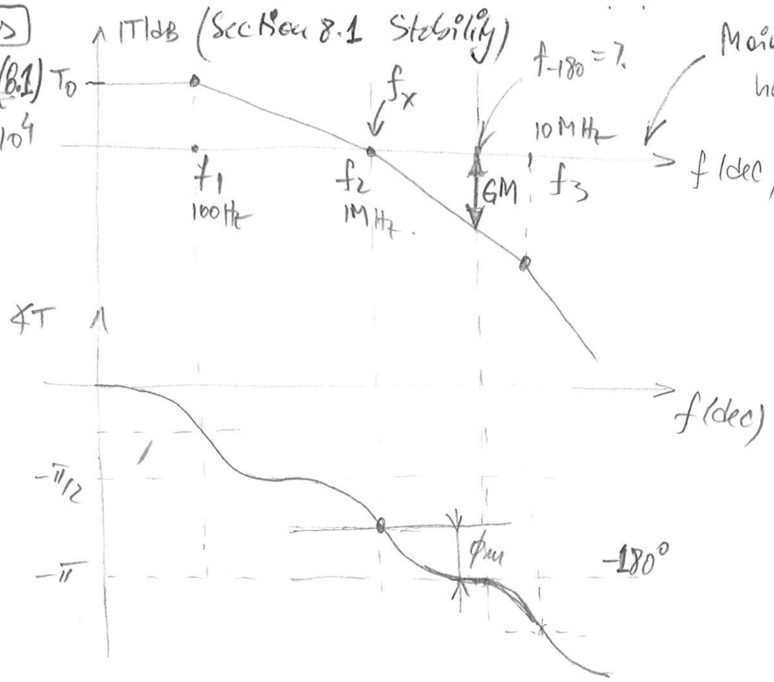
Ch8 **Examples**

Example (8.1) $T_0 = 10^4$



Main idea for $|T|$ has to have 2nd, 3rd, ... poles below zero dB line! ①

$f_1 = 100\,\text{Hz}$ $\quad T_0 = 10^4$

$f_2 = 1\,\text{MHz}$

$f_3 = 10\,\text{MHz}$

ⓐ $GM = ?$

ⓑ $\phi_m = ?$

ⓒ $T_0 = ?$ for $\phi_m = 60°$

ⓐ to find GM we need $f_{-180} = ?$ to plug into: $GM = 20 \log \frac{1}{|T(jf_{-180})|}$

where

$$|T(jf)| = \frac{T_0}{\sqrt{1+\left(\frac{f}{f_1}\right)^2}\cdot\sqrt{1+\left(\frac{f}{f_2}\right)^2}\cdot\sqrt{1+\left(\frac{f}{f_3}\right)^2}} \quad (1)$$

$$\phi_T(jf) = \measuredangle T(jf) = -\left[\arctan\left(\frac{f}{f_1}\right) + \arctan\left(\frac{f}{f_2}\right) + \arctan\left(\frac{f}{f_3}\right)\right] \quad (2)$$

1) Clearly: $f_2 < f_{-180} < f_3$

2) Start with a guess a) $f^1_{-180} = 5\,\text{MHz}$

& Use (2) to compute

$\phi_T(jf^1_{-180}) = -195.3°$ too low

b) Update guess $f^2_{-180} = 3\,\text{MHz}$, & use (2) to compute

$\phi_T(jf^2_{-180}) = -178.3°$ too small

c) Update guess $f^3_{-180} = 4\,\text{MHz}$ ...

$\boxed{f_{-180} = 3.16\,\text{MHz}}$ $\Rightarrow$ $GM = 20 \cdot \log \frac{1}{|T(jf_{-180})|} = 20 \log \frac{1}{91.04 \cdot 10^{-3}}$

$\boxed{GM = 20.82\,\text{dB}}$

ⓑ Need to find $f_x = ?$

We see that a) $f^1_x \cong f_2 = 1\,\text{MHz}$ $\Rightarrow$ Use (1) to get: $|T(jf^1_x)| \cong 0.7036$ too small

b) $f^2_x = 700\,\text{kHz}$ $\Rightarrow$ $|T(jf^2_x)| = 1.167$ too large

...

$\boxed{f_x = 784\,\text{kHz}}$ $\Rightarrow$ $\boxed{\phi_m = 47.4°}$

(c)

$\phi_m = 60°$ we want $|T(jf_{-120})| = 1$

We can find by trial and error: $\boxed{f_{-120} = 512\ kHz}$ $\Rightarrow \boxed{T_0 = 5760}$ down from $T_0 = 10^4$.

Comments: HW#7: 8.6 8.16 8.26 8.36

HW#8: 8.4 8.30 8.37

<u>Example 2</u>: (Prob. 8.11) (Section 8.2 Stability of constant-GBP op amp circuits)

An OpAmp with $a(jf) = \dfrac{10^5}{1+j\cdot f/10}$ placed in a negative feedback with

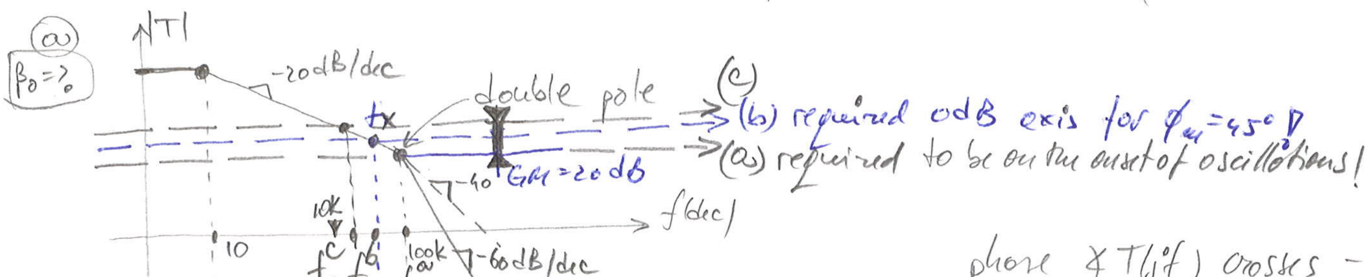
$$\beta(jf) = \frac{\beta_0}{(1+j\cdot f/10^5)^2}$$

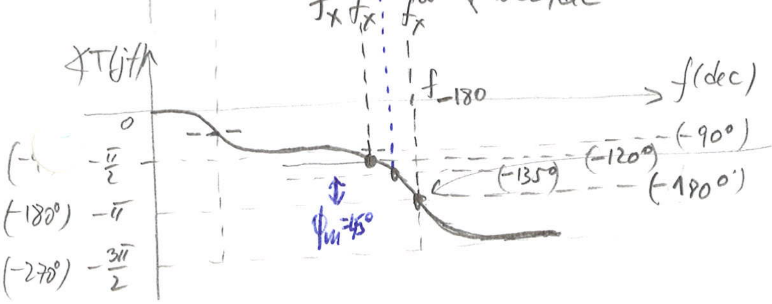
Find β_0
(a) corresponding to the onset of oscillatory behaviour.
(b) $\phi_m = 45°$
(c) $GM = 20\ dB$

$$|T(jf)| = \frac{10^5 \beta_0}{\sqrt{1+\left(\frac{f}{10}\right)^2}\cdot\left[1+\left(\frac{f}{10^5}\right)^2\right]} \quad (3)$$

$$T(jf) = a(jf)\cdot\beta(jf) = \frac{10^5\beta_0}{(1+j\cdot f/10)\underbrace{(1+j\cdot f/10^5)^2}_{\uparrow \text{double pole!}}} \quad (1)$$

$$\measuredangle T(jf) = -\left[\tan^{-1}(f/10) + \tan^{-1}(f/10^5) + \tan^{-1}(f/10^5)\right] = -\left[\tan^{-1}\left(\frac{f}{10}\right) + 2\tan^{-1}(f/10^5)\right] \quad (2)$$





phase $\measuredangle T(jf)$ crosses $-180°$ exactly at $f_{-180} \cong 100\ kHz$, the double pole freq.

where $|T| \cong \dfrac{10^5\beta_0}{10^4\cdot(1+1^2)} = 5\beta_0$

The onset of oscillations occurs for $|T| = 1 \Rightarrow \boxed{\beta_0 = \frac{1}{5}} = 0.2\ V/V$

(b) $\boxed{\beta_0 = ?}$

$\phi_M = 45^\circ \Rightarrow \boxed{f_x = f_{-135^\circ} = 41.4 \text{ kHz}}$

found by trial and error as in example 1 (Example 8.1 page 350)

$$\Rightarrow \underbrace{|T(jf)|\Big|_{f_x = 41.4\text{kHz}}}_{=1} = 20.6\,\beta_0 \Rightarrow \boxed{\beta_0 = \frac{1}{20.6}} \text{ V/V}$$

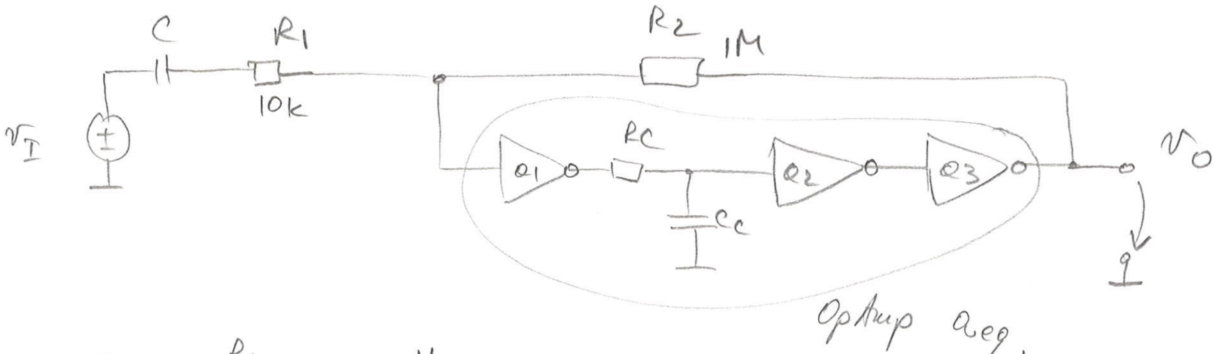
(c) $\boxed{\beta_0 = ?}$

$GM = 20\text{ dB} \Rightarrow \boxed{f_x = f_{-120^\circ} = 26.8\text{kHz}}$

$$\Rightarrow \underbrace{|T(jf)|\Big|_{f_x = 26.8\text{kHz}}}_{=1} = 34.8\,\beta_0 \Rightarrow \boxed{\beta_0 = \frac{1}{34.8}} \text{ V/V}$$

Example 3 (8.29) (Section 8.3 internal freq. compensation)

- 3 CMOS inverters cascaded to create an Op Amp.
- inverting configuration: $A_v = -100 \frac{V}{V}$



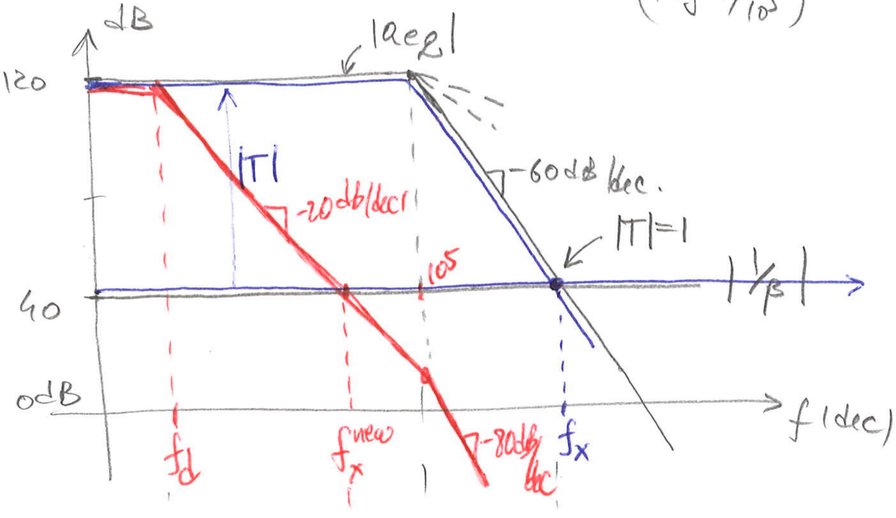
$$A_v = -\frac{R_2}{R_1} = -100 \frac{V}{V}$$

$$a_1 = a_2 = a_3 = -\frac{10^2}{1 + j\cdot f/10^5}$$

a) if $R_c = C_c = 0 \Rightarrow$ circuit is unstable

b) $R_c = ?$ $C_c = ?$ such that $\phi_M = 45^\circ$

(a) $R_c = C_c = 0 \Rightarrow a_{eq} = a_1 a_2 a_3 = -\dfrac{10^6}{(1+j\,f/10^5)^3}$



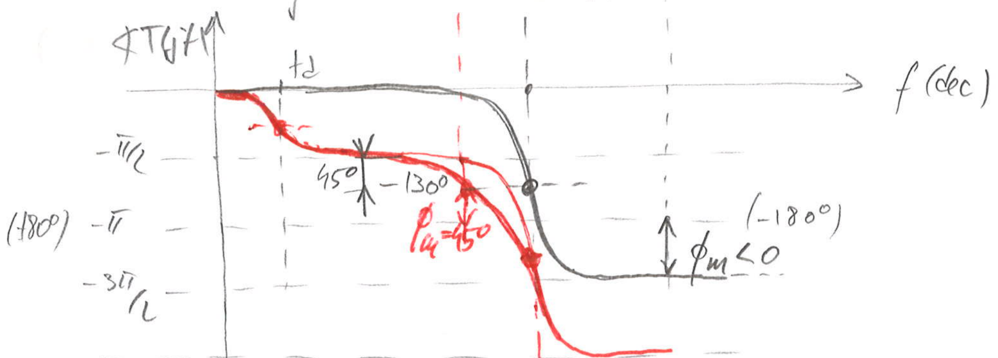
$\dfrac{1}{\beta} = 1 + \dfrac{R_2}{R_1} = 101 \cong 40\,dB$

▷ Using the rate of closure (ROC) technique, we get (page 353)

$$ROC = \left| \text{Slope}(|a_{eq}|) - \text{Slope}(|1/\beta|) \right| \Big|_{f=f_x} = |-60\,dB/dec - 0| = 60\,dB/dec$$

(Eq. 8.9) $\Rightarrow \phi_m \cong -90^\circ \Rightarrow$ unstable!

▷ Alternatively: draw $\measuredangle T(jf)$



(b) We introduce a dominant pole at f_d and want a phase margin $\phi_m = 45^\circ$

- f_d introduces (or contributes) -90° at $f = f_x^{new}$
- we want $\phi_m = 45^\circ$

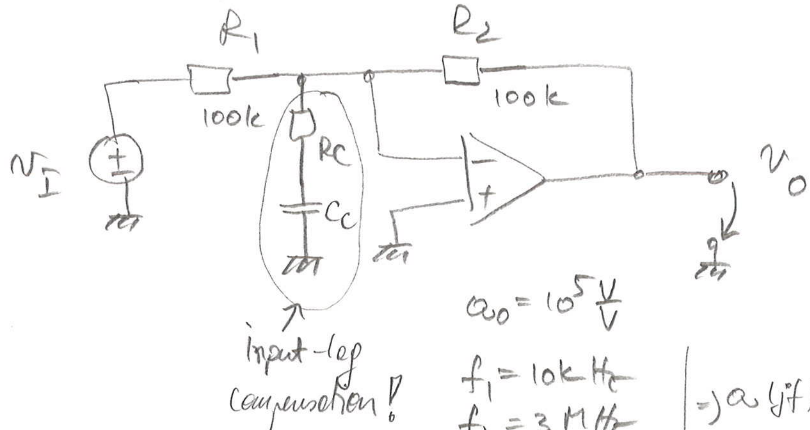
$\Rightarrow$ the phase contribution of $a_{eq}(jf)\Big|_{f=f_x^{new}}$ should be -45°. Since a_{eq} has 3 identical poles, the contribution of each pole at f_x^{new} must be $-\dfrac{45^\circ}{3} = -15^\circ$.

$\Rightarrow$ impose $-15^\circ = \tan^{-1}\dfrac{f_x^{new}}{10^5} \Rightarrow \boxed{f_{x\,new} = 26.8\ kHz}$

Finaly, $f_d = \frac{f_x^{new}}{\beta \cdot a_0} = \frac{26.8 \times 10^3}{\left(\frac{10^6}{101}\right)} = 2.7\ Hz$

$\Rightarrow$ Use $\boxed{C_c = 100\,nF}$ & $\boxed{R_c = 620\,k\Omega}$

Example 4: (8.35) (Section 8.4 External freq. compensation)



$a_0 = 10^5\ V/V$

$f_1 = 10\,kHz$

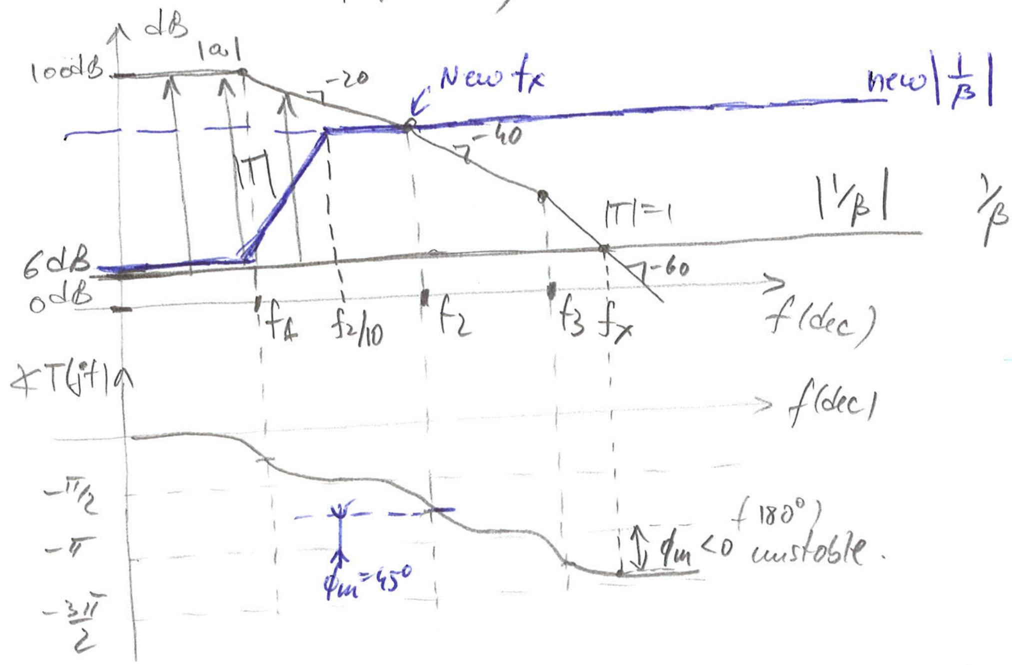
$f_2 = 3\,MHz$

$f_3 = 30\,MHz$

$\Rightarrow a(jf) = \frac{10^5}{(1+jf/10^4)(1+jf/3\cdot10^6)(1+jf/30\cdot10^6)}$

(a) Use input-lag compensation to stabilize for $\phi_m = 45°$ (b) Find $A(jf)$ (see page 375)

(a)



(See Fig. 8.25)(page 376)

$1/\beta = 1 + \frac{R_2}{R_1} = 2 \simeq 6dB$

Use eq. (8.30) to get:

$$R_c = \frac{R_2}{|a(jf_2)| - (1+\frac{R_2}{R_1})} = \frac{100\,k\Omega}{234.5 - (1+\frac{100}{100})} \cong \boxed{430\,\Omega}$$

Use eq. (8.31) to get:

$$C_c = \frac{5}{\pi R_c \cdot f_2} = \frac{5}{\pi \cdot 430 \cdot 3 \cdot 10^6} \cong \boxed{1.2\,nF}$$

(b) $\boxed{A(jf) = ?}$

Refer to fig. 8.25b and note that for $f < f_2/10$, $|T|$ is fairly large, indicating that $A(jf) \cong A_{ideal} = -1 \frac{V}{V}$.

For $f > f_2/10$ we can write

$$\left\{ \begin{array}{l} 1/\beta \cong |a(jf_2)| \\ a(jf) \cong \dfrac{a_0}{(jf/f_1)(1+jf/f_2)(1+jf/f_3)} \end{array} \right. \Rightarrow$$

$$\Rightarrow \frac{1}{T} = \frac{1}{a \cdot \beta} \cong \frac{|a(jf_2)|}{a_0} \cdot (jf/f_1)(1+jf/f_2)(1+jf/f_3)$$

$$\Rightarrow A(jf) = A_{ideal} \times \frac{1}{1+1/T} = (-1) \times \frac{1}{1+(jf/f_2)(1+jf/f_2)(1+jf/f_3)}$$

use fact:
$|a(jf_2)| \cdot f_2 = a_0 \cdot f_1$

or: $$A(jf) = -\frac{1}{1 + jf/f_2 - (f/f_2)^2 + (jf/f_3)\left[j\,f/f_2 - (f/f_2)^2 \right]}$$

in the neighborhood of f_2 ($f_2 \ll f_3$) we can approximate:

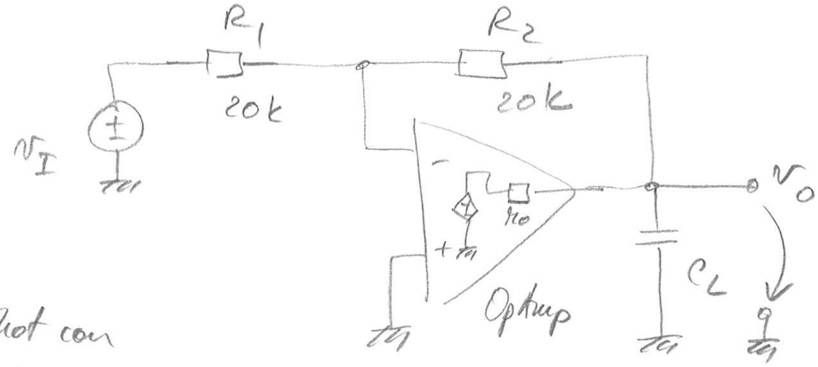
$$A(jf) \cong \frac{-1}{1 + jf/f_2 - (f/f_2)^2} = -H_{LP} \quad f_0 = f_2;\ Q = 1$$

Example 5

$f_t = 10\ MHz$

$f_b = 1\ Hz$

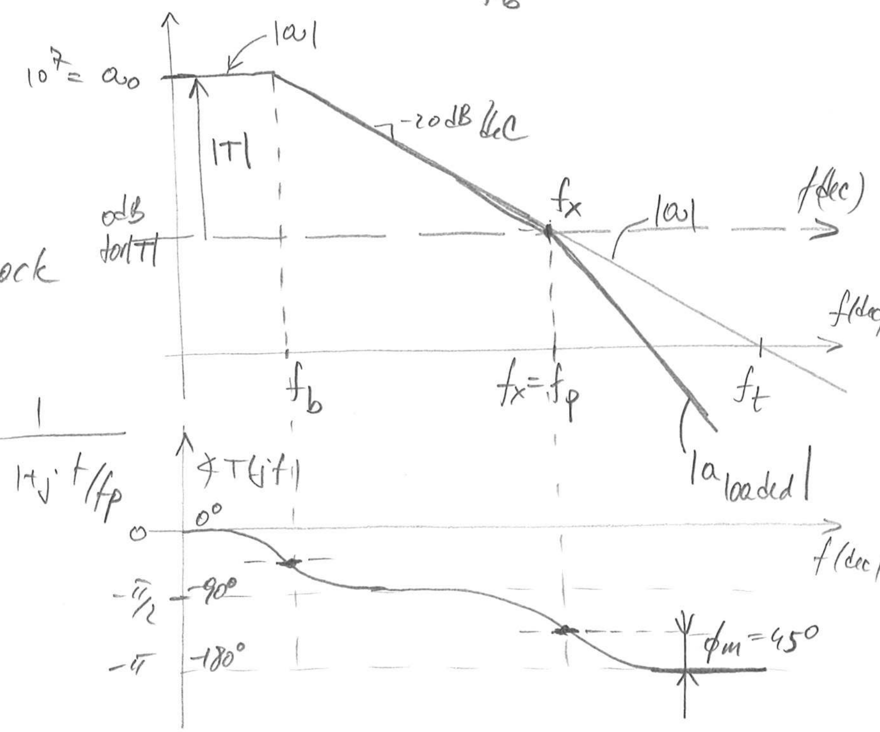
$r_o = 100\Omega$.



Find maximum C_L that can be connected at the output and still allow a phase margin of $\phi_m \geq 45^\circ$

$$a(jf) \cong \frac{a_0}{j f/f_b}$$

$$a_0 \cdot f_b = 1 \cdot f_t \Rightarrow \boxed{a_0 = 10^7}$$



C_L introduces a pole f_P

ignoring the loading of the feedback network, the loaded gain is:

$$a_{loaded} \cong a \cdot \frac{1}{1+j\,f/f_p} = \frac{a_0}{j f/f_b} \cdot \frac{1}{1+j\,f/f_p}$$

where $\boxed{f_p = \frac{1}{2\pi r_o C_L}}$

$$T(jf) = a(jf)\cdot\beta = \frac{a_0}{j\,f/f_b \cdot (1+j\,f/f_p)} \cdot \frac{R_1}{R_1+R_2}$$

From Bode diagram, impose crossover frequency $\boxed{f_x \leq f_p}$ at which $|T(jf_x)| = 1$ for equality.

$$|T(jf_x)| = 1 \Leftrightarrow \frac{a_0}{\left(\frac{f_x}{f_b}\right)\cdot\sqrt{1+1}} \cdot \frac{R_1}{R_1+R_2} = 1 \Rightarrow \boxed{f_x = \frac{a_0 f_b}{\sqrt{2}} \cdot \frac{R_1}{R_1+R_2}}$$

$$\boxed{f_p = \frac{1}{2\pi r_o C_L}} \Rightarrow$$

$$\Rightarrow \frac{a_0 \cdot f_b}{\sqrt{2}} \cdot \frac{R_1}{R_1+R_2} \leq \frac{1}{2\pi r_o C_L} \Rightarrow \boxed{C_L \leq 318pF}$$