▸ First, do a summary of things learned so far!!!

Transient response of first-order circuits

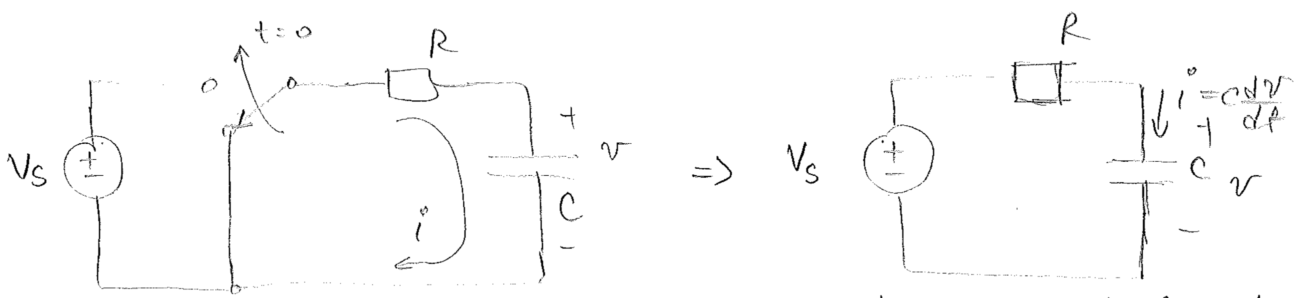
- We had found that the complete response to a dc forcing function consists of a transient component & a dc steady-state component.
- The transient response has the same functional form as the natural response, which is an exponentially decaying function!
- The dc steady-state component has the same functional form as the forcing function which is a constant.

Comparison of C and L behaviour:

Element:	C	L
• dc steady-state condition	v_C = const	i_L = const
• dc steady-state behaviour	open-circuit	short-circuit.
• cannot change instantaneously	v_C	i_L
• can change instantaneously	i_C	v_L

1 The RC circuit

(a)



Capacitance is initially discharged!
$i(0^-) = 0$ $v(0^-) = 0$.

Recall: (1) $\boxed{RC \cdot \frac{dv}{dt} + v = V_S}$ which begins at $t = 0^+$!

and property (or "crucial observation"): $v(0^-) = v(0^+) = 0$ V

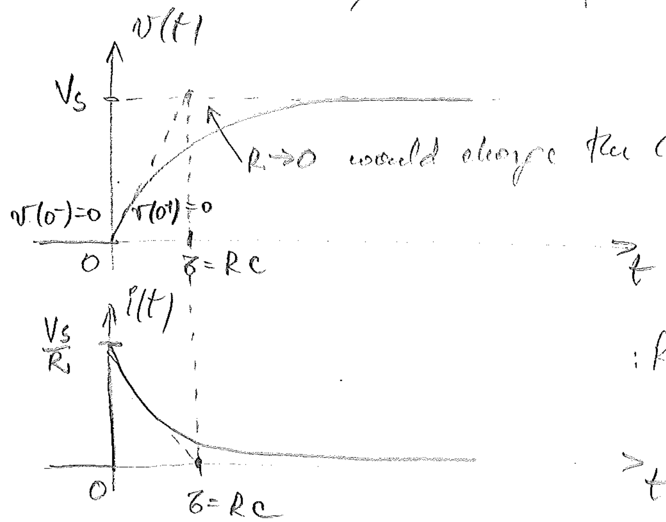
Also recall: $\boxed{y(t) = \underbrace{y(0)}_{=v(0^+)=0} \cdot e^{-\frac{t}{\tau}} + \underbrace{X_S}_{=V_S} \left(1 - e^{-\frac{t}{\tau}}\right)}$ $\quad \tau = RC$

So, for $t \geq 0$, the solution is: ← basically just the forced component!

$\boxed{v(0^-) = 0; t \leq 0}$ $\quad \boxed{v(t) = V_S \left(1 - e^{-\frac{t}{RC}}\right)}$ (2) ; $t \geq 0$!

Also: $i = C \frac{dv}{dt} = \cancel{C} \cdot V_S \cdot \frac{1}{R\cancel{C}} e^{-\frac{t}{RC}} = \boxed{\frac{V_S}{R} e^{-\frac{t}{RC}} = i(t)}$ (3) ; $t \geq 0$!

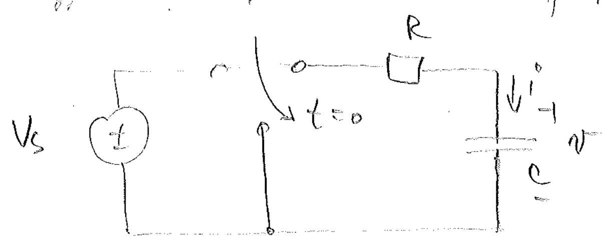
$\boxed{i(0^-) = 0; \quad t \leq 0}$



R→0 would charge the capacitance "instantaneously" but this would require an ∞ current spike!

: Referred to as a "spike"

ⓑ Suppose now, the switch is flipped back!



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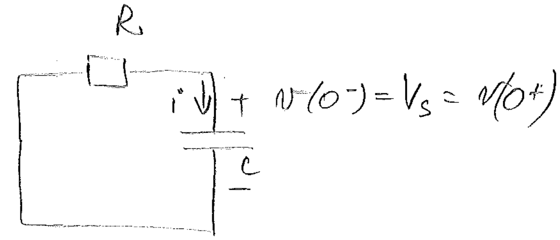


- And let's choose again $t=0$ this instant for convenience.
- By the continuity rule for capacitance:

$$v(0^-) = v(0^+) = V_S$$

- This is the source-free circuit! with the solution:

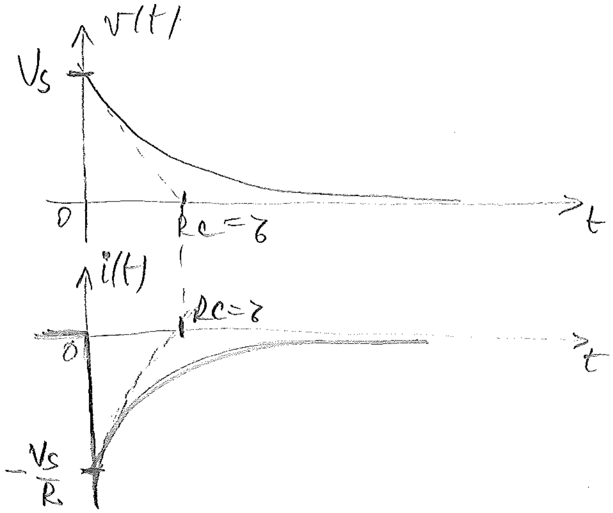
(4) $\boxed{RC \frac{dv}{dt} + dv = 0}$

(5) $\boxed{v(t) = V_S \cdot e^{-\frac{t}{RC}}}$; $t \geq 0$

$\boxed{v(0^-) = V_S}$

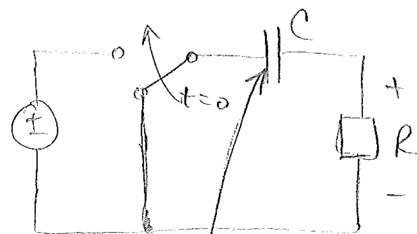
Also: $i = C \cdot \frac{dv}{dt} = -\cancel{C} \cdot V_S \cdot \frac{1}{R\cancel{C}} \cdot e^{-\frac{t}{RC}} = \boxed{-\frac{V_S}{R} \cdot e^{-\frac{t}{RC}} = i(t)}$ (6) $t \geq 0$

$\boxed{i(0^-) = 0}$, $t \leq 0$

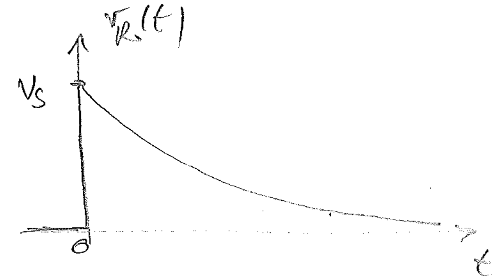


1' The CR circuit [illegible] OR give it as HW assignment

(a)



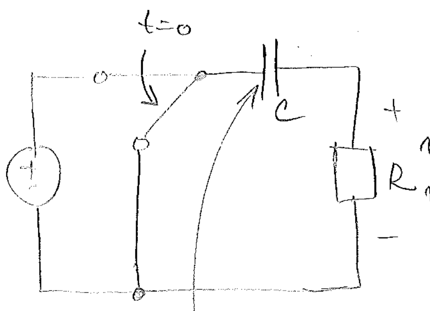
$v_R(0^-) = 0$; $v_R(0^+) = V_s$



- At t=0
  left plate jumps from $0V \to V_s$
- By the continuity rule right plate also jumps to V_s!

From equation (3): $i(t) = \frac{V_s}{R} e^{-\frac{t}{RC}} \Rightarrow \boxed{v_R(t) = V_s \cdot e^{-\frac{t}{RC}}}$ $\underline{t \geq 0}$

(b)



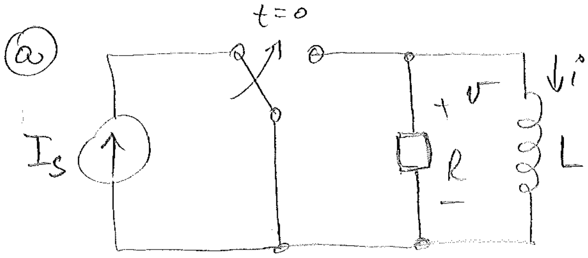
$v_R(0^-) = 0$
$v_R(0^+) = -V_s$

Jumps $V_s \to 0$; right plate jumps from $0 \to -V_s$ due to the continuity rule! So, $v_R(0^+) = -V_s$; R draws current and discharges.

By equation (6): $i(t) = -\frac{V_s}{R} e^{-\frac{t}{RC}} \Rightarrow \boxed{v_R(t) = -V_s e^{-\frac{t}{RC}}}$ $t \geq 0$.

2 The RL circuit

(a)



$\downarrow i(0^-) = 0 = i(0^+)$ by continuity rule!

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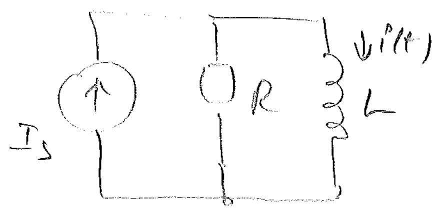


Recall:

$$(1)\quad \boxed{\frac{L}{R}\frac{di}{dt} + i = I_S}$$

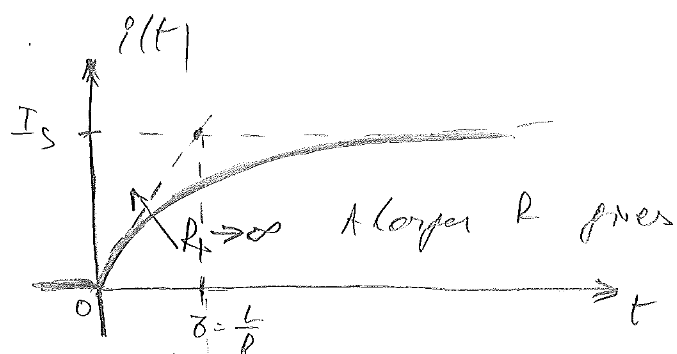
Also recall: $$\boxed{\underbrace{y(t)}_{i(t)} = \underbrace{y(0)}_{=i(0^+)=0}\cdot e^{-\frac{t}{\tau}} + \underbrace{X_S}_{I_S}\left(1-e^{-\frac{t}{\tau}}\right)}\ ;\ \tau = \frac{L}{R}$$

Hence $$\boxed{i(t) = I_S\left(1 - e^{-\frac{R}{L}t}\right)}\ (2)\quad \underline{\underline{t \geq 0}}$$

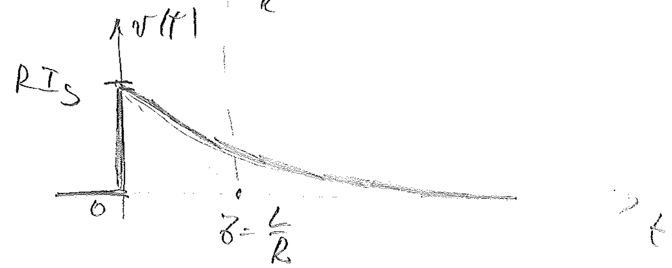
$$\boxed{i(0^-) = 0,\ t \leq 0}$$

Also: $$v = L\frac{di}{dt} = -\cancel{L}\cdot\left(-\frac{R}{\cancel{L}}\right)\cdot I_S\cdot e^{-\frac{R}{L}t} = \boxed{R I_S\cdot e^{-\frac{R\cdot t}{L}} = v(t)}\ (3)\quad \underline{\underline{t \geq 0}}$$

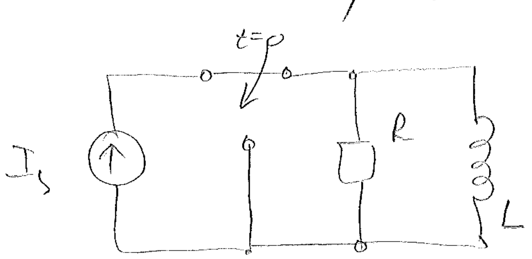
$$\boxed{v(0^-) = 0\ ;\ t \leq 0}$$



A larger R gives a larger RI_s a voltage, which gives a faster rate of current buildup!



(b) Leave (a) for very long time, then flip back the switch!



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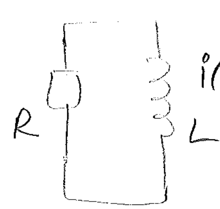

$i(0^-) = I_s = i(0^+)$

This is the source-free circuit! We have just the natural-response!

(4) $\boxed{\frac{L}{R}\frac{di}{dt} + i = 0}$

$\Downarrow$

(5) $\boxed{i(t) = I_S \cdot e^{-\frac{R}{L}t}}$ $t \geq 0.$

$\boxed{i(0^-) = I_S}$

Also: $v = L\frac{di}{dt} = -I_S \cdot \frac{R}{\cancel{L}} \cdot \cancel{L} \cdot e^{-\frac{R}{L}t} = \boxed{-RI_S \cdot e^{-\frac{R}{L}t} = v(t)}$ (6) $\underline{t \geq 0}$

$\boxed{v(0^-) = 0}$