Ch3 Part 3

1

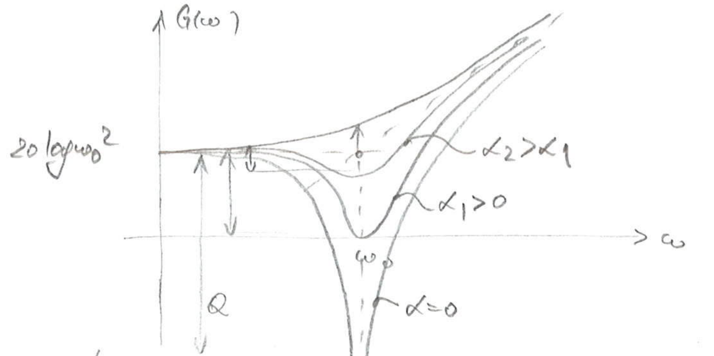
- $\boxed{H(s) = s^2 + 2\alpha s + \omega_0^2}$

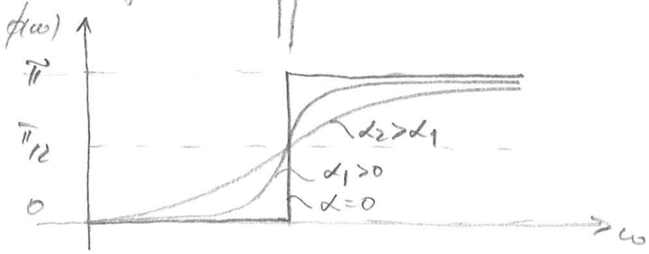
$H(j\omega) = (\omega_0^2 - \omega^2) + 2j\omega\alpha = Re(H(j\omega)) + j\,Im(H(j\omega))$

asymptotes

Gain: $G(\omega) = |H(j\omega)|_{dB} = 20\log\sqrt{(\omega_0^2-\omega^2)^2 + 4\alpha^2\omega^2} = \begin{cases} 20\cdot\log\omega_0^2, & \omega\to 0,\ \omega \ll \omega_0 \\ 20\cdot\log\omega^2, & \omega\to\infty,\ \omega\gg\omega_0 \end{cases}$

Phase: $\phi(\omega) = \arg(H(j\omega)) = \operatorname{arctg}\dfrac{2\alpha\omega}{\omega_0^2-\omega^2} = \begin{cases} 0, & \omega\to 0,\ \omega\ll\omega_0 \\ \pi/2, & \omega=\omega_0 \\ \pi - \underbrace{\operatorname{arctg}\frac{2\alpha\omega}{\omega^2}}_{=0} \to \pi, & \omega\gg\omega_0,\ \omega\to\infty. \end{cases}$





Quality factor $\boxed{Q = \dfrac{\omega_0}{2\alpha}}$

$Q = \dfrac{|H(0)|}{|H(\omega_0)|} = \dfrac{\omega_0^2}{2\alpha\omega_0} = \dfrac{\omega_0}{2\alpha}.$

- Teoreme constantelor de timp?

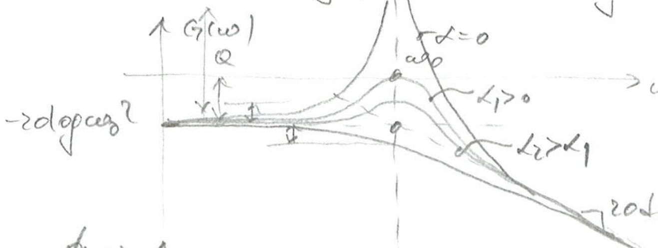
Rep in circuit will give me the pos. of zero or pole!

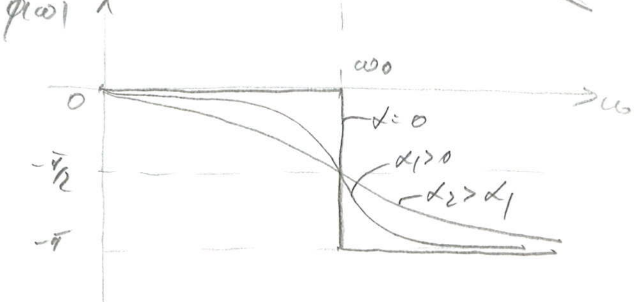
Known as "open-circuit time constant method" or "zero-value time constant technique"

- $\boxed{H(s) = \dfrac{1}{s^2+2\alpha s+\omega_0^2}} \Rightarrow H(j\omega) = \dfrac{1}{(\omega_0^2-\omega^2) + j2\alpha\omega}$

Gain: $G(\omega) = -20\cdot\log\sqrt{(\omega_0^2-\omega^2) + 4\alpha^2\omega^2} = \begin{cases} -20\log\omega_0^2, & \omega\to 0,\ \omega\ll\omega_0 \\ -20\log\omega^2, & \omega\to\infty,\ \omega\gg\omega_0 \end{cases}$

Phase: $\phi(\omega) = -\arg(H(j\omega)) = -\operatorname{arctg}\dfrac{2\alpha\omega}{\omega_0^2-\omega^2} = \begin{cases} 0, & \omega\to 0 \\ -\pi/2, & \omega=\omega_0 \\ -\pi, & \omega\to\infty. \end{cases}$





Quality factor $\boxed{Q = \dfrac{\omega_0}{2\alpha}}$

$Q = \dfrac{|H(\omega_0)|}{|H(0)|} = \dfrac{\frac{1}{2\alpha\omega_0}}{\frac{1}{\omega_0^2}} = \dfrac{\omega_0^2}{2\alpha\omega_0} = \dfrac{\omega_0}{2\alpha}$

Obs: The textbook uses this notation:

$$H(s) = \frac{N(s)}{\left(\frac{s}{\omega_0}\right)^2 + 2\xi\cdot\left(\frac{s}{\omega_0}\right) + 1} = \frac{\overset{\text{const}}{\omega_0^2}\cdot N(s)}{s^2 + \underbrace{2\xi\cdot\omega_0}_{2\alpha_i}\cdot s + \omega_0^2} \triangleq N'(s)$$

with $\boxed{\alpha_i = \xi\,\omega_0}$ notation our discussion so far is easier to understand

Depending on the $\boxed{N(s)}$, we can have various filters:

- Low-pass response $\boxed{LP}$
- High-pass $\boxed{HP}$
- Band-pass $\boxed{BP}$
- Notch $\boxed{N}$
- All-pass $\boxed{AP}$

$$H_{LP}(j\omega) = \frac{1}{1-\left(\frac{\omega}{\omega_0}\right)^2 + j\cdot\frac{\omega}{\omega_0}\cdot\frac{1}{Q}}$$

$$H_{HP}(j\omega) = \frac{-\left(\frac{\omega}{\omega_0}\right)^2}{1-\left(\frac{\omega}{\omega_0}\right)^2 + j\cdot\frac{\omega}{\omega_0}\cdot\frac{1}{Q}}$$

$$H_{BP}(j\omega) = \frac{j\frac{\omega}{\omega_0}\cdot\frac{1}{Q}}{1-\left(\frac{\omega}{\omega_0}\right)^2 + j\cdot\frac{\omega}{\omega_0}\cdot\frac{1}{Q}}$$

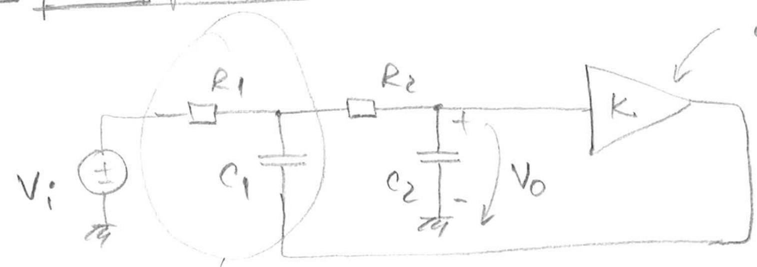
$$H_{N}(j\omega) = \frac{1-\left(\frac{\omega}{\omega_0}\right)^2}{1-\left(\frac{\omega}{\omega_0}\right)^2 + j\cdot\frac{\omega}{\omega_0}\cdot\frac{1}{Q}}$$

$$H_{AP}(j\omega) = \frac{1-\left(\frac{\omega}{\omega_0}\right)^2 - j\cdot\frac{\omega}{\omega_0}\cdot\frac{1}{Q}}{1-\left(\frac{\omega}{\omega_0}\right)^2 + j\cdot\frac{\omega}{\omega_0}\cdot\frac{1}{Q}}$$

These are standard forms!

$$\boxed{H_{LP} + H_{HP} = 1 - H_{BP} = H_N}$$

3.5 KRC filters



controls the magnitude of ω_0

$\omega_0 = \sqrt{\omega_1 \omega_2}$

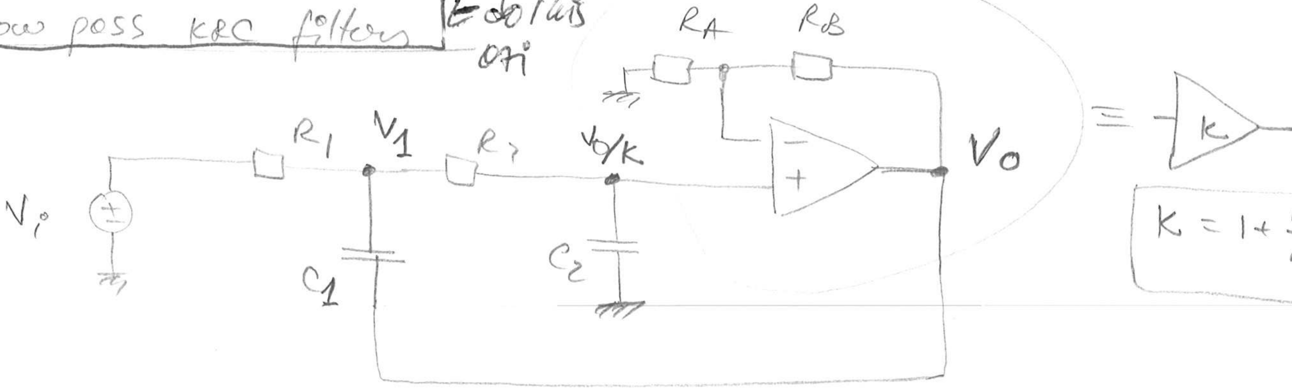
it increases Q!

$$H_1(s) = \frac{\frac{1}{sC_1}}{R_1 + \frac{1}{sC_1}} = \frac{1}{sR_1C_1 + 1} \overset{s \to j\omega}{\Rightarrow} \frac{1}{j\omega R_1 C_1 + 1} \xrightarrow{\omega \to \infty} \frac{1}{j\frac{\omega}{\omega_1}}$$

$$H(s) \xrightarrow{\omega \to \infty} \frac{1}{j\frac{\omega}{\omega_1}} \cdot \frac{1}{j\frac{\omega}{\omega_2}} = -\frac{1}{(\omega/\omega_0)^2}, \quad \omega_0 = \sqrt{\omega_1 \omega_2}$$

Near $\frac{\omega}{\omega_0} \to 1$ the positive feedback realized by K will control the amplitude!

Low pass KRC filters ← do this on?



$$\boxed{K = 1 + \frac{R_B}{R_A}}$$

$$\left\{ \begin{aligned} & V_o = K \cdot \frac{1}{sR_2C_2 + 1} \cdot V_1 \\ & KCL: \frac{V_i - V_1}{R_1} + \frac{V_o/K - V_1}{R_2} + \frac{V_o - V_1}{\frac{1}{sC_1}} \end{aligned} \right| \Rightarrow$$

$$\Rightarrow H(s) = \frac{V_o}{V_i} = \frac{K}{R_1C_1R_2C_2 s^2 + [(1-K)R_1C_1 + R_1C_2 + R_2C_2] s + 1}$$

Use standard form: $s \to j\omega$, $\boxed{H(j\omega) = H_{0LP} H_{LP}(j\omega)}$

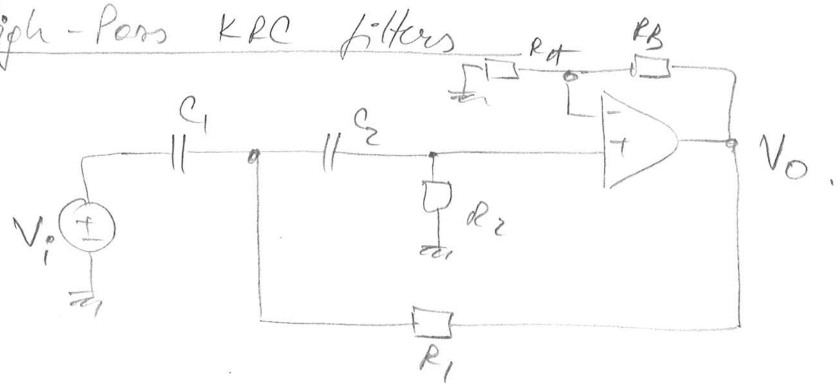
where:

$$\begin{cases} H_{OL8} = K \\ \omega_0 = \dfrac{1}{\sqrt{R_1C_1R_2C_2}} \\ Q = \dfrac{1}{(1-K)\sqrt{R_1C_1/R_2C_2} + \sqrt{R_1C_2/R_2C_1} + \sqrt{R_2C_2/R_1C_1}} \end{cases}$$

Use $\begin{cases} R_1 = R_2 = R \\ C_1 = C_2 = C \end{cases}$ commonly!

$K = 3 - \dfrac{1}{Q}$, Design for $Q=5$, $K=?$.

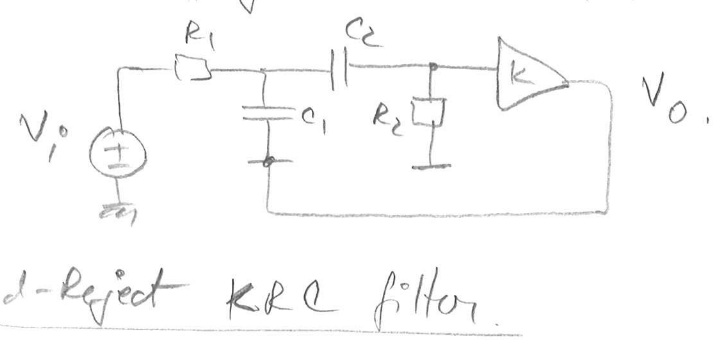
(b) High-Pass KRC filters



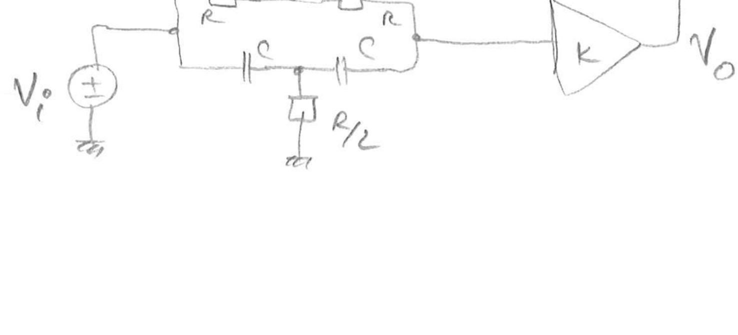
R's and C's are swapped!

(c) Band-Pass KRC filter

How would you connect R_1C_1 R_2C_2?



(d) Band-Reject KRC filter

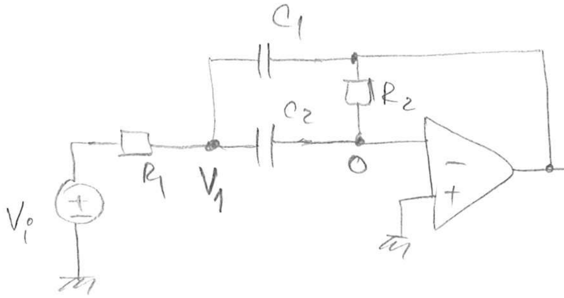


3.6 Multiple-feedback filters

Utilize more than one feedback path!

Some of the most popular Op Amp filters.

(a) Band-Pass



$$\frac{V_0 - 0}{R_2} = -\frac{V_1 - 0}{\frac{1}{sC_1}} \Rightarrow V_0 = -sR_2C_2 \cdot V_1$$

KCL: $$\frac{V_i - V_1}{R_1} + \frac{V_0 - V_1}{\frac{1}{sC_1}} + \frac{0 - V_1}{\frac{1}{sC_2}} = 0.$$

$\Rightarrow$

$\Rightarrow$ $s \to j\omega$

$$H(j\omega) = \frac{V_0}{V_i} = \frac{-j\omega R_2 C_2}{1 - \omega^2 R_1 C_1 R_2 C_2 + j\omega R_1 (C_1 + C_2)}$$ BP

Standard form: $$\boxed{H(j\omega) = H_{0BP} \cdot H_{BP}(j\omega)}$$

where:

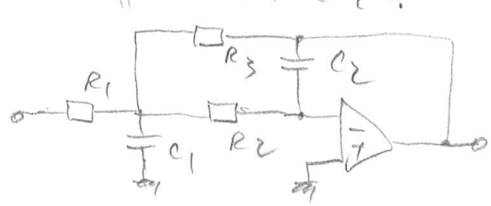
$$\omega_0 = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}$$

$$Q = \frac{\sqrt{\frac{R_2}{R_1}}}{\sqrt{\frac{C_1}{C_2}} + \sqrt{\frac{C_2}{C_1}}}$$

$$H_{0BP} = -\frac{\frac{R_2}{R_1}}{1 + \frac{C_1}{C_2}}$$

(b) Low-Pass

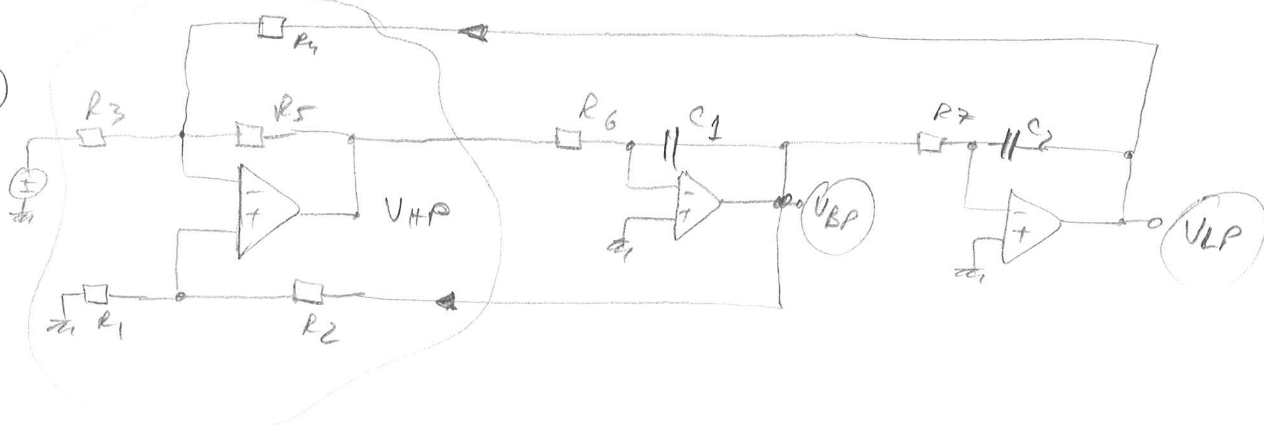
How would you shuffle R_1C_1 R_2C_2?



(c) Notch filters

3.7 (a) State variables (SV) Filters.
(b) Biquad Filters
} use more Op Amps.

(a)



(b) Biquad

(c) Notch filter