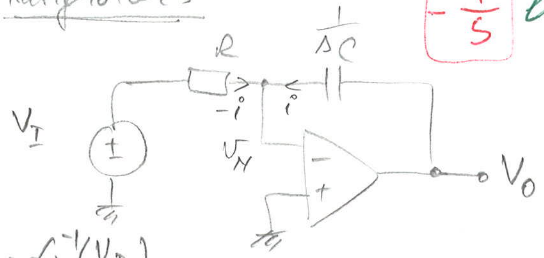
Ch. 3 Part 2

Finish stability discussion of last time.

3.2 First-order active filters ← mention only this

- Derivators → Exercise! $\boxed{s}$
- integrators $\boxed{-\frac{1}{s}}$

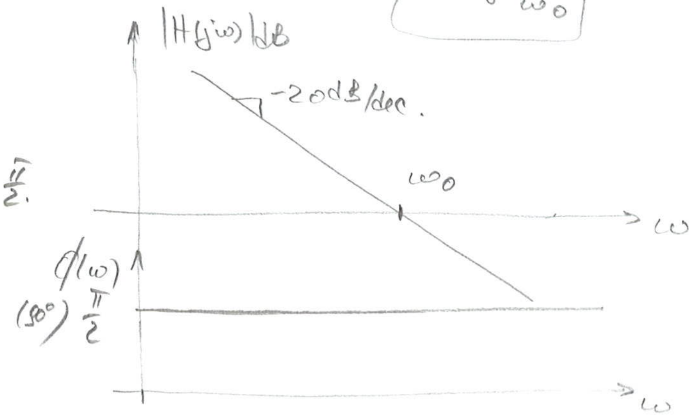


$\frac{V_O}{\frac{1}{sC}} = -\frac{V_I}{R} \Rightarrow \boxed{H(s) = \frac{V_O}{V_I} = -\frac{1}{sRC}} \Rightarrow$ inverting integrator

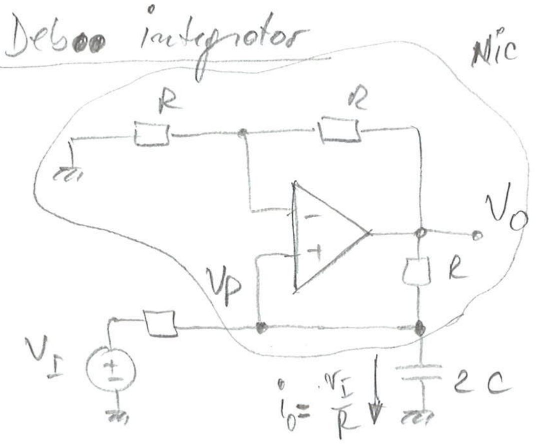
$\Rightarrow \boxed{H(j\omega) = -\frac{1}{j\frac{\omega}{\omega_0}}}$, $\omega_0 = \frac{1}{RC}$

$v_I = \mathcal{L}^{-1}(V_I)$

$\phi(\omega) = \pi - \arctg \frac{\omega/\omega_0}{0} = \pi - \frac{\pi}{2} = \frac{\pi}{2}$.



- Deboo integrator $\boxed{\frac{1}{s}}$



$H(s) = \frac{1}{sRC}$

noninverting!

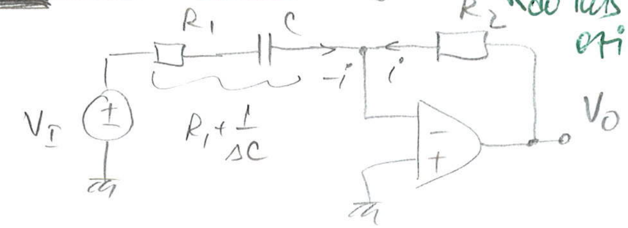
- Low-pass filter with gain → Exercise! $\boxed{\frac{1}{s+p_1}}$

High-pass filter with gain

do this on

HP

Question: how do you expect the transfer function to be? $\boxed{\frac{s}{s+p_1}}$



$$\frac{V_O}{R_2} = -\frac{V_I}{R_1+\frac{1}{sC}} \Rightarrow H(s) = \frac{V_O}{V_I} = -\frac{R_2}{R_1+\frac{1}{sC}} = -\frac{R_2}{\frac{1+sR_1C}{sC}} = -\frac{R_2}{R_1}\cdot\frac{R_1C\cdot s}{1+R_1Cs}$$

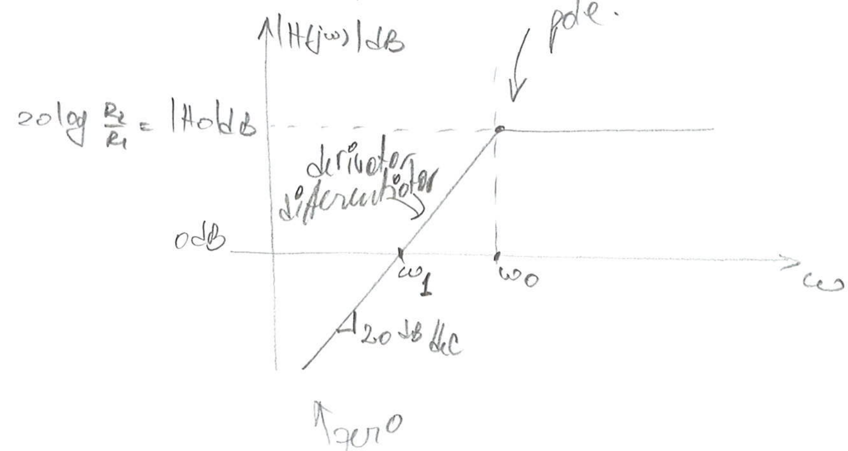
$$\boxed{H(j\omega) = H_0\cdot\frac{j\omega/\omega_0}{1+j\cdot\omega/\omega_0}}$$

← zero in the origin

← pole at $s = -\frac{1}{R_1C}$

$$\omega_0 = \frac{1}{R_1C}$$

$$H_0 = -\frac{R_2}{R_1}$$



$$\boxed{H(j\omega) = -\frac{R_2}{R_1}\cdot\frac{j\frac{\omega}{\omega_0}}{1+j\frac{\omega}{\omega_0}}}$$

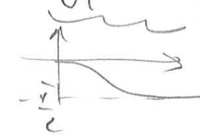
$$|H(j\omega)| = \frac{\left|\frac{R_2}{R_1}\right|\cdot\sqrt{\left(\frac{\omega}{\omega_0}\right)^2}}{\sqrt{1+\left(\frac{\omega}{\omega_0}\right)^2}}$$

derivator character

Gain: $G(\omega) = |H(j\omega)|_{dB}$

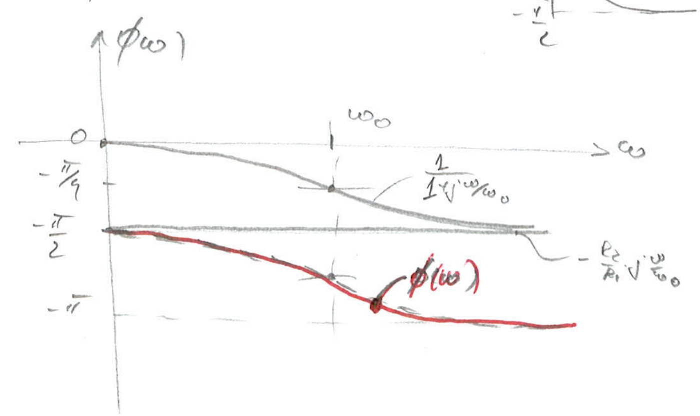
$$= 20\log\frac{R_2}{R_1} + 20\log\frac{\omega}{\omega_0} - 20\log\sqrt{1+\left(\frac{\omega}{\omega_0}\right)^2} = \begin{cases} 20\log\frac{R_2}{R_1}\cdot\frac{\omega}{\omega_0}, & \omega\to 0 \\ 20\log\frac{R_2}{R_1}, & \omega\to\infty. \end{cases}$$

Phase: $\phi(\omega) = \arg\left(-\frac{R_2}{R_1}\right) + \arg\left(j\frac{\omega}{\omega_0}\right) - \arg\left(1+j\frac{\omega}{\omega_0}\right) = \underbrace{\arg\left(-\frac{R_2}{R_1}j\frac{\omega}{\omega_0}\right)}_{=-\frac{\pi}{2}} - \arg\left(1+j\frac{\omega}{\omega_0}\right)$



$$= -\pi + \underbrace{\mathrm{arctg}\left(\frac{\omega/\omega_0}{0}\right)}_{=\pi/2} - \mathrm{arctg}\,\frac{\omega/\omega_0}{1}$$

$$= -\frac{\pi}{2} - \mathrm{arctg}\,\frac{\omega}{\omega_0}$$

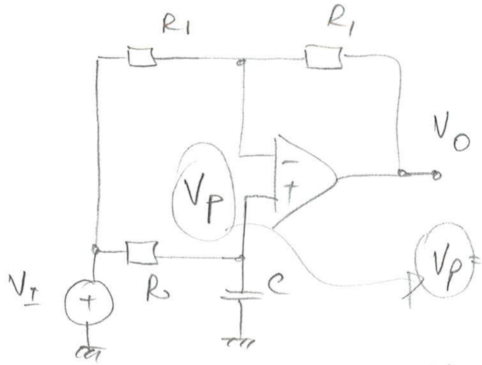


- Wideband pass filter - Exercise.

$$\boxed{\frac{s}{(s+p_1)(s+p_2)}}$$

obtained from HP and adding one more pole at higher frequencies.

- Phase Shifters



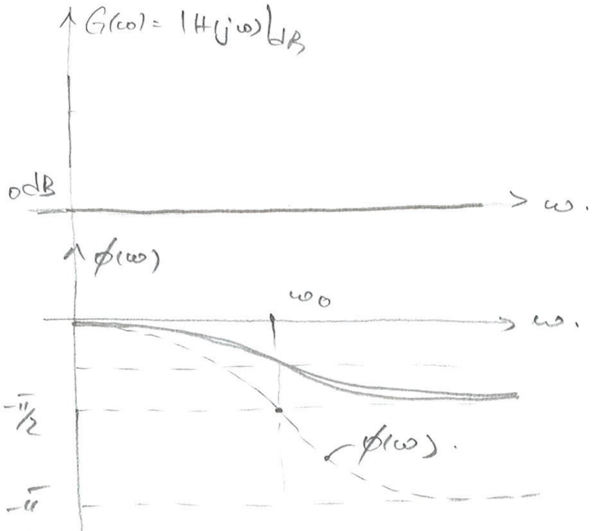
$$V_P = \frac{\frac{1}{sC}}{R+\frac{1}{sC}} V_I = \frac{1}{1+sRC} \cdot V_I$$

$$V_O = \left(1+\frac{R_1}{R_1}\right) \cdot V_P - \frac{R_1}{R_1} V_I =$$

$$= 2 \cdot \frac{1}{1+sRC} V_I - V_I = \frac{1-sRC}{1+sRC} V_I$$

$$H(s) = \frac{V_O}{V_I} = \frac{1-sRC}{1+sRC}$$

$$\boxed{H(j\omega) = \frac{1-j\frac{\omega}{\omega_0}}{1+j\frac{\omega}{\omega_0}}}, \quad \omega_0 = \frac{1}{RC}.$$



$$\phi(\omega) = -\operatorname{arctg}\frac{\omega}{\omega_0} - \operatorname{arctg}\frac{\omega}{\omega_0}$$

3.3 Audio filter applications

- Phono preamplifier.
- Tape preamplifier
- Active tone control
- Graphic Equalizer.

3.4 Standard Second-order Responses

Standard form: $$H(s) = \frac{N(s)}{\left(\frac{s}{\omega_0}\right)^2 + 2\xi\left(\frac{s}{\omega_0}\right) + 1}$$

(Last time: $H(s) = s^2 + 2\alpha s + \omega_0^2$ $\boxed{\alpha = \xi \cdot \omega_0.}$)

Poles: $\boxed{p_{1,2} = \omega_0\left(-\xi \pm \sqrt{\xi^2 - 1}\right)}$

$\xi \triangleq$ damping ratio

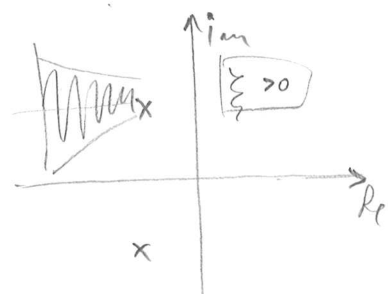
- $\xi > 1$ => poles are real => Natural response are two decaying exponentials. $\boxed{\text{is overdamped}}$

- $0 < \xi < 1$ => poles are complex and conjugate in the LHP of the complex plane.

$$p_{12} = -\xi\omega_0 \pm j\omega_0\sqrt{1-\xi^2}$$

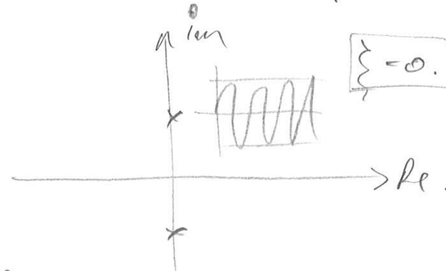
Natural response $\boxed{\text{is underdamped}}$

$$x_0(t) = 2|A| \cdot e^{-\xi\omega_0 t} \cdot \cos\left(\omega_0\sqrt{1-\xi^2}\,t + \measuredangle A\right)$$



- $\xi = 0$ => poles on an imaginary axis

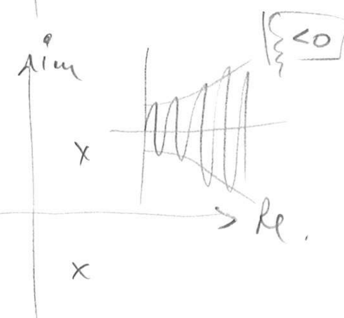
Natural response $\boxed{\text{is undamped}}$ => oscillations occur.



- $\xi < 0$ => poles are complex and conjugate in the RHP

Natural response is diverging.

$$x_0(t) = 2|A|\, e^{-\xi\omega_0 t} \cdot \cos\left(\omega_0\sqrt{1-\xi^2}\cdot t + \measuredangle A\right)$$



$$\Rightarrow e^{\overset{<0}{-\xi}\omega_0 t}$$

$s \to j\omega$

$$\boxed{H(j\omega) = \frac{N(j\omega)}{1-\left(\frac{\omega}{\omega_0}\right)^2 + j\frac{1}{Q}\left(\frac{\omega}{\omega_0}\right)}}$$

with new notation: $\boxed{Q = \frac{1}{2\xi}}$

$Q = \frac{\omega_0}{2\alpha} = \frac{1}{2\xi}$

My notation $\boxed{\alpha = \xi\omega_0}$

Use this to obtain the discussion in the textbook!

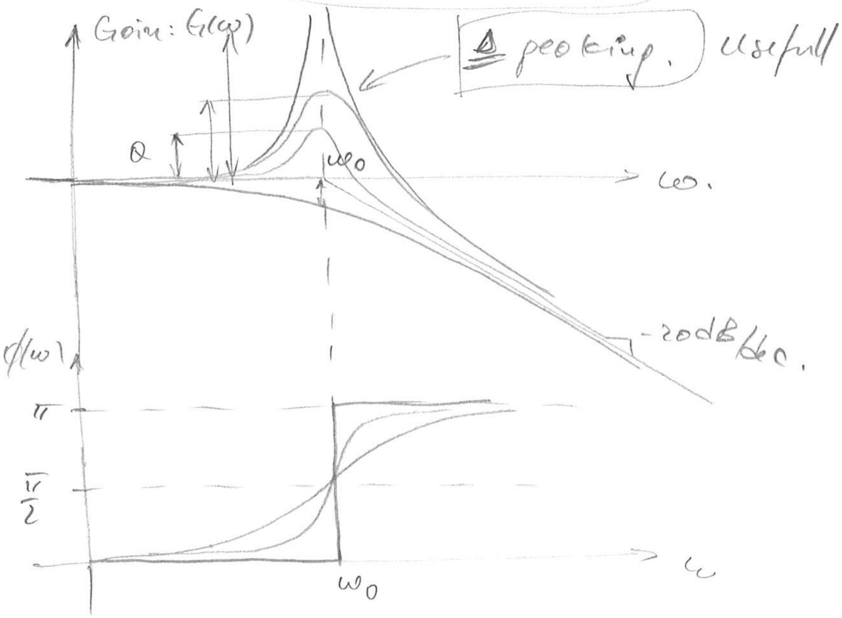
- The Lowpass (LP) Response $\boxed{H_{LP}}$

$$H_{LP}(j\omega) = H_{0LP}\cdot H_{LP}(j\omega) \qquad H_{0LP} \triangleq \text{dc. gain.}$$

$$\boxed{H_{LP}(j\omega) = \frac{1}{1-\left(\frac{\omega}{\omega_0}\right)^2 + j\frac{\omega}{\omega_0}\cdot\frac{1}{Q}}}$$

Using last time notation

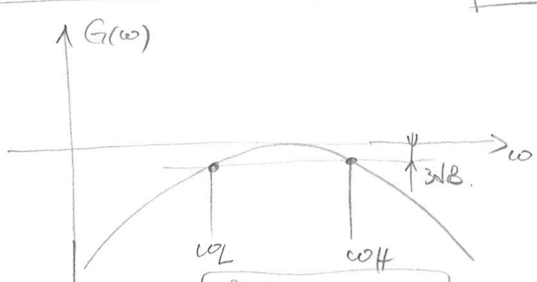
$$H(s) = \frac{1}{s^2+2\alpha s+\omega_0^2}$$



- The High-Pass (HP) Response $\boxed{H_{HP}}$

$$H(s) = \frac{s^2}{s^2+2\alpha s+\omega_0^2} \qquad H_{HP}(j\omega) = \frac{-\left(\frac{\omega}{\omega_0}\right)^2}{1-\left(\frac{\omega}{\omega_0}\right)^2 + j\frac{1}{Q}\cdot\frac{\omega}{\omega_0}}$$

- The Band-Pass (BP) Response $\boxed{H_{BP}}$



$$H_{BP}(j\omega) = \frac{j\frac{\omega}{\omega_0}\cdot\frac{1}{Q}}{1-\left(\frac{\omega}{\omega_0}\right)^2 + j\frac{1}{Q}\cdot\frac{\omega}{\omega_0}}$$

$\boxed{BW = \omega_H - \omega_L}$

– The Notch Response $\boxed{H_N}$

$$H_N(j\omega) = \frac{1 - \left(\frac{\omega}{\omega_0}\right)^2}{1 - \left(\frac{\omega}{\omega_0}\right)^2 + j\frac{1}{Q}\cdot\frac{\omega}{\omega_0}} = \boxed{H_{LP} + H_{HP} = 1 - H_{BP} = H_N}$$

– The all-pass response $\boxed{H_{AP}}$

$$H_{AP}(j\omega) = \frac{1 - \left(\frac{\omega}{\omega_0}\right)^2 - j\frac{\omega}{\omega_0}\cdot\frac{1}{Q}}{1 - \left(\frac{\omega}{\omega_0}\right)^2 - j\frac{\omega}{\omega_0}\cdot\frac{1}{Q}}$$