$\boxed{\text{Ch.10}}$ $\boxed{\text{Sinusoidal steady-state analysis}}$

(1) $\begin{cases} \cos(u \pm v) = \cos u \cdot \cos v \mp \sin u \cdot \sin v \\ \sin(u \pm v) = \sin u \cdot \cos v \pm \cos u \cdot \sin v. \end{cases}$

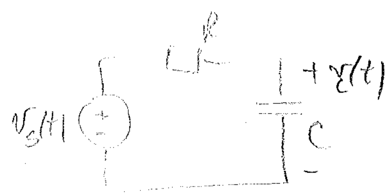
(4) Conversion between "rad" and "°";

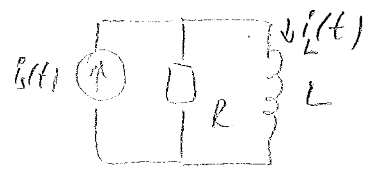
(2) $\begin{cases} A\cos(\omega t) + B\sin(\omega t) = M \cdot \cos(\omega t + \phi) \\ M = \sqrt{A^2+B^2}; \quad A = M\cos\phi \\ \phi = \tan^{-1}\left(\frac{B}{A}\right); \quad B = M\sin\phi \end{cases}$

$\omega = 2\pi f = \frac{2\pi}{T}$, T = period.

(3) "ELI the ICE man" : voltage leads current for L
current leads voltage for C.

Recall that we studied:





General form: $\boxed{\tau \frac{dy(t)}{dt} + y(t) = x(t)}$

$\begin{cases} y(t) = i_L(t) \\ \tau = \frac{L}{R} \end{cases}$

$\begin{cases} y(t) = v_C(t) \\ \tau = RC \end{cases}$

We derived for $\boxed{x(t) = X_m \cos \omega t}$ as forcing-function, the general (complete) solution:

$$y(t) = \underbrace{y(0) \cdot e^{-\frac{t}{\tau}}}_{= \text{natural}} + \underbrace{\frac{X_m}{1+(\omega\tau)^2}\left(\cos\omega t + \omega\tau \sin\omega t - e^{-\frac{t}{\tau}}\right)}_{= \text{forced}}$$

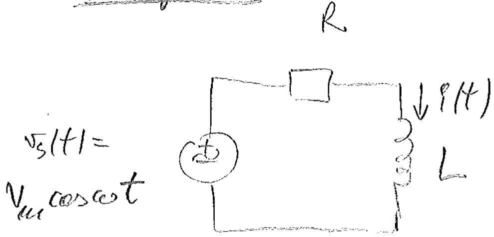
$$\boxed{y(t) = \underbrace{\left[y(0) - \frac{X_m}{1+(\omega\tau)^2}\right]e^{-\frac{t}{\tau}}}_{= \text{xsient}} + \underbrace{\frac{X_m}{\sqrt{1+(\omega\tau)^2}} \cdot \cos\left(\omega t - \tan^{-1}\omega\tau\right)}_{= \text{ss (steady-state)}}}$$

$= Y_m \cdot \cos(\omega t + \phi)$

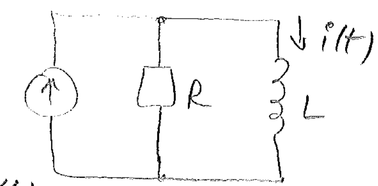
Today, we are interested in the <u>steady-state</u> response <u>only</u>!

Note: the textbook refers to the steady-state response as the forced-response (we'll tolerate that for the time-being).

Example 1:



$\Rightarrow$



$$i(t) = \underbrace{\frac{V_m}{R}}_{I_m} \cos\omega t = I_m \cdot \cos\omega t$$

This is an equivalent circuit for which we know the steady-state solution: $X_m = I_m$; $\tau = \frac{L}{R}$.

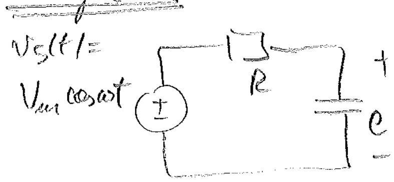
$$i_{ss}(t) = \frac{I_m}{\sqrt{1+(\omega\tau)^2}} \cdot \cos(\omega t - \tan^{-1}\omega\tau) \quad ; \quad \tau = \frac{L}{R}$$

$$i_{ss}(t) = \frac{V_m}{R\sqrt{1+\omega^2 \cdot \frac{L^2}{R^2}}} \cdot \cos\left(\omega t - \tan^{-1}\frac{\omega L}{R}\right)$$

$$(1) \quad \boxed{i_{ss}(t) = \underbrace{\frac{V_m}{\sqrt{R^2+(\omega L)^2}}}_{=I_m^{ss}} \cdot \cos\Big(\omega t - \underbrace{\tan^{-1}\frac{\omega L}{R}}_{=\phi}\Big)} = \underbrace{I_m^{ss}}_{\text{magnitude}} \cdot \cos(\omega t + \underbrace{\phi}_{\text{angle or phase}})$$

(eq. [4] in textbook)

Example 2:



$$v_C(t) = v_C^{\text{transient}}(t) + \boxed{v_C^{ss}(t)}$$

we are interested in today!

Solution is:

$$(2) \quad \boxed{v_C^{ss}(t) = \underbrace{\frac{V_m}{\sqrt{1+(\omega RC)^2}}}_{=V_m^{ss}} \cdot \cos\big[\omega t - \underbrace{\tan^{-1}(\omega RC)}_{=\phi}\big]} = \underbrace{V_m^{ss}}_{\text{magnitude}} \cdot \cos[\omega t + \underbrace{\phi}_{\text{phase}}]$$

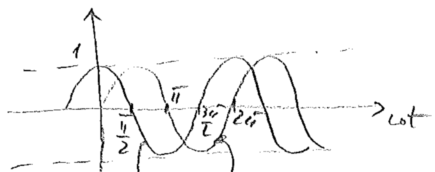
Observations:

(1) - Steady-state response has the same functional form as the forcing-function

(2) - Amplitude of the response (steady-state) is proportional to the amplitude of the forcing-function!
(it) changes with R, L, C, and ω!

(3) - Response $\boxed{\text{lags}}$ the applied forcing function by $\boxed{\tan^{-1}\omega\tau = -\phi}$

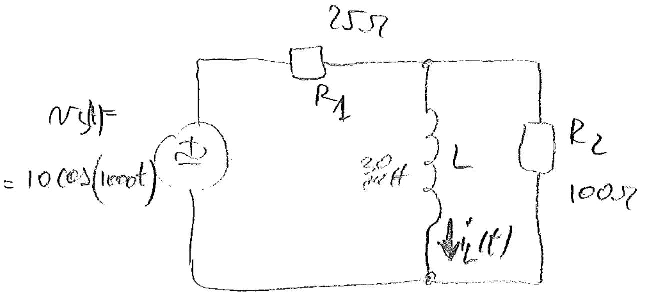
please read details in textbook on lag and lead!



$\cos\omega t$ "leads" $\sin\omega t$ by $\frac{\pi}{2}$.

$\sin\omega t = \cos(\omega t - \frac{\pi}{2})$

Example 3: (10.1 pp. 374) Find ss (steady-state) solution of $i_L(t)$!



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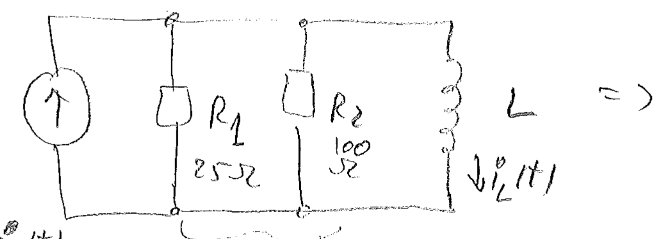
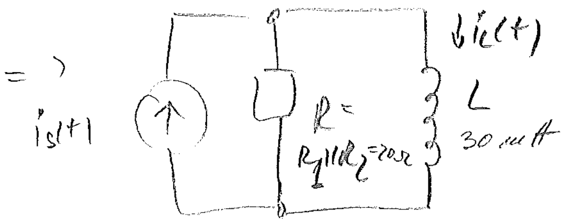


$i_s(t) = \underbrace{\frac{10}{25}}_{= I_m} \cos(1000t)$

$= R = R_1 \| R_2 = 25 \| 100 = \frac{2500}{125} = 20\,\Omega$

=>



Solution is:

$$i_L(t) = \frac{I_m}{\sqrt{1+\left(\frac{\omega L}{R}\right)^2}} \cdot \cos\left(\omega t - \tan^{-1}\frac{\omega L}{R}\right) = I_m^{ss} \cdot \cos(\omega t + \phi)$$

$$= \frac{\frac{10}{25}\cdot R = 8}{\underbrace{\sqrt{R^2+(\omega L)^2}}_{=\sqrt{20^2+30^2} = \sqrt{1300}}} \times \cos\left(\omega t - \tan^{-1}\frac{30}{20}\right)$$

$$= \frac{8}{\sqrt{1300}} \cdot \cos\left\{\omega t - \tan^{-1}\frac{30}{20}\right\} = 0.222 \cos\left(\omega t - \tan^{-1}\frac{30}{20}\right) \; [A]$$