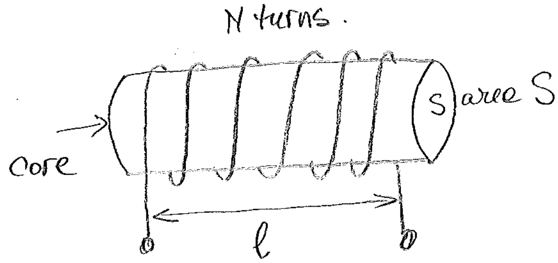
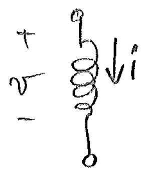
Inductors

(a) Inductance

Inductance represents the ability of a circuit element to produce magnetic flux linkage in response to current.

- Circuit elements designed for such function are called **inductors**





inductor

Symbol for inductance

- inductor realization = a coil of insulated wire wound around a core.
- Sending a current down the wire => creates magnetic field in the core, hence a magnetic flux ϕ

(0) $\boxed{\lambda = N\phi} = \psi$ is called the **magnetic linkage** expressed in **weber-turns**
other notation

- The rate at which λ varies with the applied current is denoted as L and is called **self-inductance** or simply **inductance** of the coil.

$$\boxed{L \triangleq \frac{d\lambda}{di}} \quad (1)$$

[H] Henry

- From basic magnetism, recall that inductance depends on:
  - core material
  - physical dimensions:

$$\boxed{L = \mu \cdot \frac{N^2 S}{l}} \quad (2)$$

permeability of the core material

For vacuum: $\mu_0 = 4\pi \cdot 10^{-7} \left[\frac{H}{m}\right]$

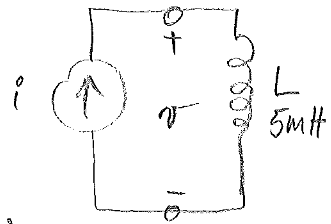
- capacitors: $\boxed{i = \frac{dq}{dt} = C\frac{dv}{dt}} \Leftarrow \boxed{C = \frac{dq}{dv}}$
- inductors: $\boxed{v = \frac{d\lambda}{dt} = L\frac{di}{dt}} \Leftarrow \boxed{L = \frac{d\lambda}{di}}$

$\lambda = \int v\,dt$

(b) The v–i characteristic

$i \to (\lambda \to \phi) \to v.$

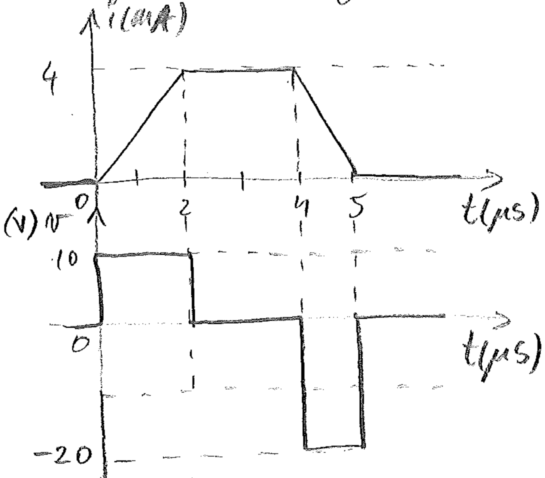
Suppose current i is increased by di => flux linkage is increased by $L\,di$



$$\boxed{d\lambda = L\,di} \overset{(0)}{=} N\,d\phi \quad (1)$$

By Faraday's law, the change in flux linkage induces a voltage

$$\boxed{v = \frac{d\lambda}{dt} = N\frac{d\phi}{dt}} \quad (2)$$



Therefore:

$$\boxed{v = L\frac{di}{dt}} \quad (3) \qquad \boxed{v(t) = L\cdot\frac{di(t)}{dt}} \quad (3')$$

inductance is said to perform the operation of current differentiation!

Observations
- inductance voltage depends on the rate of change of the current, not on the current per-se.
- To produce voltage across an inductance, the applied current must change!
- if current is kept const., no voltage will be induced.

(c) The i–v characteristic

Turn around the equation (3) to get the current carried by an inductance in response to ~~an~~ applied voltage:

$$\boxed{i(t) = \frac{1}{L}\int_0^t v(\xi)\,d\xi + i(0)} \quad (4)$$

inductance current at $t=0$

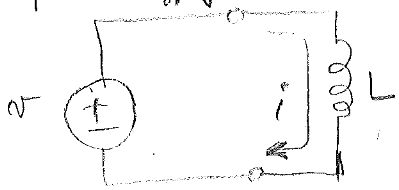
Seen like this, an inductance performs the operation of voltage integration!

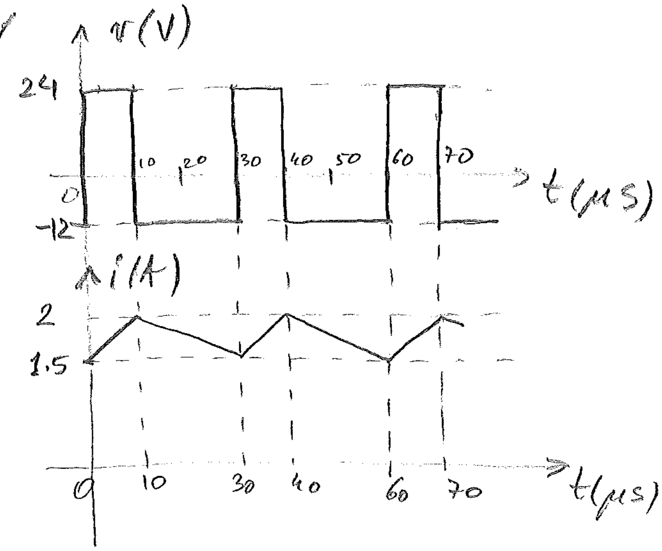
Particular case: voltage is constant: $v(t) = V$

$$\boxed{i(t) = \frac{V}{L}\cdot t + i(0)} \quad (4')$$

Forcing a constant voltage yields a linear current or a current ramp!

Example: inductance used in a switching power supply.





Given the wave forms: what is $L = ?$

$L = 480\,\mu H$

$i(0) = 1.5\,A$

$i(10\mu s) = 2\,A$

$i(t) = \frac{V}{L}t + i(0)$; $t = 10\mu s \Rightarrow 2 = \frac{24}{L}\cdot 10\cdot 10^{-6} + 1.5$

$0.5\cdot L = 240\cdot 10^{-6}$

$L = 480\cdot 10^{-6}\,H = \boxed{480\,\mu H = L}$

d) Inductive energy

- The process of establishing magnetic flux inside an inductor involves an expenditure of energy! This is found by integrating power:

$$p = v\cdot i = i\cdot\left(L\cdot\frac{di}{dt}\right)$$

variable change $j = i(\xi)$

$$\boxed{w(t) = \int_0^t p(\xi)\,d\xi = \int_0^t i\cdot L\cdot\underbrace{\left(\frac{di}{d\xi}\right)\cdot d\xi}_{=\underbrace{i'(\xi)d\xi}_{=dj}} = \int_0^{i(t)} L\,j\,dj} \quad (5)$$

- For a linear inductance: $\boxed{w(t) = \frac{1}{2}L\,i^2(t)}$ (6)

- Inductive energy is stored in the form of potential energy in the magnetic field inside the core!. inductance is the ability of a device to store energy in the form of moving charge, or current!

HW: Parallel & Serial C's and L's.