frac{\omega L}{R}}_{= \phi_i}\Big)$$

- Finally: $$\boxed{i(t) = I_m \cdot \cos(\omega t + \phi_i)}$$ where
$$\begin{cases} I_m = \dfrac{V_m}{\sqrt{R^2 + (\omega L)^2}} \\ \phi_i = -\tan^{-1}\dfrac{\omega L}{R} = -\tan^{-1}\omega\tau \\ \tau = \dfrac{L}{R} \end{cases}$$

which is the <u>steady-state</u> component of the complete-response to a sinusoid forcing function
(Note: see 1st lecture on Ch. 10)

Impedance

$$\underbrace{R}_{=Z_R} = \frac{V}{I} \qquad \underbrace{j\omega L}_{=Z_L} = \frac{V}{I} \qquad \underbrace{\frac{1}{j\omega C}}_{=Z_C} = \frac{V}{I}$$

- complex quantities, which we treat the same way we treat resistances.
- we'll call these, $\boxed{\text{impedance}}$, denoted as $\boxed{Z}$ $[\Omega]$

<u>OBS</u>:
- <u>not</u> a phasor!
- cannot be transformed to the time domain by multiplying with $e^{j\omega t}$ and taking the real part!

- R, L, C are for time-domain
- Z_R, Z_L, Z_C are for the frequency-domain.

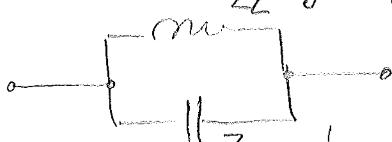
The beauty is that impedances can be combined in series or in parallel the same way as resistances!

<u>Examples</u>:

$$\underbrace{Z_L = j\omega L = j50\ \Omega \qquad Z_C = \frac{1}{j\omega C} = -\frac{j}{\omega C} = -j1\ \Omega}$$

$$Z_{eq} = Z_L + Z_C = j50 - j1 = j49\ \Omega.$$ ← <u>OBS</u>: depends on frequency!

$$Z_L = j\omega L = j50\ \Omega$$



$$Z_C = \frac{1}{j\omega C} = -\frac{j}{\omega C} = -j1\ \Omega$$

$$Z_{eq} = \frac{Z_L \cdot Z_C}{Z_L + Z_C} = \frac{(j50) \times (-j1)}{j50 - j1} = \frac{50}{j49} = -j\frac{50}{49} = -j\,1.02\ \Omega$$

Definitions

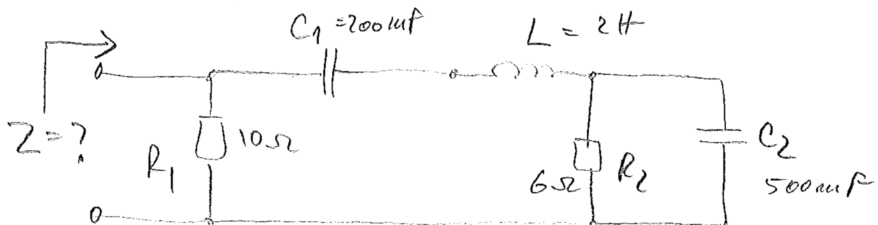
$$\boxed{Z = R + j \cdot X} = |Z| \angle \theta$$

impedance ↑ resistance ↑ reactance ↑ [Ω] all.

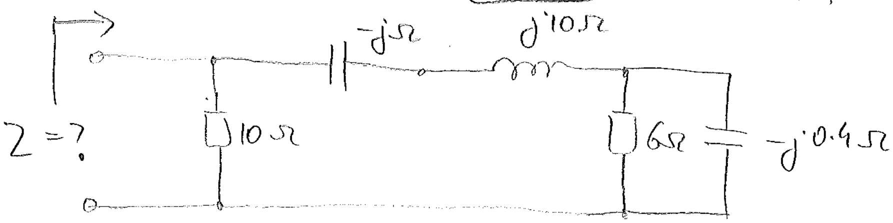
$$\begin{cases} Z_R = R & \text{(R has zero reactance)} \\ Z_L = j\omega L \\ Z_C = \dfrac{1}{j\omega C} = -j\dfrac{1}{\omega C} \end{cases}$$

Example: $Z = \underbrace{50}_{\text{resistance of } 50\Omega} - j\underbrace{86.6}_{\text{reactance of } -86.6\,\Omega}\ \Omega$

Example: Find equivalent impedance of given network for $\omega = 5$ rad/s.



$\Downarrow$ $\boxed{\omega = 5\ \text{rad/s}}$ convert R, L, C into impedances.



$$= -j + j10 = j9\,\Omega \qquad = 6 \parallel (-j0.4) = \frac{6 \times (-j0.4)}{6 - j0.4} = 0.026 - j\,0.398\,\Omega$$

$$= 0.026 - j0.398 + j9 = 0.026 + j\,8.602\,\Omega$$

$$= 10 \parallel (0.026 + j8.602)$$

$$= \frac{10 \times (0.026 + j8.602)}{10 + 0.026 + j8.602}$$

$$\boxed{= 4.255 + j\,4.929\,\Omega = Z\big|_{\omega = 5\,\text{rad/s}}}$$

Alternatively: find $Z(\omega)$ as general solution and then evaluate for desired ω!