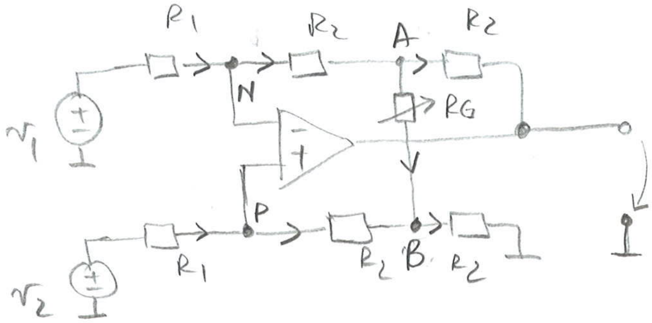
Examples | Ch. 2 | page. 77 (Problem 2.27)

Variable gain difference amplifier



$$\boxed{v_0 = \frac{2R_2}{R_1}\left(1+\frac{R_2}{R_G}\right)(v_2-v_1)}$$

Method 1

KCL at node N: $\dfrac{v_1-v_N}{R_1} = \dfrac{v_N-v_A}{R_2}$

KCL at node P: $\dfrac{v_2-v_P}{R_1} = \dfrac{v_P-v_B}{R_2}$

Subtract with $v_N = v_P$

$$\boxed{\frac{v_2-v_1}{R_1} = \frac{v_A-v_B}{R_2}} \Rightarrow \boxed{v_A - v_B = \frac{R_2}{R_1}(v_2-v_1)}$$

key trick.

KCL node A: $\dfrac{v_N-v_A}{R_2} = \dfrac{v_A-v_0}{R_2} + \dfrac{v_A-v_B}{R_G}$

KCL node B: $\dfrac{v_P-v_B}{R_2} = \dfrac{v_B}{R_2} - \dfrac{v_A-v_B}{R_G}$

Subtract with $v_N = v_P$

$$\boxed{\frac{v_A-v_B}{R_2} = -\frac{v_A-v_B}{R_2} + \frac{v_0}{R_2} - \frac{2}{R_G}(v_A-v_B)}$$

$$\boxed{\left(\frac{2}{R_2}+\frac{2}{R_G}\right)(v_A-v_B) = \frac{v_0}{R_2}}$$

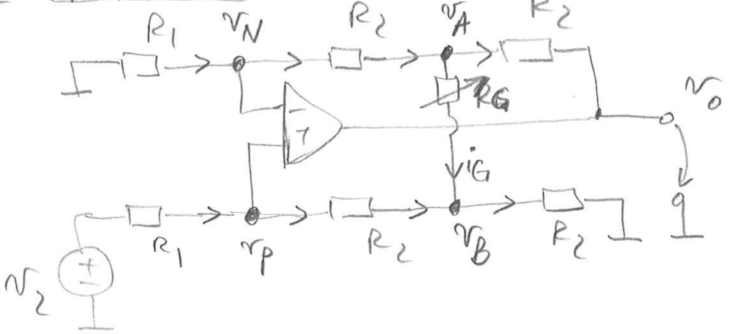
$\Rightarrow$

$$\frac{v_0}{R_2} = \left(\frac{2}{R_2}+\frac{2}{R_G}\right)\cdot\frac{R_2}{R_1}(v_2-v_1)$$

$$\frac{v_0}{\cancel{R_2}} = \frac{2(R_G+R_2)}{\cancel{R_2}R_G}\cdot\frac{R_2}{R_1}(v_2-v_1) \Rightarrow \boxed{v_0 = 2\cdot\frac{R_2}{R_1}\left(1+\frac{R_2}{R_G}\right)(v_2-v_1)}$$

Method 2 Superposition

Case 1: $\boxed{v_1 = 0,\ v_2 \neq 0}$



$$\left\{\begin{array}{l} v_P = \dfrac{R_2}{R_1+R_2} v_2 + \dfrac{R_1}{R_1+R_2} v_B \\ \\ v_N = \dfrac{R_1}{R_1+R_2} v_A \end{array}\right| \Rightarrow$$

$$\Rightarrow v_P = v_N \Leftrightarrow R_2 v_2 = R_1(v_A - v_B) \Rightarrow v_2 = \frac{R_1}{R_2}(v_A - v_B)$$

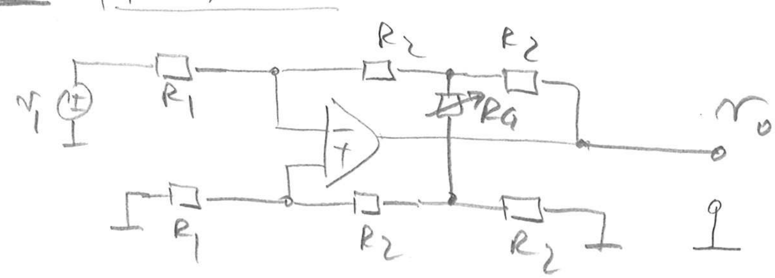
KCL node A: $\dfrac{v_N - v_A}{R_2} = \dfrac{v_A - v_B}{R_G} + \dfrac{v_A - v_o}{R_2}$

KCL node B: $\dfrac{v_P - v_B}{R_2} = -\dfrac{v_A - v_B}{R_G} + \dfrac{v_B}{R_2}$ subtract $v_N = v_P$

$$\frac{v_A - v_B}{R_2} = -2\frac{v_A - v_B}{R_G} - \frac{v_A - v_B}{R_2} + \frac{v_o}{R_2} \Rightarrow \frac{v_o}{R_2} = \left(\frac{2}{R_2} + \frac{2}{R_G}\right)(v_A - v_B) \Rightarrow$$

$$\Rightarrow \frac{v_o}{R_2} = \left(\frac{2}{R_2} + \frac{2}{R_G}\right) \cdot \frac{R_2}{R_1} v_2 \Rightarrow \boxed{v_o = 2\frac{R_2}{R_1}\left(1 + \frac{R_2}{R_G}\right) v_2}$$

Case 2: $\boxed{v_1 \neq 0,\ v_2 = 0}$

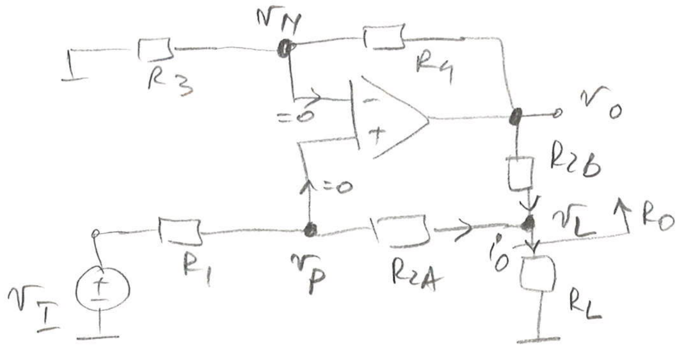


Similarly: $\boxed{v_o = -2\frac{R_2}{R_1}\left(1 + \frac{R_2}{R_G}\right) v_1}$

Finally by superposition $\Rightarrow$

$$\boxed{v_o = 2\frac{R_2}{R_1}\left(1 + \frac{R_2}{R_G}\right)(v_2 - v_1)}$$

Example 2.12 improved Howland circuit.



Prove that when: $\frac{R_4}{R_3} = \frac{R_{2A}+R_{2B}}{R_1}$

(a) $R_O = \infty$

(b) $i_O = \frac{R_2}{R_1 \cdot R_{2B}} v_I$

(a)

$$\begin{cases} \boxed{v_O = \left(1+\frac{R_4}{R_3}\right) v_P = \left(1+\frac{R_4}{R_3}\right)\cdot\frac{R_{2A}v_I + R_1 v_L}{R_1+R_{2A}}} \\ \boxed{v_P = \frac{R_{2A}}{R_1+R_{2A}} v_I + \frac{R_1}{R_1+R_{2A}} v_L} \text{ by superposition.} \end{cases}$$

KCL at v_L: $\boxed{i_O} = \frac{v_I - v_L}{R_1+R_{2A}} + \frac{v_O - v_L}{R_{2B}} = v_I\left(\frac{1}{R_1+R_{2A}}\right) - v_L\left(\frac{1}{R_1+R_{2A}}+\frac{1}{R_{2B}}\right) + \frac{v_O}{R_{2B}} =$

$$= v_I\left[\frac{1}{R_1+R_{2A}} + \frac{1}{R_{2B}}\left(1+\frac{R_4}{R_3}\right)\cdot\frac{R_{2A}}{R_1+R_{2A}}\right] \triangleq \frac{1}{R}$$

because for $v_I=0$ we get: $\boxed{i_O = -\frac{v_O}{R_O}}$

$$- v_L\left[\frac{1}{R_1+R_{2A}} + \frac{1}{R_{2B}} - \frac{1}{R_{2B}}\cdot\left(1+\frac{R_4}{R_3}\right)\cdot\frac{R_1}{R_1+R_{2A}}\right] \triangleq \frac{1}{R_O} \qquad = \frac{v_I}{R} - \frac{v_L}{R_O}$$

This is key trick!

related because $\boxed{R_O = \frac{v_L}{-i_O}}$!

$$\frac{1}{R} = \frac{R_3R_{2B} + R_{2A}(R_3+R_4)}{R_{2B}R_3(R_1+R_{2A})} = \frac{R_{2A}R_4 + R_3(R_{2A}+R_{2B})}{R_{2B}R_3(R_1+R_{2A})}$$

$$\frac{1}{R_O} = \frac{R_{2B} + \cancel{R_1}+R_{2A} - \left(\cancel{1}+\frac{R_4}{R_3}\right)R_1}{R_{2B}(R_1+R_{2A})} = \frac{R_{2A}+R_{2B} - \frac{R_4}{R_3}R_1}{R_{2B}(R_1+R_{2A})} = \frac{\frac{R_{2A}+R_{2B}}{R_1} - \frac{R_4}{R_3}}{R_{2B}\left(1+\frac{R_{2A}}{R_1}\right)}$$

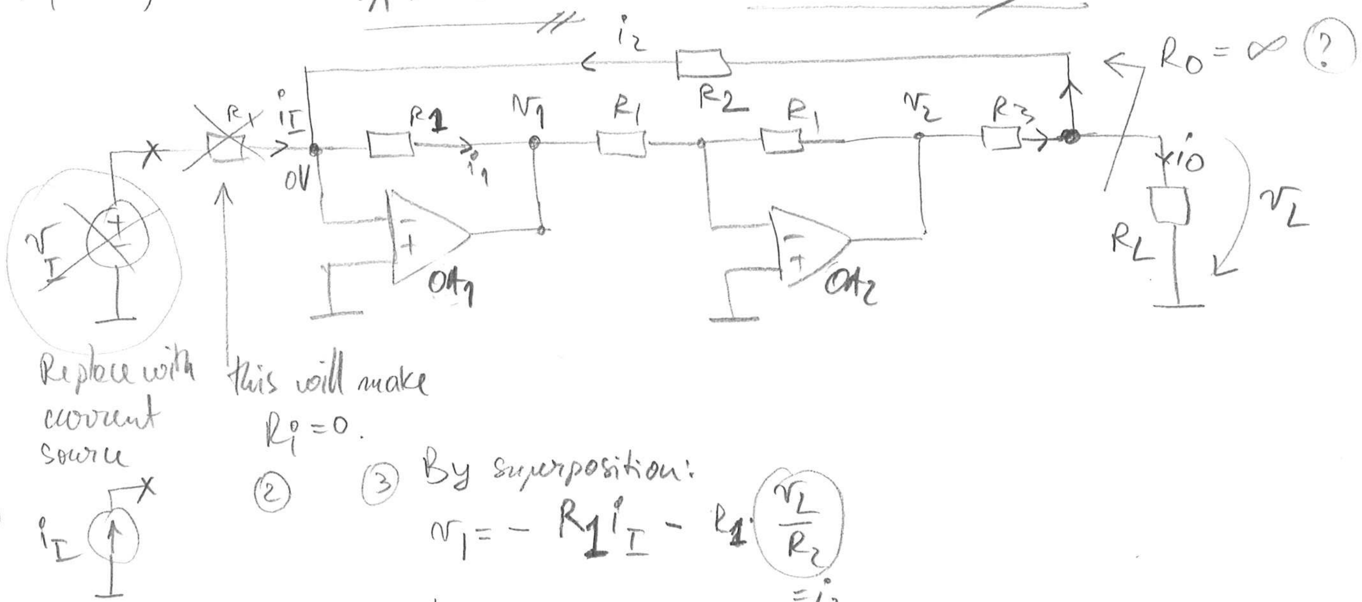
we can see that $R_O \to \infty$ if $\boxed{\frac{R_{2A}+R_{2B}}{R_1} - \frac{R_4}{R_3} = 0}$! $\Leftrightarrow \frac{R_2}{R_1} = \frac{R_4}{R_3}$ when

(b) $\boxed{R_{2A}+R_{2B} \triangleq R_2} \Rightarrow R = \frac{R_3R_{2B}(R_1+R_{2A})}{R_3R_2 + R_{2A}R_4} = \frac{R_3R_{2B}(R_1+R_{2A})}{\underbrace{\frac{R_4R_1}{R_2}}_{=R_3}\cdot R_2 + R_{2A}\cdot R_4} = \frac{R_3R_{2B}(\cancel{R_1+R_{2A}})}{R_4(\cancel{R_1+R_{2A}})} = \frac{R_3}{R_4}R_{2A}$

Finally: $\boxed{i_O = \frac{v_I}{R} = \frac{R_4}{R_3}\cdot\frac{1}{R_{2B}}\cdot v_I}$

Example $\boxed{2.24}$

Modify given circuit so that it becomes a current amplifier with $R_i = 0$, $A = 10\ A/A$, $R_o = \infty$. Assume ideal OpAmps.



③ By superposition:

$$v_1 = -R_1 i_I - R_1 \cdot \underbrace{\frac{v_L}{R_2}}_{=i_2}$$

④

$$\boxed{v_2 = -v_1 = R_1 i_I + R_1/R_2\, v_L}$$

$$i_O = \frac{v_2 - v_L}{R_3} - \frac{v_L}{R_2} = \frac{1}{R_3}\left(R_1 i_I + \frac{R_1}{R_2} v_L\right) - \left(\frac{1}{R_3} + \frac{1}{R_2}\right) v_L$$

I needed this "−" to introduce R_o!

$$i_O = \frac{R_1}{R_3} i_I + \left[\frac{R_1}{R_2 R_3} - \frac{R_2 + R_3}{R_2 R_3}\right] v_L = \frac{R_1}{R_3} i_I - \underbrace{\left[\frac{R_2 + R_3}{R_2 R_3} - \frac{R_1}{R_2 R_3}\right]}_{\triangleq \frac{1}{R_o}} v_L$$

that is same key trick!

because: $i_O = \left.\frac{-v_L}{R_o}\right|_{i_I = 0}$

=> To get $R_o = \infty$ we need: $\boxed{R_1 = R_2 + R_3}$

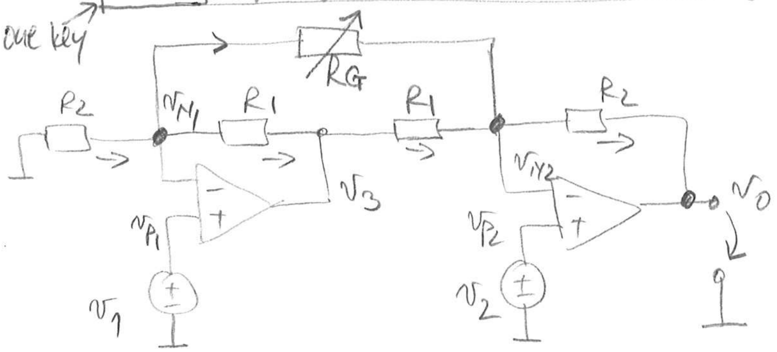
Also, To get a gain of $A = 10\ \frac{A}{A}$ we need: $\boxed{\frac{R_1}{R_3} = 10}$

Use $\boxed{R_1 = 10k}$, $\boxed{R_3 = 1k}$ and $\boxed{R_2 = 9.09k}$

Example. $\boxed{2.38}$ (Figure 2.24, pp. 83)

$\boxed{\text{Dual}}$ Op Amp IA with variable gain

one key



$$v_0 = A(v_2 - v_1)$$

$$A = 1 + \frac{R_2}{R_1} + \frac{2R_2}{R_G}$$

(a)

$$\boxed{v_{N_1} = v_{P_1} = v_1} \qquad \boxed{v_{N_2} = v_{P_2} = v_2}$$

KCL: $\frac{0V - v_1}{R_2} = \frac{v_1 - v_2}{R_G} + \frac{v_1 - v_3}{R_1}$ want to get rid of !

KCL: $\frac{v_2 - v_0}{R_2} = \frac{v_1 - v_2}{R_G} + \frac{v_3 - v_2}{R_1}$ add. equations.

$$\frac{v_2 - v_1}{R_2} - \frac{v_0}{R_2} = 2\frac{v_1 - v_2}{R_G} + \frac{v_1 - v_2}{R_1}$$

$$(v_2 - v_1)\left(\frac{1}{R_2} + \frac{1}{R_1} + \frac{2}{R_G}\right) = \frac{1}{R_2} v_0$$

$$\boxed{v_0 = \left(1 + \frac{R_2}{R_1} + 2\frac{R_2}{R_G}\right)(v_2 - v_1)}$$

however gain does not vary linearly with R_G.

(b) specify components to get: $10\ V/V \le A \le 100\ V/V$ by means of $10\ k\Omega$.

Let $R_G = R_{GA} + R_{GB} = 10\ k\Omega$

Let arbitrarily $\frac{R_2}{R_1} = 1 \Rightarrow A = 2\left(1 + \frac{R_2}{R_G}\right)$.

$$5 \le \left(1 + \frac{R_2}{R_G}\right) \le 50 \Rightarrow 4 \le \frac{R_2}{R_G} \le 49$$

case: $R_G = 0 + R_{GB} \Rightarrow \frac{R_2}{R_{GB}} = 49$

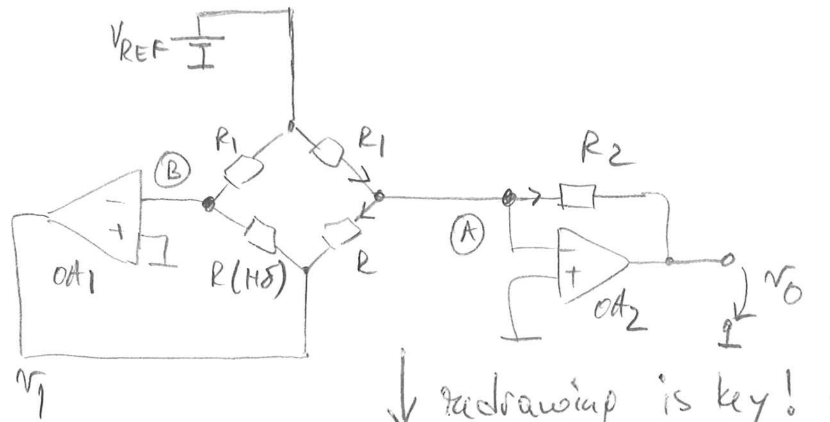
case: $R_G = 10 + R_{GB} \Rightarrow \frac{R_2}{10 + R_{GB}} = 4$

$\Rightarrow$ $R_{GB} = 889\ \Omega$ (use 887Ω, 1%)

$R_2 = 49 R_{GB} = 43.5\ k\Omega = R_1$

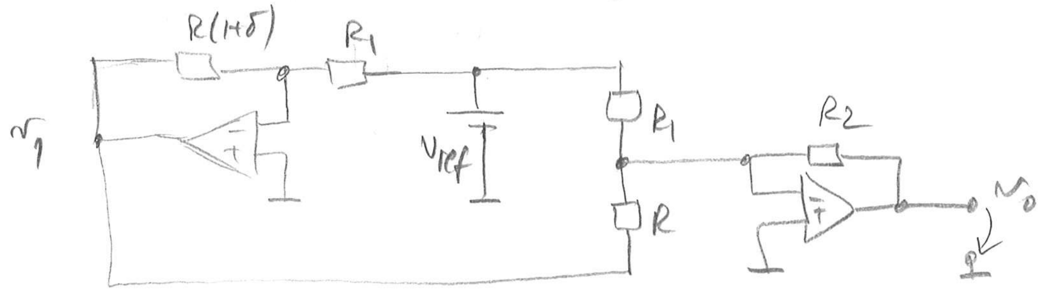
(use $R_1 = R_2 = 43.2\ k\Omega$, 1%)

Example Problem 2.49 Simple transducer circuit with linear response



$$v_O = \frac{R_2}{R_1} \cdot \delta \cdot V_{REF}$$

↓ redrawing is key! to notice configuration.



$$v_1 = -\frac{R(1+\delta)}{R_1} V_{ref}$$

By superposition: $v_O = -\frac{R_2}{R_1} V_{ref} - \frac{R_2}{R} v_1 = -\frac{R_2}{R_1} V_{ref} + \frac{R_2}{\cancel{R}} \cdot \frac{\cancel{R}}{R_1}(1+\delta) V_{ref}.$

$$v_O = V_{ref}\left(-\frac{R_2}{R_1} + \frac{R_2}{R_1}(1+\delta)\right) = \frac{R_2}{R_1}\delta \cdot V_{ref}.$$

$$\boxed{v_O = \frac{R_2}{R_1}\delta \cdot V_{ref}.}$$