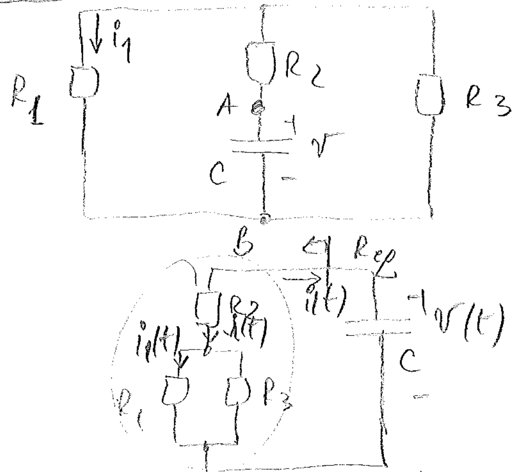
Seminar Nov. 8.

Example 8.5 (pp. 273)



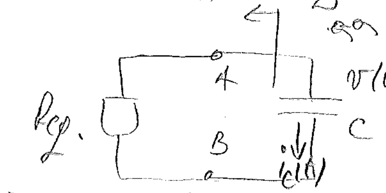
$R_{eq} = R = R_2 + (R_1 \| R_3)$

(=>)

$v(0^-) = V_0$

Find $v(0^+)$, $i_1(0^+)$

$R_{eq} = R = ?$



$v(0^-) = V_0 = v(0^+)$

$= R_2 + (R_1 \| R_3)$

This is the natural-response case with initial condition $v(0^+) = V_0$

so: the voltage $v_c(t)$:

$$v_c(t) = v(0^+) \cdot e^{-\frac{t}{\tau}}$$

$$\boxed{v_c(t) = V_0 \cdot e^{-\frac{t}{R_{eq}C}}} \; ; \; t \geq 0$$

$$i(t) = C\frac{dv_c(t)}{dt} = \boxed{-\frac{V_0}{R_{eq}} e^{-\frac{t}{R_{eq}C}} = i_c(t)} \; ; \; t \geq 0$$

$i_1(t) = ?$, $t \geq 0$

By current division:

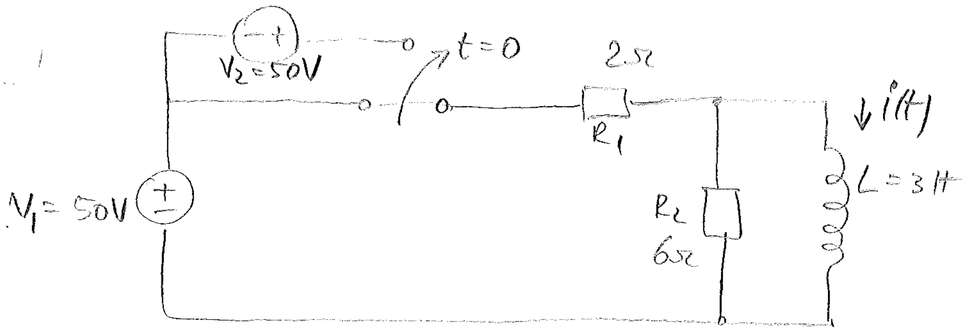
$$i_1(t) = \frac{R_3}{R_1 + R_3}(-i(t))$$

$$i_1(t) = \frac{R_3 V_0}{R_1 + R_3 \, R_{eq}} e^{-\frac{t}{R_{eq}C}} = \boxed{\frac{R_3}{R_1 + R_3} \cdot \frac{1}{R_2 + R_1 \| R_3} \cdot V_0 \cdot e^{-\frac{t}{R_{eq}C}} = i_1(t)}$$

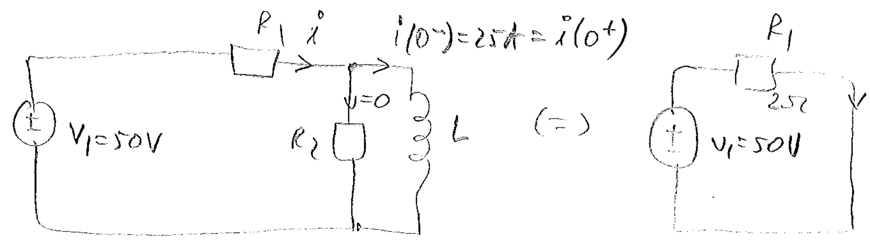
$$t = 0^+ \Rightarrow i_1(0^+) = \frac{R_3}{R_1 + R_3} \cdot \frac{V_0}{R_2 + \frac{R_1 R_3}{R_1 + R_3}}$$

$$= \frac{R_3 V_0}{R_1 R_3 + R_1 R_2 + R_2 R_3} = i_1(0^+)$$

Example 8.8 (pp. 286 ← Do it in class too ; to illustrate $u(t)$ step function!



(i) $\boxed{t \le 0}$



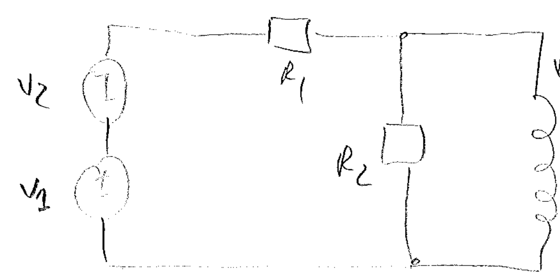
$i = \frac{V_1}{R_1} = \frac{50}{2} = 25\,[A]$

$\boxed{dc}$ circuit; in dc, L is a short-circuit.

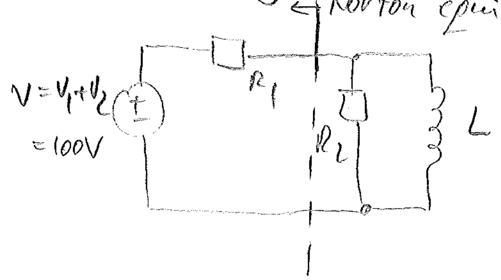
For $t \le 0$; $i(t) = 25$

(ii) $\boxed{t \ge 0}$

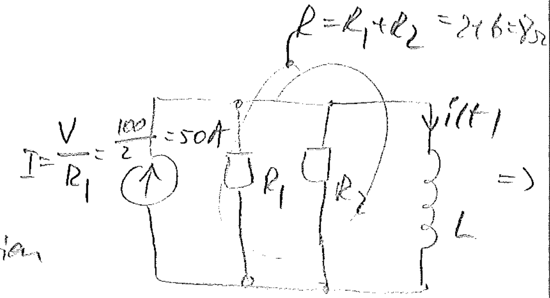
$i(0^-) = 25A = i(0^+)$ by continuity rule ('crucial obs #2')



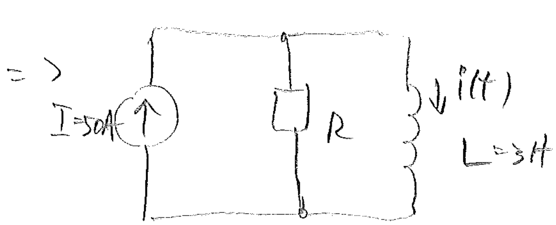
⇓ ← Norton equivalent



by simple source transformation ⟹



=>



Recall: Summary. $y(t) = y(0) \cdot e^{-\frac{t}{\tau}} + X_s\left(1 - e^{-\frac{t}{\tau}}\right)$

$i(t)$; $i(0^+) = 25A$; $I = 50A = y(\infty)$; $\tau = \frac{L}{R} = \frac{3}{8}$

OR: $y(t) = [y(0) - y(\infty)]\, e^{-\frac{t}{\tau}} + y(\infty)$

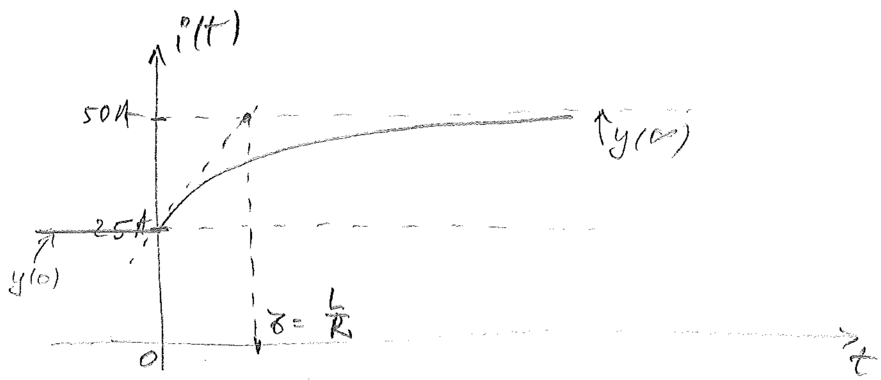
Therefore:
$$i(t) = [25 - 50]\, e^{-\frac{t}{\tau}} + 50$$

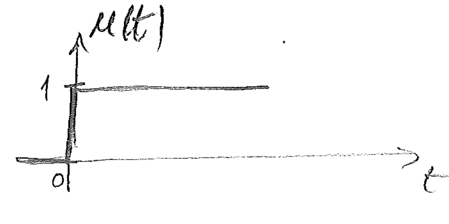
$$i(t) = \underbrace{50}_{i(t)_{ss}} - \underbrace{25\, e^{-\frac{R}{L}t}}_{i(t)_{transient}} = \boxed{50 - 25\cdot e^{-\frac{8}{3}t} = i(t)}\;;\; \underline{t \geq 0}$$

One can use $u(t)$ the step-function to write the complete solution:

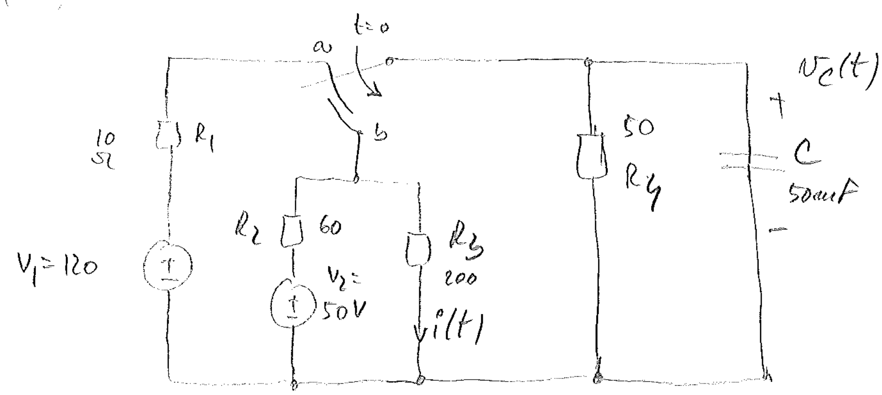
$$i(t) = \begin{cases} 25A, & t \leq 0 \\ 50 - 25\cdot e^{-\frac{8}{3}t}, & t \geq 0 \end{cases} \quad \text{or:} \quad \boxed{i(t) = \left(50 - 25 e^{-\frac{8}{3}t}\right)\cdot u(t)}$$

where:

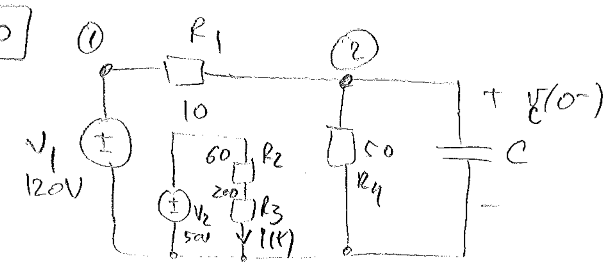




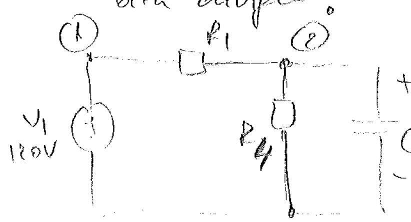
Example $\boxed{8.10.}$ pp. 290



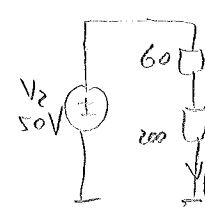
(i) $\boxed{t \leq 0}$



$\boxed{dc}$ circuit, C is an open after it has been charged!



$$+v_C(0^-) = V_2 = \frac{R_4}{R_1 + R_4} V_1 = \frac{50}{10+50} \times 120\,V = \boxed{100\,V = v_C(0^-)}$$



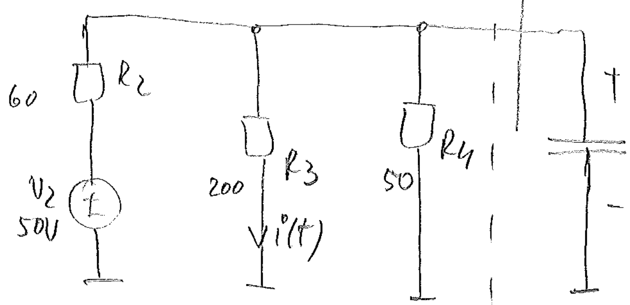
$$\boxed{i(0^-) = \frac{V_2}{R_2 + R_3} = \frac{50}{260} = \frac{5}{26}A}$$

(ii) $\boxed{t \geq 0}$

equivalent circuit:

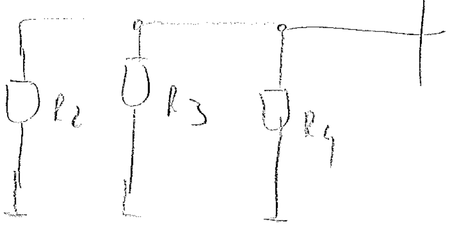
$\boxed{v_C(t) = ?}$

Thevenin's equivalent.



$v_C(0^+) = v_C(0^-) = 100\,V$. ("Crucial obs. # 1") continuity rule.

(a) $R_{eq} = ? = R_{TH}$



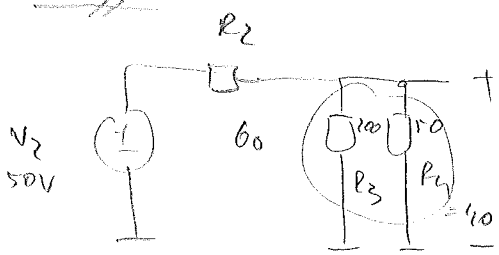
$R_{eq} = R_2 \| R_3 \| R_4 = 24\,\Omega = R_{TH}$

$60 \| 200 \| 50$

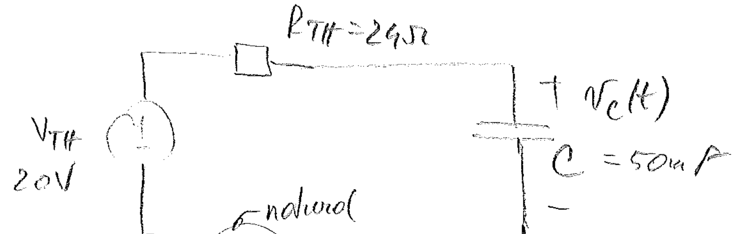
$\frac{200 \times 50}{250} = \frac{1000}{250} = 40$

$60 \| 40 = \frac{60 \times 40}{100} = \frac{2400}{100} = 24$

(b) $V_{TH} = ?$



$v_{OC} = V_{TH} = \frac{40}{\underset{2}{40+60}} \times 50\,V = 20\,V.$



Recall: $y(t) = \underbrace{y(0) \cdot e^{-\frac{t}{\tau}}}_{\text{natural}} + \underbrace{X_S\left(1 - e^{-\frac{t}{\tau}}\right)}_{\text{forced}}$

$v_C(t)$ $\quad v_C(0^+) = 100V$ $\quad V_{TH} = 20V = y(\infty)$ $\quad \tau = R_{TH} \cdot C = 24 \times 50 \times 10^{-3}$ sec.

or: $\boxed{y(t) = [y(0) - y(\infty)]\, e^{-\frac{t}{\tau}} + y(\infty)}$

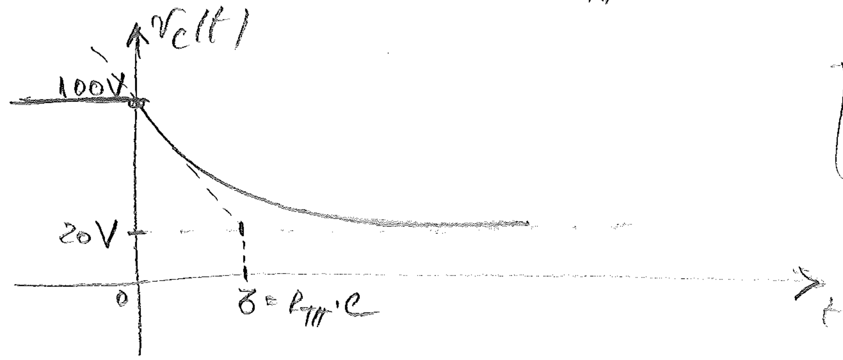
$v_C(t) = [100 - 20] \cdot e^{-\frac{t}{\tau}} + 20 = 80\, e^{-\frac{t}{\tau}} + 20$

transient $\quad$ steady-state.

see verso !!!

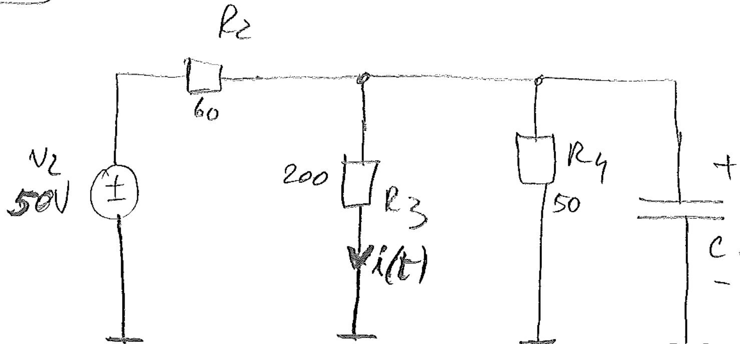
$$\boxed{v_c(t) = 20 + 80\,e^{-\frac{t}{\tau}}};\ \underline{t \geq 0}$$

$$\tau = R_{TH} \cdot C = 1.2 \text{ sec.}$$



$$\boxed{v_c(t) = \begin{cases} \left[20 + 80 \cdot e^{-\frac{t}{\tau}}\right]; & t \geq 0. \\ 100V\ ; & t < 0 \end{cases}}$$

$\boxed{i(t) = ?}$

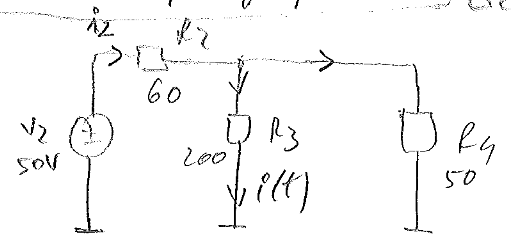


$$\boxed{v_c(t) = 20 + 80 \cdot e^{-\frac{t}{\tau}}}$$

acts as an open for $\boxed{dc}$

$$i(0^-) = \frac{5}{26}\,A$$

Think of superposition: here.



Skip this

$\boxed{\text{Textbook}}\boxed{\text{Method}}$

Forced response:

$$i_f(t) = \frac{R_4}{R_3 + R_4} \cdot i_2 = \frac{R_4}{R_3 + R_4} \cdot \frac{V_2}{R_2 + R_3 \| R_4} = \frac{50}{50 + 200} \cdot \frac{50V}{60 + \frac{50 \times 200}{50 + 200}} = 0.1\,A = 100\,mA$$

$$i_2 = \frac{V_2}{R_2 + R_3 \| R_4}$$

The natural response: has form: $i_n(t) = A \cdot e^{-\frac{t}{1.2}}$

$$i(t) = i_f(t) + i_n(t) = 0.1 + A \cdot e^{-\frac{t}{1.2}}$$

– to find A we need $i_t(0^+)$

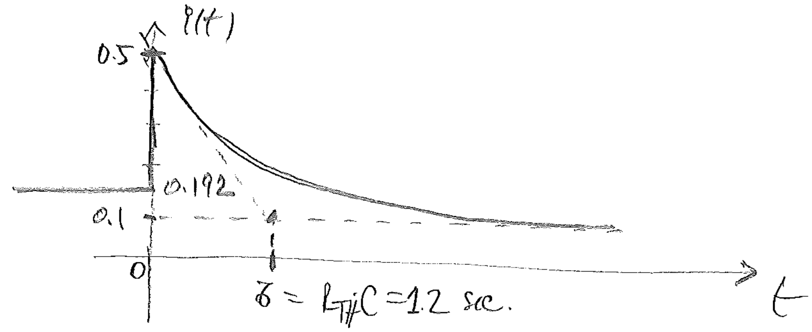
$v_C(0^-) = 100V = v_C(0^+)$ needs to be used here!

$$i(0^+) = \frac{100V}{R_3} = \frac{100}{200} = 0.5A$$

$$\Rightarrow i(t=0^+) = 0.1 + A \cdot e^0 = 0.5 \Rightarrow A = 0.4A$$

Therefore:

$$\boxed{\begin{array}{l} i(t) = 0.1 + 0.4e^{-\frac{t}{1.2}}; \ t \geq 0. \\ i(t) = \frac{5}{26}A; \quad t < 0 \end{array}}$$



Method | Lecturer's way — do this

Think of the solution as: (this is key idea!)

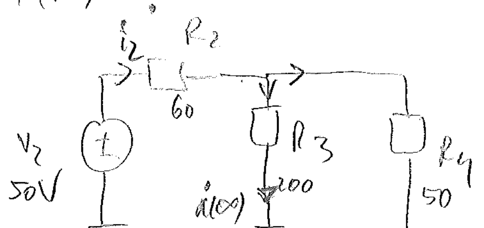
$$y(t) = [y(0) - y(\infty)] \cdot e^{-\frac{t}{\tau}} + y(\infty)$$

$$\boxed{i(t) = [i(0^+) - i(\infty)]e^{-\frac{t}{\tau}} + i(\infty)}$$

(1) $i(0^+)$ must be obtained from $v_C(0^-) = 100V = v_C(0^+)$

$$i(0^+) = \frac{v_C(0^+)}{R_3} = \frac{100V}{200\Omega} = 0.5A$$

(2) $i(\infty) = ?$



$$i(\infty) = \frac{R_4}{R_3 + R_4} \cdot i_2 = \frac{R_4}{R_3 + R_4} \cdot \frac{V_2}{R_2 + R_3 \| R_4} = 0.1A.$$

Therefore: $\boxed{i(t) = [0.5 - 0.1] \cdot e^{-\frac{t}{\tau}} + 0.1 = 0.1 + 0.4e^{-\frac{t}{1.2}}; \ t \geq 0}$

$$\boxed{i(t) = \frac{5}{26}A, \quad t < 0}$$

~~Plot~~ is the same!