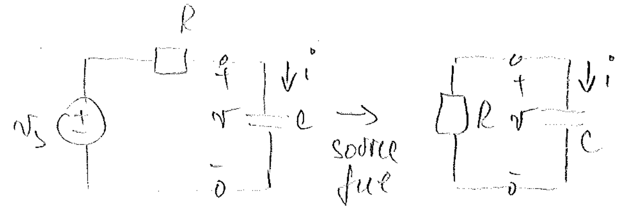
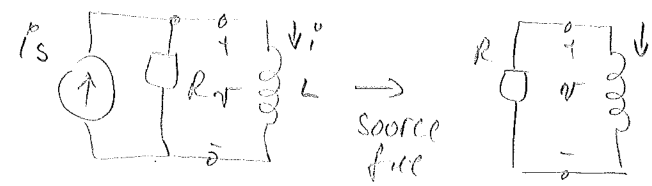
Last time:

$\boxed{\text{Natural}}$ or $\boxed{\text{source-free}}$ response





$$(1)\quad \boxed{\tau \frac{dv}{dt} + v = v_s} \to 0 \text{ source free} \qquad \boxed{\tau = RC}$$

$$(2)\quad \boxed{\tau \frac{di}{dt} + i = i_s} \to 0 \text{ source free} \qquad \boxed{\tau = \frac{L}{R}}$$

General form:

$$\boxed{\tau \frac{dy}{dt} + y = x} \quad (3)$$

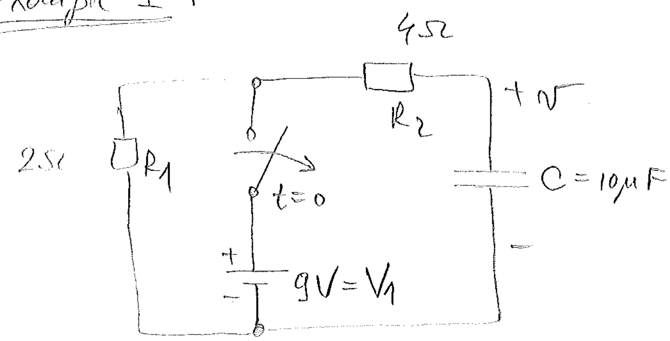
Homogeneous solution, source-free response, or natural-response $(x=0)$

$$\boxed{y(t) = y(0) e^{-\frac{t}{\tau}}} \quad (10)$$

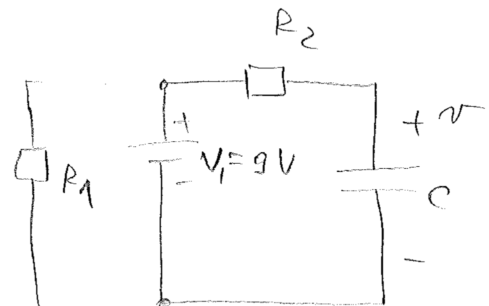
$$\boxed{v(t) = v(0) \cdot e^{-\frac{t}{RC}}} \quad (10') \qquad \boxed{i(t) = i(0) \cdot e^{-\frac{Rt}{L}}} \quad (10'')$$

Examples on how to have/create initial conditions non-zero.

Example 1:

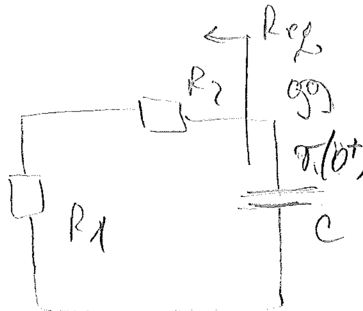


$\boxed{t \le 0}$



We assume that any transients (studied later in course) have died out. This is a DC circuit, current thru C is zero and voltage $v = V_1 = 9V$. So, $v(0) = 9V$

$\boxed{t \ge 0}$



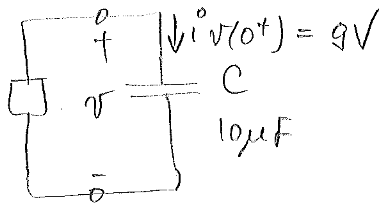
$v(0^+) = v(0^-) = 9V$

$\boxed{\text{Crucial Observation } \boxed{1}}$ Voltage across a capacitor cannot change abruptly! it takes time to bring or take charge from its plates!

Continuity condition: $v_C(t_0^-) = v_C(t_0^+)$

$\Rightarrow$ equivalent circuit:

$R = R_{eq} = R_1 + R_2 = 6\Omega$

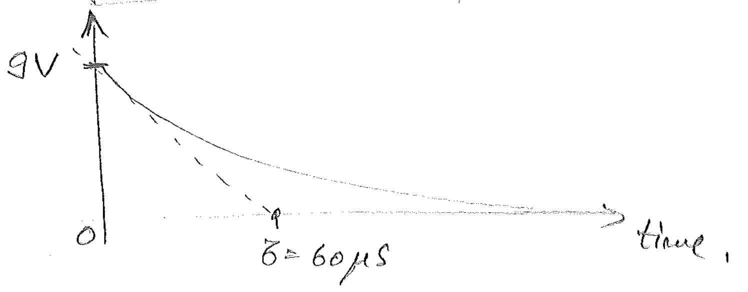


Solution by eq(10'):

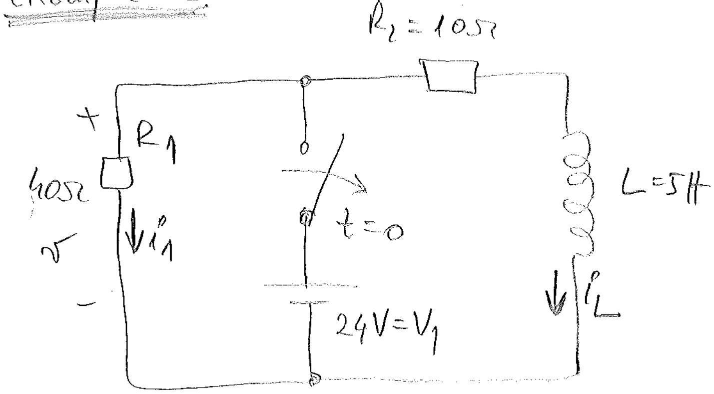
$v(t) = \underbrace{v(0)}_{=9V} \cdot e^{-\frac{t}{RC}}$

$$\boxed{v(t) = 9 \cdot e^{-\frac{t}{RC}}}$$

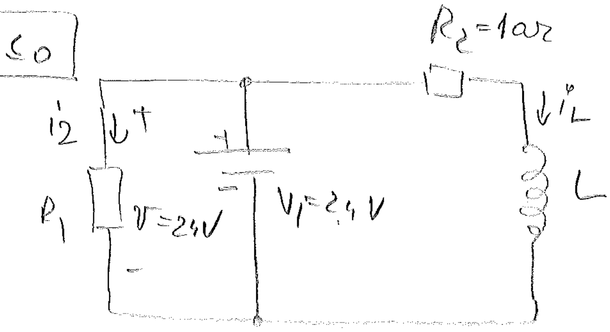
$RC = \tau = 6 \times 10 \cdot 10^{-6} = 60\,\mu sec.$



Example 2:



$\boxed{t \le 0}$



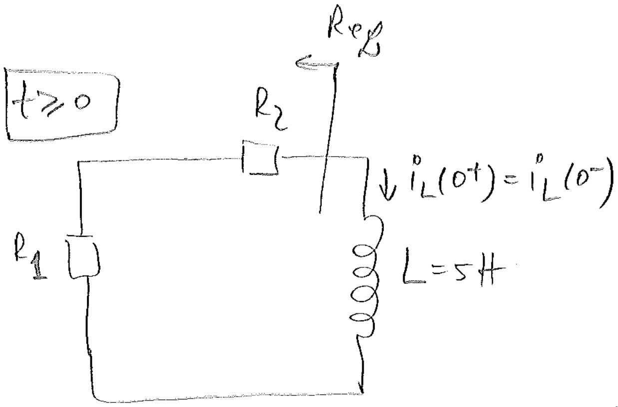
Also assume this has been like this forever long. This is a DC circuit with voltage across L zero (L is just a short-circuit for DC current; it's just a wire!). So:

$$i_L(0^-) = \frac{V_1}{R_2} = \frac{24V}{10} = 2.4A.$$

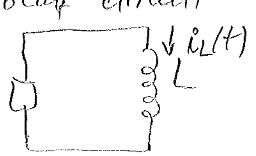
Continuity condition: $i_L(0^-) = i_L(0^+)$

$\boxed{\text{Crucial observation} \mid 2}$ Current through an inductor cannot change abruptly (all of a sudden)! It takes time to change the magnetic flux. (picture it or visualize it as something that is going thru the coil...)

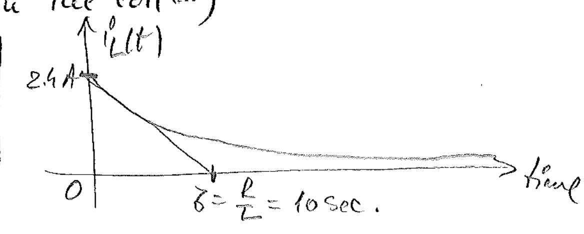
$\boxed{t \ge 0}$



$\Rightarrow$ equivalent circuit

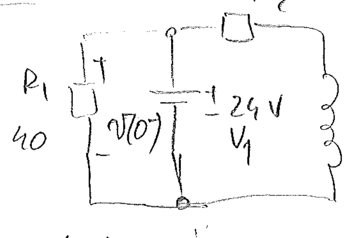


$(10'') \Rightarrow \boxed{i_L(t) = i_L(0^+) \cdot e^{-\frac{R}{L}t}}$



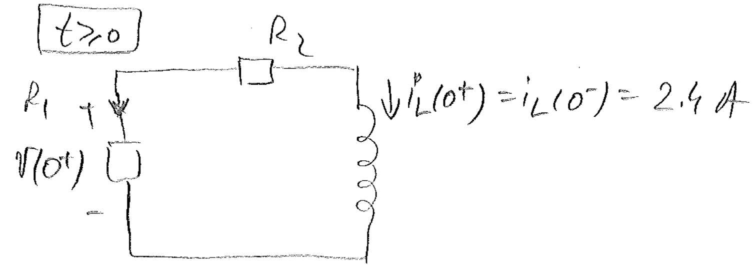
· Let's say we are asked about $v(0^-)$, $v(0^+)$ across R_1.

$\boxed{t \le 0}$



$v(0^-) = V_1 = 24V.$

$\boxed{t \ge 0}$



$v(0^+) = R_1 \cdot (-i_L(0^+)) = -40 \times 2.4\ V = -96\ V$ (!)

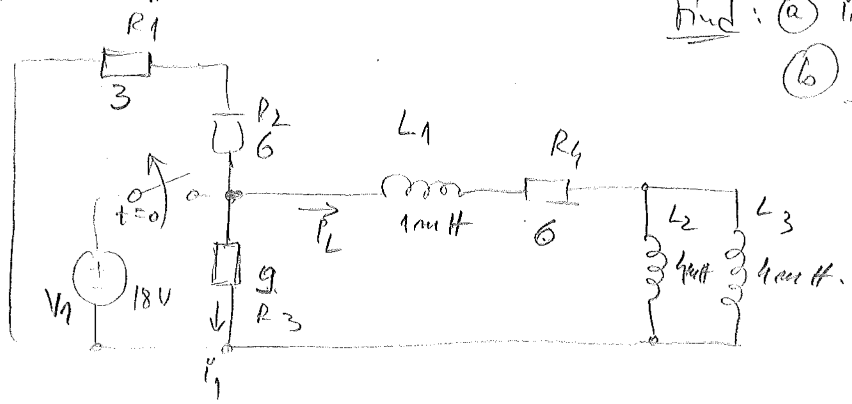
▸ the inductance is the "culprit" for this!

$\boxed{\text{Example 8.4. pp. 241}}$

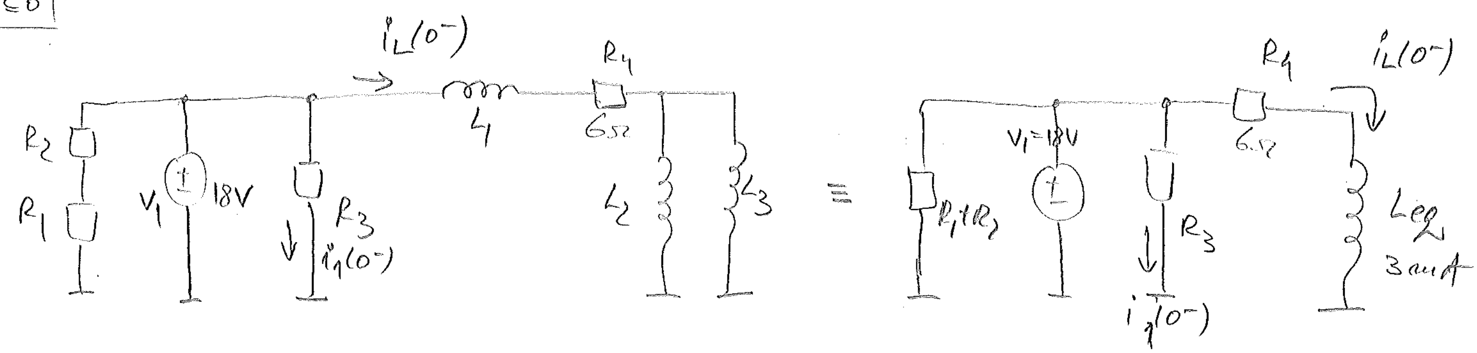
Find: (a) $i_1(0^-)$, $i_L(0^-)$

(b) for $t \ge 0$, $i_1 = i_1(t) = ?$

$i_L = i_L(t) = ?$



(a) $\boxed{t < 0}$



$L_{eq} = L_1 + L_2 \parallel L_3 =$

$= 1 + \frac{4 \times 4}{8} = 3\ mH$

Note: we cannot simplify more than this because we look for i_1, i_L!

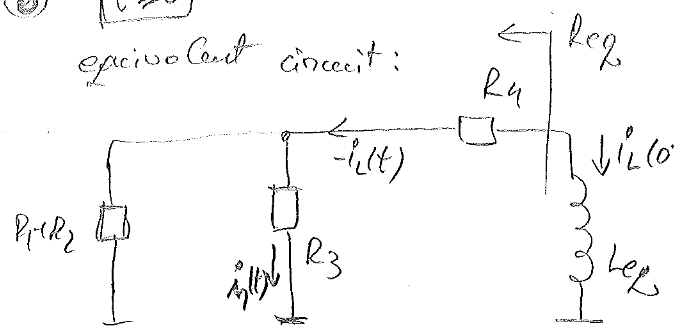
So, $i_1(0^-) = \frac{V_1}{R_3} = \frac{18V}{9\Omega} = 2A$

$i_L(0^-) = \frac{V_1}{R_4} = \frac{18V}{6\Omega} = 3A.$

assuming all transients had died out! L is short circuit.

(b) $\boxed{t \geq 0}$

equivalent circuit:



$i_L(0^+) = i_L(0^-)$ Recall the "crucial observation #2"

⇕

$$i_L(t) = i_L(0^+) \cdot e^{-\frac{t}{\tau}} \quad \tau \stackrel{\wedge}{=} \frac{L_{eq}}{R_{eq}}$$

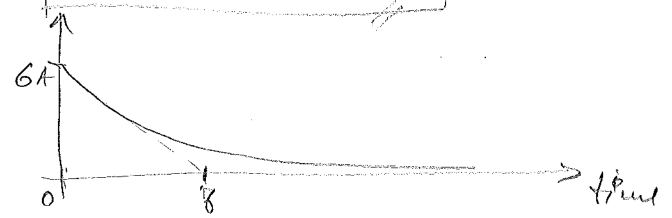
$$\boxed{i_L(0^+) \cdot e^{-\frac{R_{eq} t}{L_{eq}}} = i_L(t)}$$

$i_L(0^+)$: 6A

$L_{eq} = 3\,mH$

$R_{eq} = 6 + \underbrace{9 \| 9}_{4.5} = 10.5\,\Omega$

$$\boxed{i_L(t) = 6 \cdot e^{-\frac{10.5}{3 \cdot 10^{-3}} \cdot t}} \quad , t \geq 0$$



$i_1(t) = ?$

Use current division:
(because L "acts" as source of energy and supplies the current $-i_L(t)$!)

$$i_1(t) = \frac{R_1+R_2}{(R_1+R_2)+R_3} \times \left[-i_L(t)\right] = -\frac{9}{18} \cdot i_L(t) = \boxed{-\frac{3}{2} e^{-\frac{10.5}{3 \cdot 10^{-3}} t} = i_1(t)}$$