Ch 3 | Active filters | part 1

Note: add Spectre from Cadence and hspice from Synopsis to the list.

The transfer function $\boxed{H(s)}$

$$\mathcal{L}^{-1}(X_i(s)) = x_i(t) \xrightarrow{\text{IN}} \boxed{H(s)} \xrightarrow{\text{OUT}} x_o(t) = \mathcal{L}^{-1}(X_o(s))$$

$$x_i(t) \downarrow X_i(s) \qquad x_o(t) \downarrow X_o(s)$$

$$\boxed{X_o(s) = H(s)\cdot X_i(s)}$$

$$\boxed{x_o(t) = \mathcal{L}^{-1}(H(s)\cdot X_i(s))}$$

General form: (1) $$\boxed{H(s) = \frac{k(s-z_1)(s-z_2)\dots}{(s-p_1)(s-p_2)\dots} = \frac{N(s)}{D(s)}}$$

$z_1, z_2, \dots$ Zeroes ? $\quad p_1, p_2, \dots$ Poles. $\}$ can be real or can be complex, written as:

Think of it as being the "personality" of the system.

$\boxed{k \triangleq H_0}$ $\triangleq$ scaling factor.

Complex frequency $\boxed{s = \sigma + j\omega}$ $\begin{cases} \omega = 0 \Rightarrow s = \sigma \text{ (real)} \\ \omega \neq 0 \Rightarrow s = \sigma \pm j\omega \end{cases}$

where $\begin{cases} \sigma \triangleq \text{Neper freq. } [Np/s] \\ \omega \triangleq \text{angular freq. } [rad/s] \end{cases}$

$\boxed{\omega = 2\pi f}$

$$\left.\begin{matrix} s \to j\omega \Leftrightarrow H(s) \to H(j\omega) \\ \omega = 2\pi f \end{matrix}\right\} \Rightarrow H(jf)$$

- Magnitude, absolute value; it's convenient to work with ω: $H(j\omega) = \dfrac{k(j\omega - z_1)(j\omega - z_2)\dots}{(j\omega - p_1)(j\omega - p_2)\dots}$

$$|H(j\omega)| = \frac{|k|\cdot\sqrt{\omega^2+z_1^2}\cdot\sqrt{\omega^2+z_2^2}\dots}{\sqrt{\omega^2+p_1^2}\cdot\sqrt{\omega^2+p_2^2}\dots}$$

ATTN! When a zero z_k or pole p_k is complex we have its conjugate too: $p_k = \sigma_k \pm j\omega_k$

- Phase, angle, argument:

$$\phi(\omega) = \angle H(j\omega) = \measuredangle H(j\omega) = \arg H(j\omega) =$$
$$= \arg(k) + \arg(j\omega - z_1) + \arg(j\omega - z_2) + \dots$$
$$- \arg(j\omega - p_1) - \arg(j\omega - p_2) - \dots$$

Question (!)

- Gain $\triangleq$ the magnitude in decibels:

introduced due to energy considerations!

$$G(\omega) = |H(j\omega)|_{dB} = \mathbf{10\cdot\log|H(j\omega)|^2} = 20\cdot\log\frac{|k|\cdot\sqrt{\omega^2+z_1^2}\sqrt{\omega^2+z_2^2}\dots}{\sqrt{\omega^2+p_1^2}\cdot\sqrt{\omega^2+p_2^2}\dots}$$

$$= 20\log|k| + 20\log\sqrt{\omega^2+z_1^2} + 20\log\sqrt{\omega^2+z_2^2} + \dots$$
$$- 20\log\sqrt{\omega^2+p_1^2} - 20\log\sqrt{\omega^2+p_2^2}\dots$$

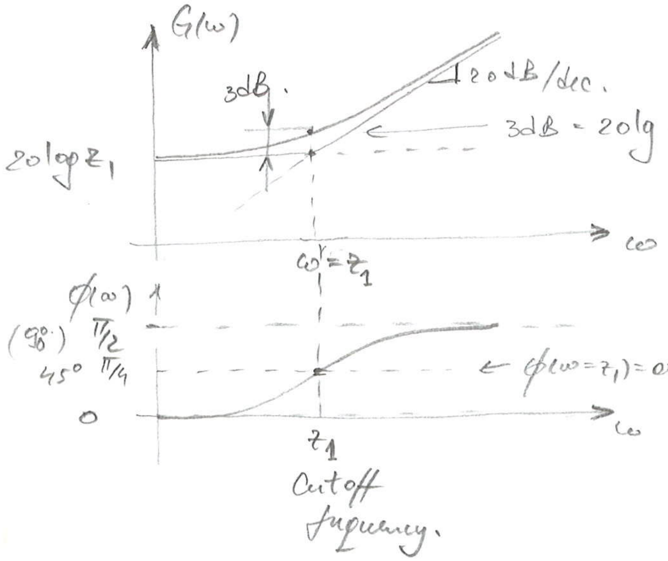
Examples:

(a) $\boxed{H(s) = s + z_1}$ => $H(j\omega) = j\omega + z_1$ $\boxed{z_1 > 0}$

$$G(\omega) = 20\cdot\log\sqrt{\omega^2+z_1^2} = \begin{cases} 20\cdot\log z_1, & \omega \ll z_1 \quad \text{(constant)}. \\ 20\cdot\log\omega, & \omega \gg z_1 \end{cases}$$

approximate using asymptots

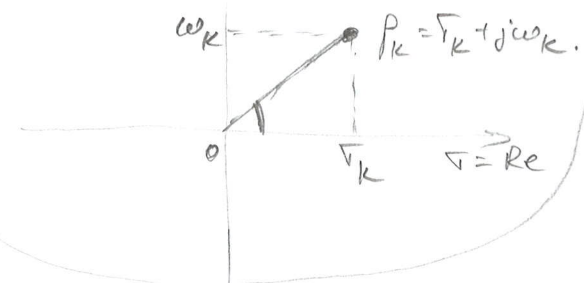
$$\phi(\omega) = \arg H(j\omega) = \operatorname{arctg}\frac{\omega}{z_1} \qquad \begin{cases} \phi(\omega)\to 0, & \omega\to 0 \\ \phi(\omega)\to\frac{\pi}{2}\ (90^\circ), & \omega\to\infty \end{cases}$$



⊖ No omractionoi oici rõ ii confusionori!

Roots (i.e. zeroes and poles) are conviniently visualized in the complex plane (s-plane).

$p = \sigma + j\omega$, $Im = j\omega$



(a.1.) Now, if $\boxed{z_1 < 0}$ => $G(\omega) = 20\cdot\log\sqrt{\omega^2+|z_1|^2}$ remains the same

But the phase changes!

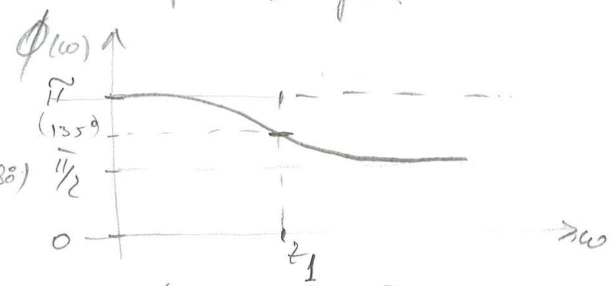
$\boxed{H(s) = s - z_1}$

$H(j\omega) = j\omega - |z_1| =$

$= j\,Im(H(j\omega)) + Re(H(j\omega))$

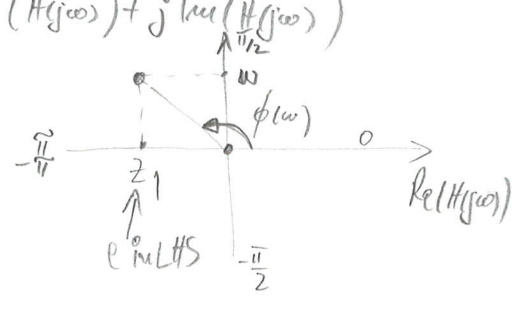
$= j\omega + (-|z_1|)$

$$\begin{cases} Im(H(j\omega)) = \omega \\ Re(H(j\omega)) = -|z_1| \end{cases}$$



because $\phi(\omega) = \operatorname{arctg}\frac{-z_1}{\omega} = \pi - \operatorname{arctg}\frac{z_1}{\omega}$

A way to remember this: keep in mind $H(j\omega)$ is complex too: $H(j\omega) = Re(H(j\omega)) + j\,Im(H(j\omega))$



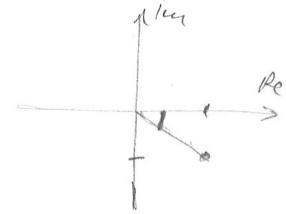
$\boxed{\text{Note}}$ that in this case the actual pole/zero is real and positive which when plugged into eq (1) gives $s-(+z_1) = s-(+\sigma_1)$.
The zero is in RHP of the s-plane, complex plane
Do not confuse angle of zero in the s-plane with angle of $H(j\omega)$ in the s-plane of it!

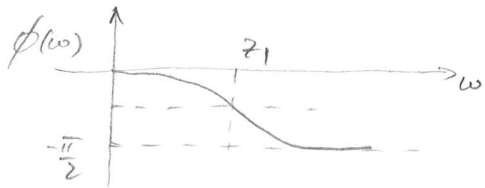
important!

(a.2) If we worked with negative frequency:

$H(s) = -s + z_1 \quad (z_1 > 0)$

$H(j\omega) = -j\omega + z_1 = j\,\mathrm{Im}(H(j\omega)) + \mathrm{Re}(H(j\omega)) = j(-\omega) + z_1$





(a.3) $H(s) = -s - z_1 \quad (z_1 < 0)$

Exersize!

(b) <u>Derivator</u>

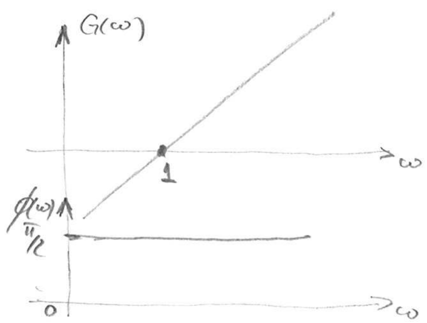
$H(s) = s$

$H(j\omega) = j\omega$

$G(\omega) = 20\log\omega$

$\phi(j\omega) = \arg(H(j\omega))$

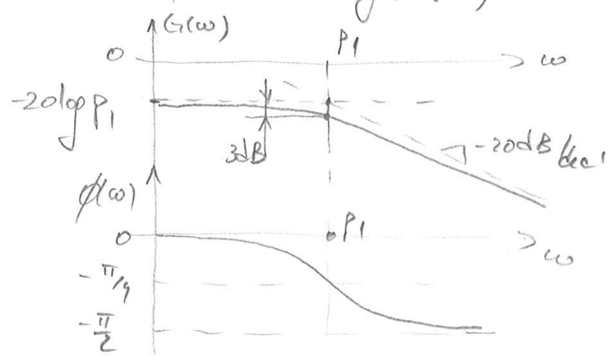
$= \mathrm{arctg}\,\frac{\omega}{0} = \frac{\pi}{2}$



(c) $H(s) = \dfrac{1}{s + p_1}$

$H(j\omega) = \dfrac{1}{j\omega + p_1} \Rightarrow G(\omega) = -20\log\sqrt{\omega^2 + p_1^2} \overset{\text{asymptotes}}{=} \begin{cases} -20\log\omega, & \omega \gg p_1 \\ -20\log p_1, & \omega \ll p_1 \end{cases}$

$\phi(j\omega) = -\arg(H(j\omega)) = -\mathrm{arctg}\,\frac{\omega}{p_1} \quad \begin{cases} \phi(\omega) \to 0, & \omega \to 0 \\ \phi(\omega) \to -\frac{\pi}{2}, & \omega \to \infty \end{cases}$



Exercize Cases: !

C1: $H(s) = \dfrac{1}{s - p_1}$

C2: $H(s) = \dfrac{1}{-s + p_1}$

C3: $H(s) = \dfrac{1}{-s - p_1}$

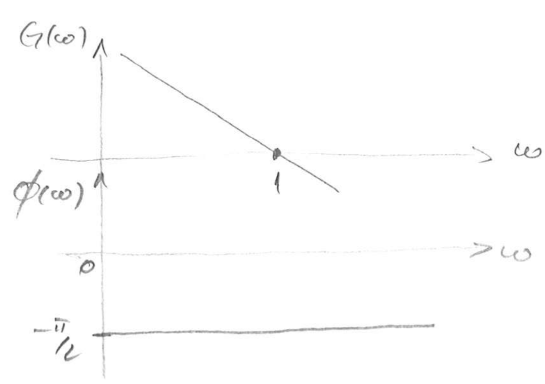
(d) <u>Integrator</u>

$H(s) = \dfrac{1}{s}$

$H(j\omega) = \dfrac{1}{j\omega}$

$G(\omega) = -20\log(\omega)$

$\phi(\omega) = -\arg(j\omega) = -\frac{\pi}{2}$



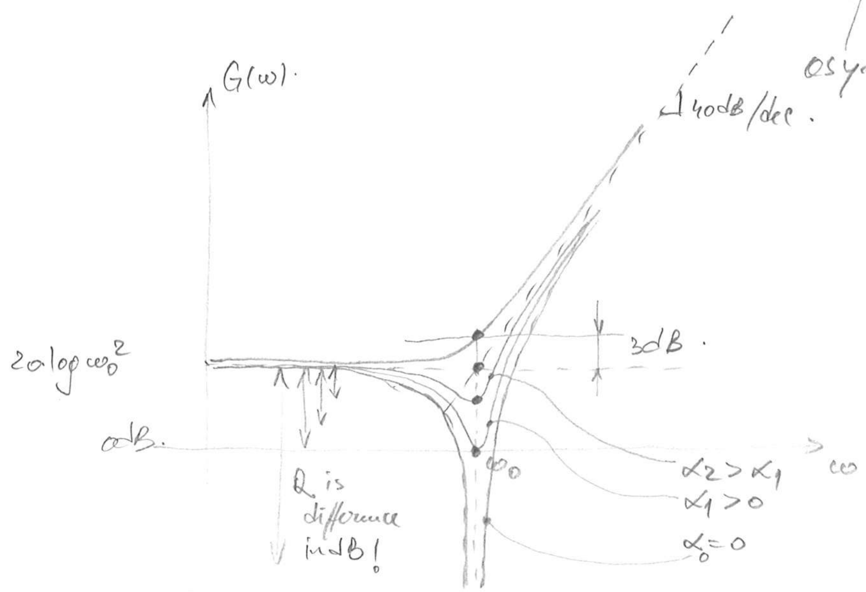
(e) $\boxed{\text{Quadratic terms}}$ ($\boxed{\text{Notch}}$ filters) ← do this ori.

$\boxed{H(s) = s^2 + 2\alpha s + \omega_0^2}$

$$H(j\omega) = -\omega^2 + 2j\omega\alpha + \omega_0^2 = (\omega_0^2 - \omega^2) + 2j\omega\alpha = Re(H(j\omega)) + j\cdot Im(H(j\omega))$$

$$G(\omega) = 20\cdot\log\sqrt{(\omega_0^2-\omega^2)^2 + 4\alpha^2\omega^2} = \begin{cases} 20\cdot\log\omega_0^2, & \omega \ll \omega_0 \\ 20\cdot\log\omega^2, & \omega \gg \omega_0 \end{cases}$$

asymptotes.



$|H(\omega\to 0)| = \omega_0^2$

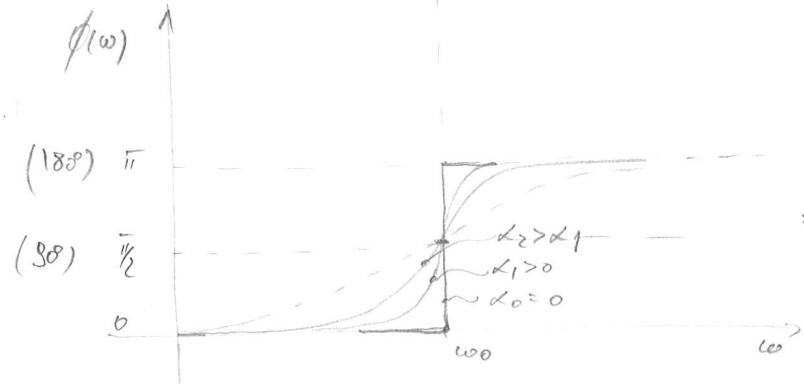
$|H(\omega=\omega_0)| = 2\alpha\omega_0$

Quality factor $\boxed{Q = \frac{\omega_0}{2\alpha}} = \frac{|H(0)|}{|H(\omega_0)|}$

$\alpha = 0 \Rightarrow$ oscillator

$G(\omega_0) = 20\cdot\log(2\alpha\omega_0)$

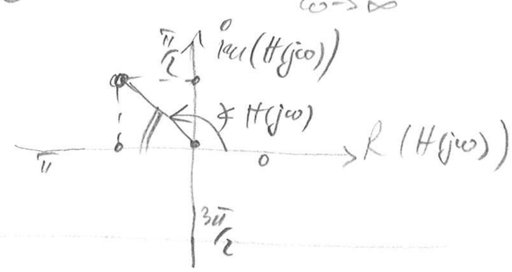
When $G(\omega_0) = 0 \Rightarrow \boxed{\omega_0 = \frac{1}{2\alpha}}$



Phase:

$$\phi(\omega) = \arg(H(j\omega)) =$$

$$= \begin{cases} \operatorname{arctg}\frac{2\alpha\omega}{\omega_0^2} \to 0, & \omega \ll \omega_0,\ \omega\to 0 \\ \frac{\pi}{2}, & \omega = \omega_0 \\ \pi - \operatorname{arctg}\frac{2\alpha\omega}{\omega^2} \to \pi, & \omega \gg \omega_0,\ \omega\to\infty \end{cases}$$



Case: $\boxed{\alpha < 0} \Rightarrow H(s) = s^2 - 2|\alpha| s + \omega_0^2$

Exercise!

(f)

$$H(s) = \frac{1}{s^2 + 2\alpha s + \omega_0^2}$$

$\boxed{H(s) \text{ and Stability}}$

- A circuit is stable if it produces a bounded output in response to any bounded input.
- $\underline{\text{Check}}$: inject some energy (like a Dirac) into one or more of its reactive elements (C, L) and observe how the circuit does on its own, in the absence of any applied sources!
- The response in this case is called "natural response" because the Laplace of $\delta(t)$ is 1.

$$h(t) = \mathcal{L}^{-1}(H(s)\cdot 1)$$

- $\boxed{\text{Natural response is determined by the } \boxed{\text{poles}}\text{! Not zeroes!}}$

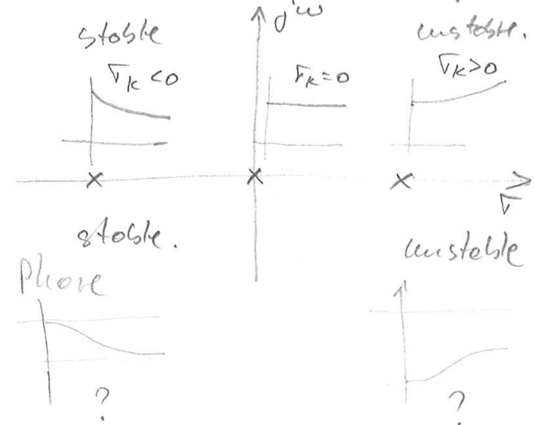
$\boxed{\text{Case 1}}$

$\boxed{\text{real}}$ pole: $\quad s = \sigma_K \pm j0 = \sigma_K = p_1$

means that $H(s)$ contains the term $\dfrac{A_K}{s-\sigma_K}$, $\quad A_K \triangleq$ residue of $H(s)$ at that pole.

where $\quad A_K = (s-\sigma_K)\cdot H(s)\Big|_{s=\sigma_K}$

and $\quad \mathcal{L}^{-1}\left(\dfrac{A_K}{s-\sigma_K}\right) = A_K \cdot e^{\sigma_K t}\cdot u(t)$



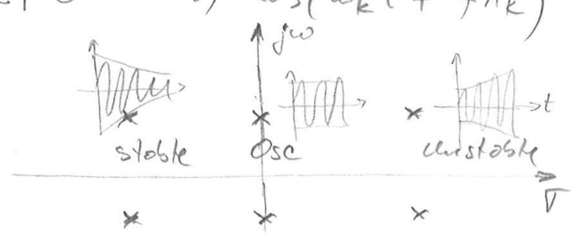
$\boxed{\text{Case 2}}$

$\boxed{\text{Complex}}$ pole pair: $\quad s = \sigma_K \pm j\omega_K$

means that $H(s)$ contains terms $\dfrac{A_K}{s-(\sigma_K+j\omega_K)}$ and $\dfrac{A_K^*}{s-(\sigma_K-j\omega_K)}$

where $\quad A_K = [s-(\sigma_K+j\omega_K)]\cdot H(s)\Big|_{s=\sigma_K+j\omega_K}$

$$\mathcal{L}^{-1}\left\{\frac{A_K}{s-(\sigma_K+j\omega_K)} + \frac{A_K^*}{s-(\sigma_K-j\omega_K)}\right\} = 2\,|A_K|\cdot e^{\sigma_K t}\cdot u(t)\cdot\cos(\omega_K t + \measuredangle A_K)$$



<u>Conclusion:</u> To be stable the system's poles have to be in the left plane of s-plane! where $\sigma < 0$.

- Passive circuits RLC meet this req.
- Circuits with active components may not!