Ch. 6. Dynamic Op Amp Limitations — PART I

ToDo: examples 6.2, 6.3, 6.4

6.1 Open-loop response – most common is the dominant-pole response.

Origins of this simplified model:

$C_C = 30pF \Rightarrow C_{eq} = 16.3nF$

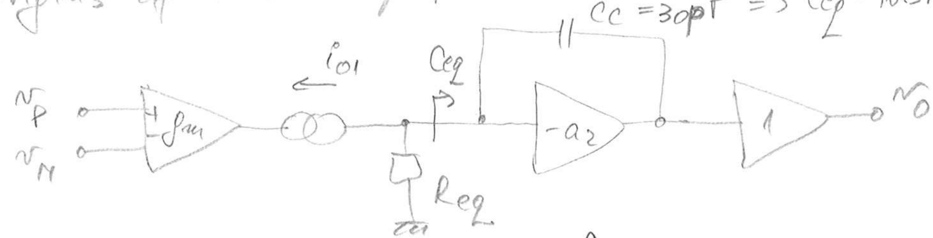


fig. 1

– Low frequencies $\Rightarrow$ C_c acts as open!

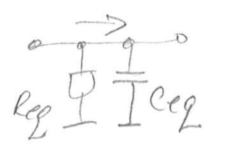
$$v_O = 1 \times (-a_2) \times (-R_{eq} \cdot i_{o1}) = g_{m1} \cdot R_{eq} \cdot a_2 (v_P - v_N)$$

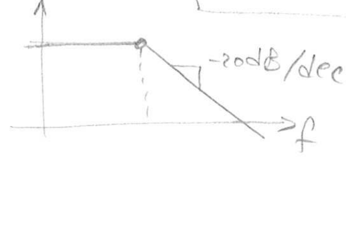
$$\boxed{a_0 = g_{m1} \cdot R_{eq} \cdot a_2} \triangleq \text{DC gain}$$

Example: μA741 $\left\{ \begin{array}{l} g_{m1} = 189 \mu A/V \\ R_{eq} = 1.95 M\Omega \\ a_2 = 544 V/V \end{array} \right. \Rightarrow a_0 \cong 200 V/mV.$

– High frequencies $\Rightarrow$ C_{eq} starts to have effect.

R_{eq}, C_{eq} = Low pass filter. $\Rightarrow$ Gain starts to roll off after $\boxed{f_b = \frac{1}{2\pi R_{eq} C_{eq}}}$





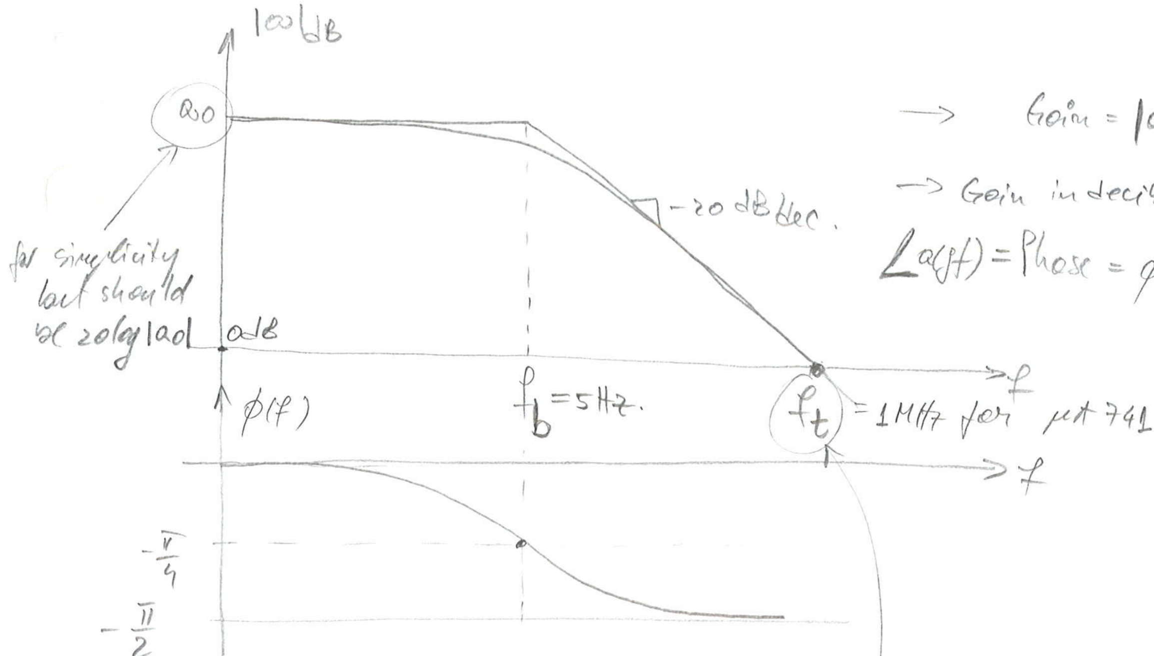
Therefore, the "dominant pole" expression for the open-loop gain/response is:

$$\boxed{a(jf) = \frac{a_0}{1 + j\, f/f_b}}$$

open-loop dc gain.

QUESTION: How does this affect the gains of Op Amp circuits (inverting/non inv, integrator, etc) and their input/output resistances?

Bode plots of this one:



→ Gain = $|a| = \dfrac{a_0}{\sqrt{1+\left(\frac{f}{f_b}\right)^2}}$

→ Gain in decibels is $20\log|a|$

$\angle a(jf) = \text{Phase} = \phi(f) = -\arctan\left(\frac{f}{f_b}\right)$

$f_b \triangleq$ open loop -3dB frequency
$\triangleq$ open loop bandwidth

$f_t \triangleq$ unity gain frequency

We want it as high as possible!

The intersection of $|a(jf)|_{dB}$ with 0dB axis:

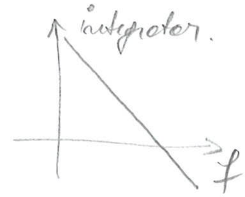
$$\frac{a_0}{\sqrt{1+\underbrace{\left(\frac{f_t}{f_b}\right)^2}_{\gg 1}}} = \underset{0dB}{1} \Rightarrow \boxed{f_t = a_0 \cdot f_b}$$

$\triangleq$ unity gain freq. or the transition freq.

For $f \gg f_b$ ⇒ the open-loop amplifier will have the <u>Gain Bandwidth Product</u>:

$$GBP \triangleq |a(jf)|_f \cdot f = f_t = \text{constant}$$

and that the Op Amp behaves as an integrator.



which is a very <u>important merit metric</u> of an Op Amp! figure

<u>Quick derivation of $\boxed{f_t}$</u>

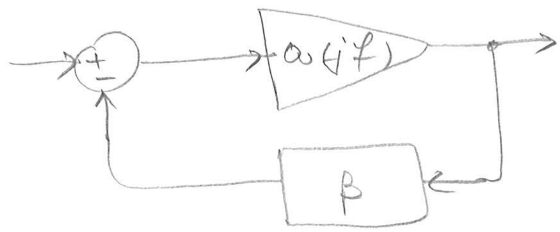
At high freq: $V_0 \cong 1 \times Z_{C_C} \cdot I_{O1} = \dfrac{1}{j2\pi f C_C} \cdot g_{m1}(v_P - v_N)$ (see fig. 1)

$= a \underset{f\to\infty}{=} \dfrac{f_t}{f} = \lim_{f\to\infty} a(jf)$ ⇒

$$\Rightarrow \boxed{f_t = \frac{g_{m1}}{2\pi C_C} = \frac{I_A}{8\pi V_T \cdot C_C}} = 1\text{MHz for } \mu A741$$

6.2 Closed-loop response

considering $\boxed{a(jf) = \dfrac{a_0}{1 + j\, f/f_b}}$ for an OpAmp.



Now, we'll have:

$$A(jf) = A_{ideal} \cdot \frac{1}{1 + \frac{1}{T(jf)}}$$

$\triangleq$ error function, has now to be

where. $\boxed{T(jf) = a(jf) \cdot \beta(jf)}$

specified in terms of
- magnitude error

$$\varepsilon_m = \left| \frac{1}{1 + \frac{1}{T(jf)}} \right| - 1$$

- phase error

$$\varepsilon_\phi = -\angle\left(1 + T(jf)\right)$$

Example ①: The noninverting amplifier

Use derived expressions from prev. chapters and replace $a \to a(jf)$

$$\Rightarrow A(jf) = \left(1 + \frac{R_2}{R_1}\right) \cdot \frac{1}{1 + \left(1 + \frac{R_2}{R_1}\right) \cdot a(jf)} = \boxed{\frac{A_0}{1 + j\frac{f}{f_B}} = A(jf)}$$

where:

$$\boxed{A_0 = \left(1 + \frac{R_2}{R_1}\right) \cdot \frac{1}{1 + \left(1 + \frac{R_2}{R_1}\right) \cdot \frac{1}{a_0}} \cong 1 + \frac{R_2}{R_1}}$$

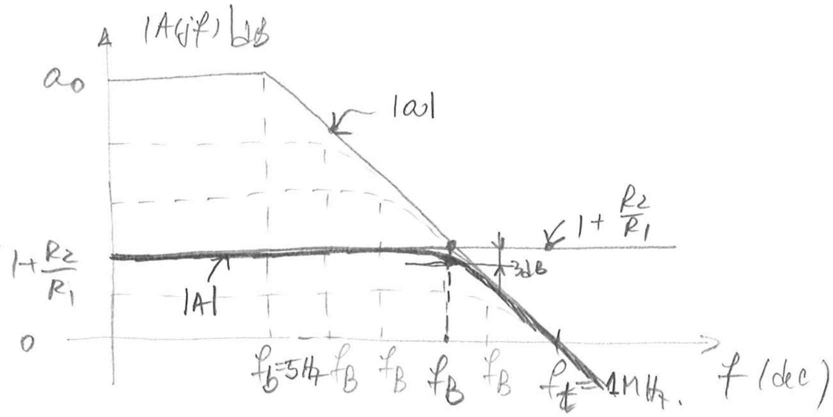
$$\boxed{f_B \triangleq f_b\left(1 + a_0 \cdot \frac{R_1}{R_1 + R_2}\right)} = f_b\,(\cancel{1} + a_0\beta) \cong$$

$$\cong \underbrace{f_b \cdot a_0}_{= f_t}\,\beta = \boxed{\beta f_t}$$

So, the noninverting amplifier has also a dominant-pole gain! $\boxed{\text{BUT}}$ the dominant pole frequency is f_B that is $\cong T_0 = a_0\beta$ times higher!

Bigger bandwidth $\boxed{\text{BUT}}$ smaller dc gain!

$f_B \triangleq$ the closed-loop bandwidth.



Conclusion: There is a bandwidth-gain tradeoff!

IDEA

$$\boxed{GBP = A_0 \cdot f_B = \boxed{f_t}} = \text{constant.}$$

The GBP remains constant after the negative feedback!!!

Same f_t as in the GBP of open-loop.
It's clear $\boxed{f_t}$ is pretty important for an OpAmp and OpAmp circuits!

Example ② | Inverting amplifier :

Exercise!

6.3 Input/output impedances

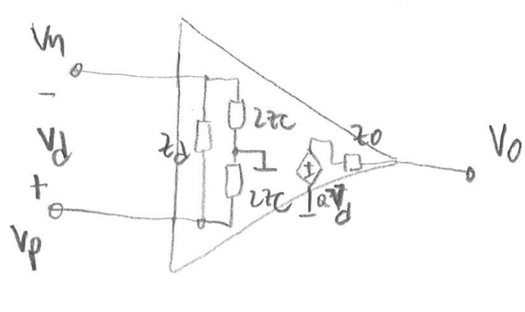
considering $\boxed{a(jf) = \frac{a_0}{1 + j\, f/f_b}}$

Recall
Negative feedback benefit:

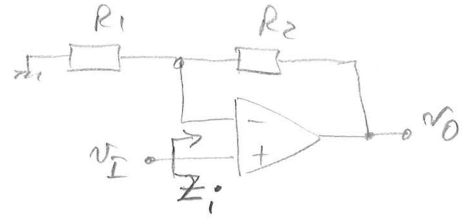
$$Z = \begin{cases} \frac{z}{1+T} & \text{for shunt configuration} \\ z(1+T) & \text{for series configuration.} \end{cases}$$

Now, $T = a(jf) \cdot \beta$ will decrease with frequency

which will make for the closed-loop amplifier to depart from ideal even more with frequency!



Case 1: Series impedances



(a) Input: $Z_d = r_d(1+T)$

dominant-pole behavior will make Z to have similar behavior! We expect.

$z \cong r_d$. open-loop.

$$\Rightarrow Z_d \cong r_d(1+T) = r_d(1+a(jf)\cdot\beta)$$

$$1+\frac{a_0\cdot\beta}{1+j\,f/f_b} = \frac{1+a_0\beta+j\frac{f}{f_b}}{1+j\frac{f}{f_b}} =$$

$f\to\infty \to 0$

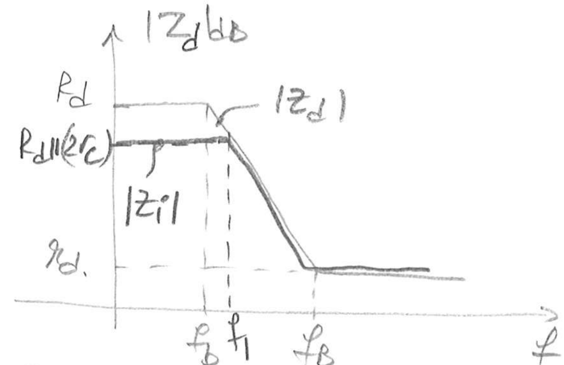
$$=(1+a_0\beta)\cdot\frac{1+j\frac{f}{f_b\cdot(1+a_0\beta)}}{1+j\cdot f/f_b} = (1+a_0\beta)\frac{1+j\cdot f/f_B}{1+j\,f/f_b}$$

$R_d \triangleq r_d\cdot(1+a_0\beta)$

$$\Rightarrow \boxed{Z_d(jf) \cong R_d\cdot\frac{1+j\,f/f_B}{1+j\cdot f/f_b}}$$

zero!

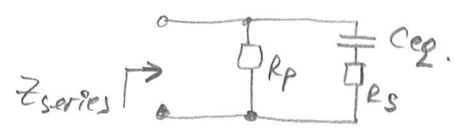
pole!



$$\boxed{Z_i(jf) = Z_d(jf)\,\|\,(2z_c)} = z_{series}$$

Conclusion: input impedance decreases with f
=> "capacitive" behavior!

Equivalent circuit:



(b) Output: Similar conclusion!

Case 2: Shunt impedances

for shunt/parallel type of negative feedback

(a) input
(b) output
→ Exercise!

Example $\boxed{6.4}$ page 271

Op Amp has: $r_d = 1\,M\Omega$; $r_c = 1\,G\Omega$; $a_0 = 10^5\ V/V$; $r_o = 100\,\Omega$; $f_t = 1\,MHz$;
Use in noninverting configuration: $R_1 = 2k$, $R_2 = 18k$ find element values in the equivalent circuit for Z_i and breakpoint frequencies of its magnitude plot.

$$\beta \cong \frac{R_1}{R_1 + R_2} = \frac{2}{2+18} = \frac{1}{10}$$

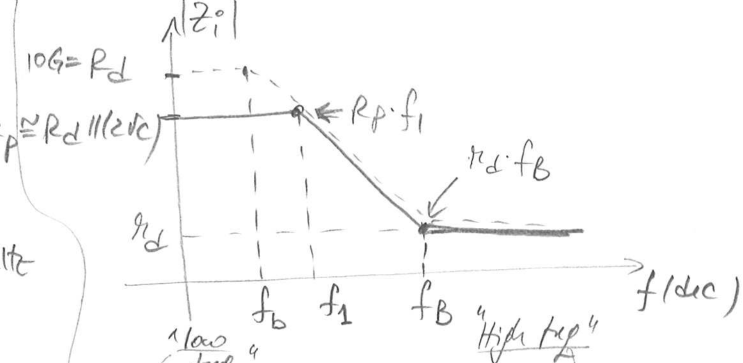
$$f_B = \beta f_t = \frac{1}{10}\cdot 10^6\ Hz = 10^5\ Hz = 100\ kHz$$

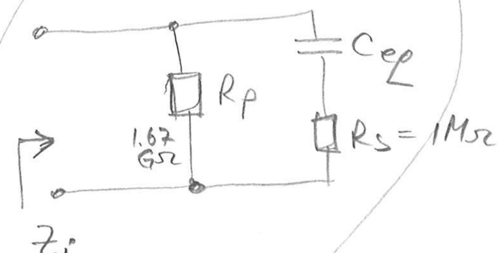
$$\boxed{R_s \underset{\uparrow (2)}{=} r_d = 1\,M\Omega}$$

$$R_d = r_d(1 + a_0\beta) = 10^6\left(1 + 10^5\cdot\frac{1}{10}\right) = 10\ G\Omega$$

$$\boxed{R_p \underset{\uparrow (1)}{=} (2r_c) \,\|\, R_d = 2\,G\Omega \,\|\, 10\,G\Omega = 1.67\,G\Omega}$$

$$C_{eq} \underset{\uparrow (3)}{=} \frac{1}{2\pi f_B r_d} = \frac{1}{2\pi\cdot 10^5\cdot 10^6} = 1.59\,pF$$





Low frequencies: $\boxed{R_p \cong (2r_c) \,\|\, R_d \cong = (2r_c) \,\|\, (r_d(1 + a_0\beta))}$ (1)

High frequencies: $\left.\begin{array}{l} R_p \| R_s \cong r_d \\ R_p \gg r_d \end{array}\right\} \Rightarrow \boxed{R_s \cong r_d}$ (2)

$$|Z_{C_{eq}}(jf_B)| = r_d \Rightarrow \boxed{C_{eq} = \frac{1}{2\pi f_B r_d}} \quad (3)$$

Finally:

$$R_p f_1 = R_s f_B \Rightarrow \boxed{f_1 = \frac{R_s f_B}{R_p} = \frac{\cancel{10^6}\cdot 10^5}{1.67\cdot \cancel{10^9}} = 60\,Hz}$$

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