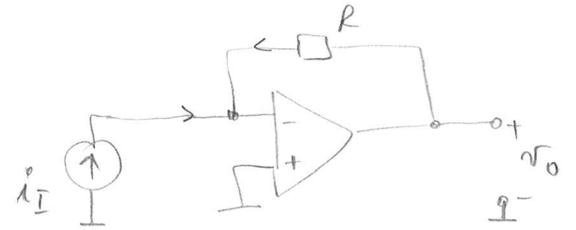
Chapter 2 Circuits with resistive feedback

I. Opamp used to implement
- IV converter (transresistance)
- VI converter (transconductance)
- current amplifier

II. instrumentation concepts
- difference amplifiers
- instrumentation amplifiers (IA)
- transducer bridge amplifiers

2.1 Current-to-voltage converter $\boxed{i} \to \boxed{v}$



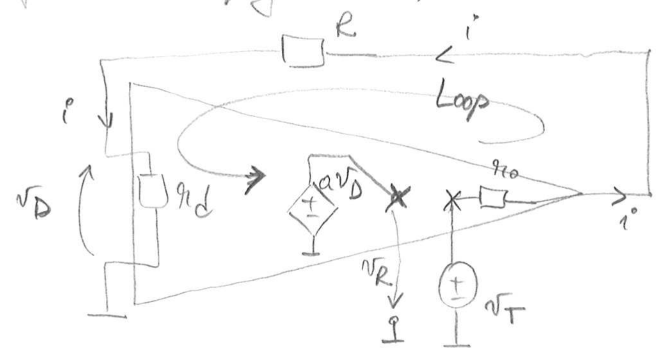
- shunt in
- compare current $\boxed{i}$

- shunt out
- sample voltage $\boxed{v}$

ideal Opamp analysis:

$$\frac{v_O}{R} = -i_I \Rightarrow \boxed{\frac{v_O}{i_I} = -R = A_r}$$

(a) To find the loop gain T, use the technique from 1.7 (chapter 1)



$$T = -\frac{v_R}{v_T}\bigg|_{x_I=0}$$

$$v_R = av_D = a(-ir_d) = -ar_d \cdot \frac{v_T}{r_o+R+r_d} \Rightarrow \boxed{T = \frac{r_d \cdot a}{r_d+R+r_o}}$$

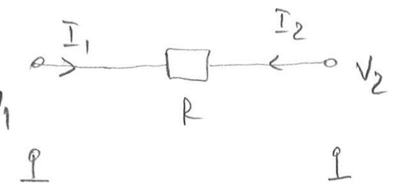
→ Loop gain T is unit-less irrespective of the type of a_v, a_i, a_g, a_r and $\beta_r, \beta_v, \beta_r, \beta_g$!

→ So, this technique is ok for any type of neg. feedback configuration

→ Note we have T but we do not know a_r and β_g! find them w/ "a", "β" circuits!

For example, finding β_g is always easier:

The "β" circuit is:



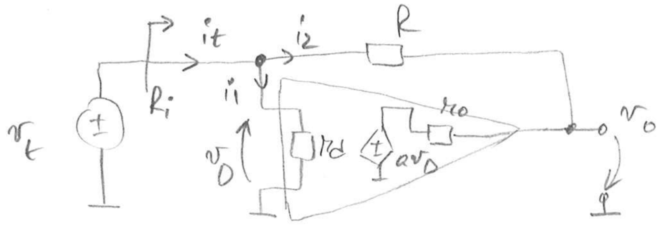
Using the reference-handout:

$$\beta_g = \left.\frac{I_1}{V_2}\right|_{V_1=0} = (y_{12}) = -\frac{1}{R}$$

Therefore the closed loop amplification of this IV converter is:

$$\boxed{A_r = \frac{1}{\beta_g}\cdot\frac{T}{1+T} = -R\cdot\frac{T}{1+T}}\ ; \quad \text{where } T = \frac{r_d\cdot a}{r_d + R + r_o}$$

input resistance → Method 1



$$v_t = -v_D = i_1 r_d \Rightarrow i_1 = \frac{v_t}{r_d}$$

$$i_2 = \frac{v_t - v_o}{R} = \frac{v_t - r_o i_2 - a v_D}{R}$$

$$(R + r_o)\, i_2 = v_t - a\underbrace{v_D}_{=-v_t} = (1+a)\, v_t$$

$$\Rightarrow i_t = i_1 + i_2 = \frac{v_t}{r_d} + \frac{1+a}{R+r_o} v_t = \left(\frac{1}{r_d} + \frac{1+a}{R+r_o}\right) v_t$$

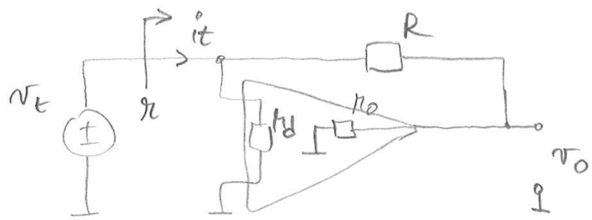
$$\boxed{R_i = \frac{v_t}{i_t} = \frac{r_d\cdot(R+r_o)}{R+r_o + r_d(1+a)}} = \underbrace{\frac{r_d\cdot(R+r_o)}{r_d+(R+r_o)}}_{= r_d \parallel (R+r_o)} \cdot \underbrace{\frac{r_d+(R+r_o)}{(R+r_o)+r_d(1+a)}}_{= \frac{1}{1+\frac{r_d\cdot a}{r_d+R+r_o}} = \frac{1}{1+T}} = \boxed{\frac{r_d \parallel (R+r_o)}{1+T}}$$

Method 2 (read page 37)

We know that one of the benefits of the neg. feedback is:

$$R_i = \begin{cases} r(1+T), & \text{for series topology at in} \\ \dfrac{r}{1+T}, & \text{for shunt/parallel topology at in} \end{cases}$$

$\boxed{r}$ is the input resistance into amplifier with $a=0$

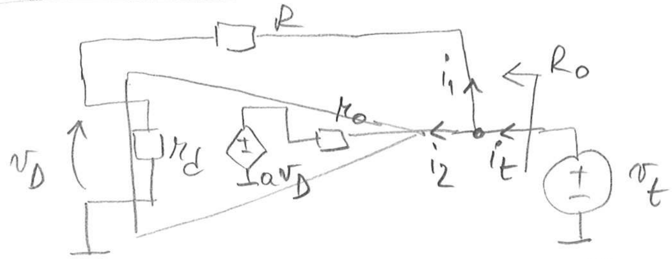


$$r = \left.\frac{v_t}{i_t}\right|_{a=0} = r_d \parallel (R+r_o)$$

Therefore: $\boxed{T = \dfrac{r}{1+T} = \dfrac{r_d \parallel (R+r_o)}{1+T}}$

Same as before!

Output resistance → Method 1



$$i_1 = \frac{v_t}{R + r_d}$$

$$i_2 = \frac{v_t - a v_D}{r_o} = \frac{v_t + a r_d \cdot i_1}{r_o}$$

$$v_D = -r_d \cdot i_1$$

$$i_t = i_1 + i_2 = \frac{v_t}{R + r_d} + \frac{v_t + a r_d \cdot \frac{v_t}{R + r_d}}{r_o}$$

$$= v_t \left(\frac{1}{R + r_d} + \frac{R + r_d + a r_d}{r_o (R + r_d)} \right) = v_t \left(\frac{r_o + R + (1+a) r_d}{r_o (R + r_d)} \right)$$

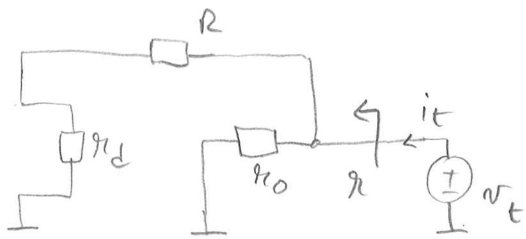
$$\Rightarrow \boxed{R_o = \frac{v_t}{i_t} = \frac{r_o (R + r_d)}{r_o + R + (1+a) r_d} = r_o \cdot \frac{1}{1 + \frac{\cancel{r_o} + a r_d}{R + r_d}} \cong \frac{r_o}{1 + T}}$$

$$\frac{r_o + a r_d}{R + r_d} \cong \frac{r_d \cdot a}{r_d + R}$$

$$\text{Recall } T = \frac{r_d \cdot a}{r_d + R + \cancel{r_o}} \cong \frac{r_d \cdot a}{r_d + R}$$

Method 2: (read page 37)

$R_o \cong \dfrac{r}{1 + T}$; $\boxed{r}$ is the output resistance from amplifier with $a = 0$.



$$r = \frac{v_t}{i_t} = r_o \,\|\, (R + r_d) \cong r_o$$

Therefore $\boxed{R_o \cong \dfrac{r_o}{1 + T}}$

Same as before!

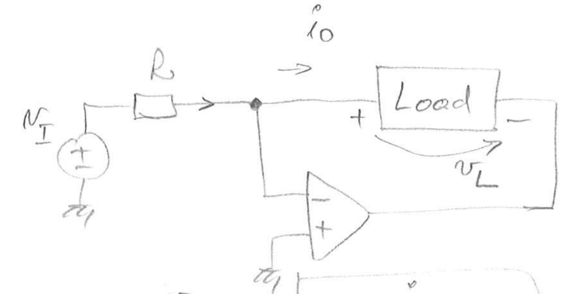
use those techniques to calculate $\beta = -\frac{v_D}{v_T}\Big|_{x_I=0}$ because here the feedback network brings for comparison a current!

▸ insert here the Notes on the theory of negative-feedback and the universal relation: $A = A_{ideal} \cdot \frac{a\beta}{1+a\beta}$.

2.2. $\boxed{\text{Voltage-to-current Converters}}$ $\boxed{v} \to \boxed{i}$

(trans-conductance "g" amplifiers)

Floating-load Converter



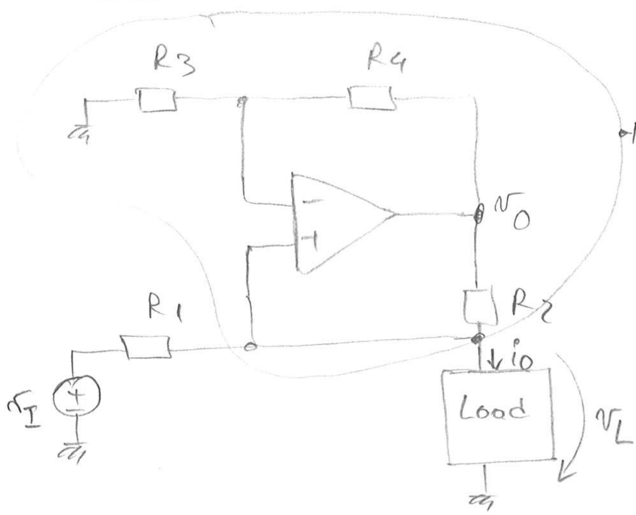
$$i_O = \frac{v_I}{R} \Rightarrow \boxed{A_g = \frac{i_O}{v_I} = \frac{1}{R}}$$

Feedback: shunt–series (compare voltage – sample current) ⇒ "g" amplifier.

See Fig. 1.26.

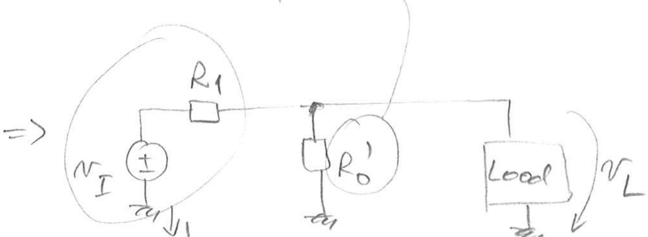
Grounded-load Converter.

$\boxed{\text{HOWLAND}}$ current pump: Negative feedback topology: "series-series": more appropriate for "g" amplifier!

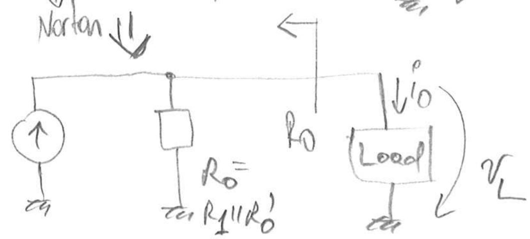


Negative resistor $= -\frac{R_2 R_3}{R_4}$

⇒



Norton ⇓



$i_I = \frac{v_I}{R_1}$

$$R_o = R_1 \parallel \left(-\frac{R_2 R_3}{R_4}\right) = \frac{R_1 \cdot \left(-\frac{R_2 R_3}{R_4}\right)}{R_1 + \left(-\frac{R_2 R_3}{R_4}\right)} =$$

$$= \frac{-\frac{R_1 R_2 R_3}{\cancel{R_4}}}{\frac{R_1 R_4 - R_2 R_3}{\cancel{R_4}}} = \frac{R_1 R_2 R_3}{R_2 R_3 - R_1 R_4} = \boxed{\frac{R_2}{\frac{R_2}{R_1} - \frac{R_4}{R_3}} = R_o}$$

$\frac{R_2}{R_1} - \frac{R_4}{R_3} \to 0$

Observe: $\lim\limits_{\frac{R_2}{R_1} \to \frac{R_4}{R_3}} R_o = \infty$

This is a "g" amplifier = at the output it has to be seen as a current source $\Rightarrow$ ideally $R_o \to \infty$. That is the case for

$$\boxed{\frac{R_2}{R_1} = \frac{R_4}{R_3}}$$

The gain: $A_g = \frac{i_o}{v_I} = ?$

$$i_o = \frac{R_o}{R_o + R_L} \cdot i_I = \frac{R_o}{R_o + R_L} \cdot \frac{v_I}{R_1} \Rightarrow \boxed{A_g = \frac{i_o}{v_I} = \frac{1}{R_1} \cdot \frac{R_o}{R_o + R_L}}$$

Ideally: $\lim\limits_{R_o \to \infty} A_g = \boxed{\frac{1}{R_1} = A_{ideal}.}$

$\boxed{\text{important}}$ Howland circuit has both <u>negative</u> and <u>positive</u> feedbacks:

$$\beta_N = \frac{R_3}{R_3 + R_4} = \frac{1}{1 + R_2/R_1}$$

$$\beta_P = \frac{R_1 \parallel R_L}{R_1 \parallel R_L + R_2} = \frac{1}{1 + R_2/R_1 + R_2/R_L}$$

$\Rightarrow \beta_P \overset{\text{always}}{<} \beta_N \Rightarrow$ Negative feedback prevails!

$\Rightarrow$ We can still apply the "virtual short" concept.

Also the circuit will be stable!

(a) Effect of resistance mismatches

$$\frac{R_4}{R_3} = \frac{R_2}{R_1}(1-\varepsilon) \qquad \varepsilon \triangleq \text{imbalance factor.}$$

$$\Rightarrow \boxed{R_O = \frac{R_1}{\varepsilon}}$$ the smaller the imbalance the higher R_O.

(b) Effect of finite open-loop gain

$$\begin{cases} i_O = \dfrac{v_I - v_L}{R_1} + \dfrac{v_O - v_L}{R_2} \\ v_O = v_L \cdot \dfrac{a}{1 + a\dfrac{R_3}{R_3+R_4}} \end{cases}$$

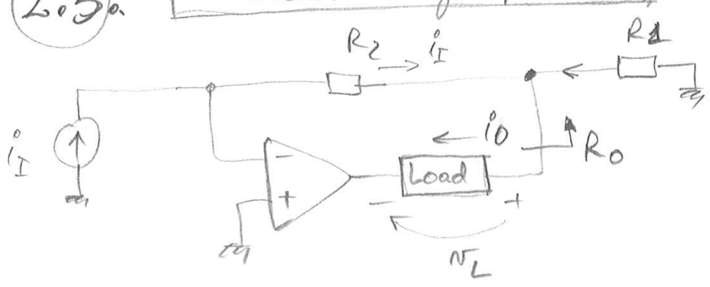
: (look at it as a noninverting amplifier $v_L \to v_O$)

Assume perfect $\frac{R_4}{R_3} = \frac{R}{R_1}$

$$\Rightarrow i_O = \frac{1}{R_1} v_I - \frac{1}{R_O} v_L$$

where $R_O = (R_1 \| R_2)\left(1 + \dfrac{a}{1 + \frac{R_2}{R_1}}\right)$

2.3 $\boxed{\text{Current amplifiers}}$ $\boxed{i} \to \boxed{i}$



$$A_i = \frac{i_O}{i_I}$$

Typically : $\boxed{i_O = A\, i_I - \frac{1}{R_O} v_L}$

Feedback: "shunt-series"
compare $\boxed{i}$, sample $\boxed{i}$

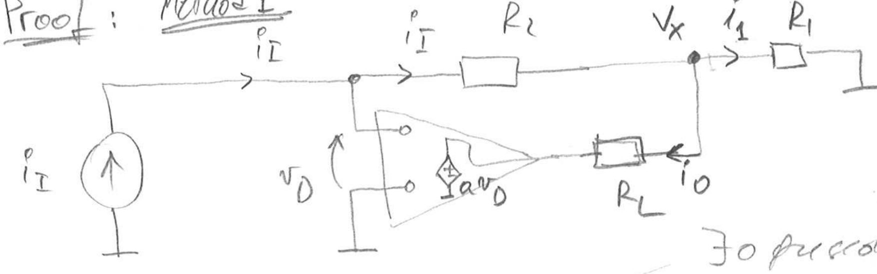
R_O = output resistance as seen by the load.

$$i_O = i_I + \frac{R_2 i_I}{R_1} \Rightarrow \boxed{A_i = \frac{i_O}{i_I} = 1 + \frac{R_2}{R_1}}$$

Holds regardless of v_L
=> circuit provides $R_O \to \infty$

if a is finite, it can be proven: $\boxed{A = 1 + \dfrac{R_2/R_1}{1 + \frac{1}{a}}}$ $\boxed{R_O = R_1(1+a)}$

Proof: Method 1



Jo piccolo. il ritorno.

$$\begin{cases} v_x = R_L i_O + a v_D = -R_2 i_I - \underline{v_D} \\ v_D = -R_2 i_I - R_1 i_1 = -R_2 i_I - R_1 (i_I - i_O) \end{cases} \Rightarrow R_L i_O + a\left[-R_2 i_I - R_1(i_I - i_O)\right] = -R_2 i$$

$i_1 = i_I - i_O$

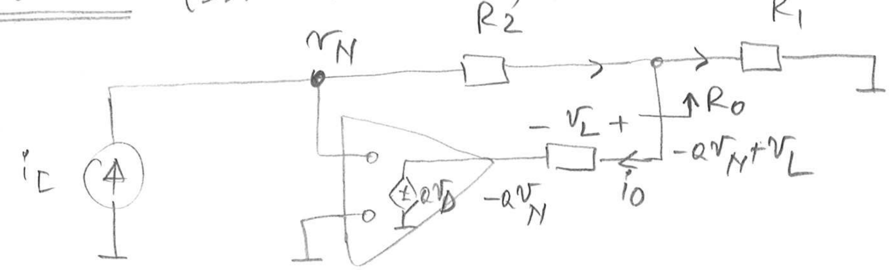
$$\Rightarrow (\cancel{R_L} + aR_1)\, i_O = (aR_2 + aR_1 - \cancel{R_2})\, i_I$$

$$\boxed{A_i = \frac{i_O}{i_I} = \frac{a(R_1+R_2) - R_2}{aR_1 + \cancel{R_L}} = 1 + \frac{(a-1)}{a}\frac{R_2}{R_1} = 1 + \frac{R_2}{R_1}\underbrace{\left(1 - \frac{1}{a}\right)}_{\cong \frac{1}{1+1/a}} \cong 1 + \frac{R_2}{R_1}\cdot\frac{1}{1+1/a}}$$

because $\frac{1}{a} \ll 1$

See verso for $R_O = ?$

Method 2: (S. Franco's solution)



$$v_N - (-a v_N + v_L) = R_2 i_I$$

$$\Rightarrow v_N = (R_2 i_I + v_L)/1+a$$

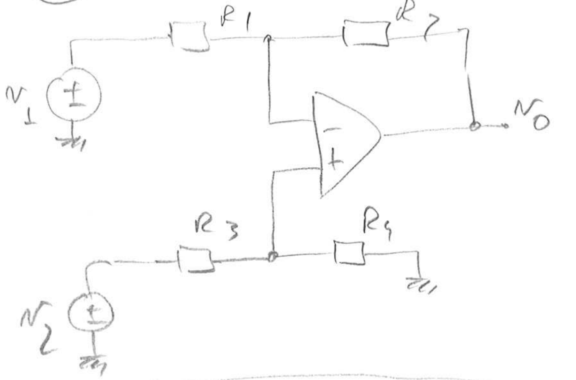
$$i_O = i_I + \frac{a v_N - v_L}{R_1} = i_I + \frac{a \cdot \frac{R_2 i_I + v_L}{1+a}}{R_1} - \frac{v_L}{R_1}$$

$$= \underbrace{i_I\left(1 + \frac{R_2/R_1}{1+1/a}\right)}_{= A_i} - \frac{v_L}{R_1}\left(1 - \frac{a}{1+a}\right) = A_i i_I - \frac{v_L}{R_O}$$

$$\frac{1}{R_1(1+a)} = \frac{1}{R_O}$$

$$\boxed{A_i = 1 + \frac{R_2}{R_1}\cdot\frac{1}{1+1/a}} \qquad \boxed{R_O = R_1(1+a)}$$

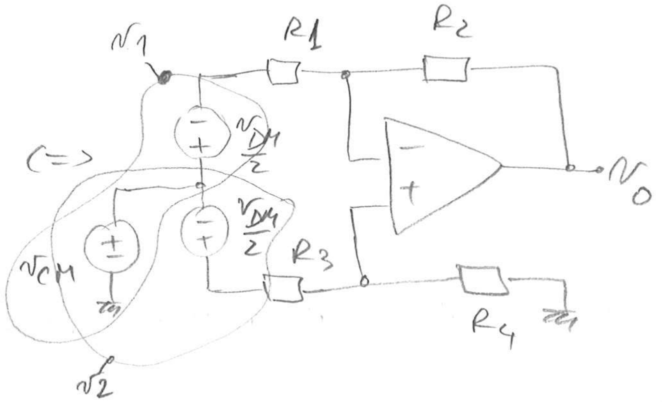
2.4. Difference amplifiers



$$\boxed{v_O = \frac{R_2}{R_1}(v_2 - v_1)}$$

if $\frac{R_4}{R_3} = \frac{R_2}{R_1}$

<=>

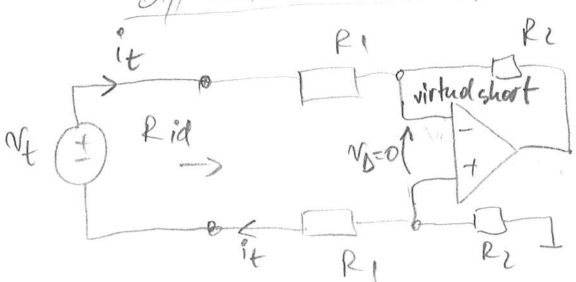


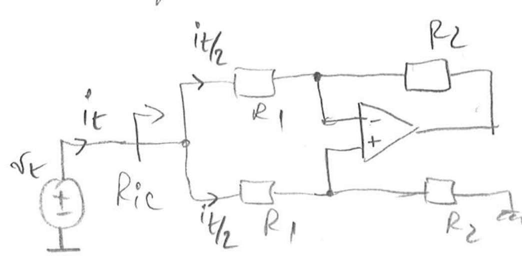
$$\begin{cases} v_{DM} = v_2 - v_1 \\ v_{CM} = \frac{v_1 + v_2}{2} \end{cases} \iff \begin{cases} v_1 = v_{CM} - \frac{v_{DM}}{2} \\ v_2 = v_{CM} + \frac{v_{DM}}{2} \end{cases}$$

Using this arrangement it's easier to study it, more insightful!

=> A true difference amplifier responds only to v_{DM}! and completely ignores v_{CM}

Differential-mode, common-mode input resistances





$\boxed{R_{id} = 2R_1}$

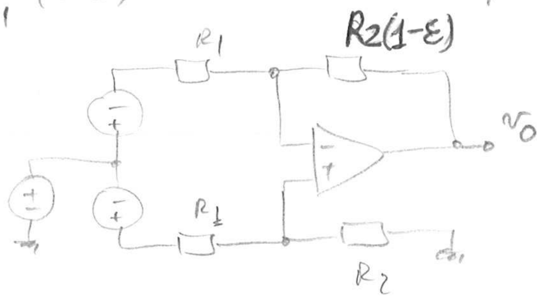
$$v_t = R_1 i_t - \underbrace{v_D}_{=0} + R_1 i_t \Rightarrow \boxed{\frac{v_t}{i_t} = 2R_1 = R_{id}}$$

$\boxed{R_{ic} = \frac{R_1 + R_2}{2}}$

$$v_t = \frac{i_t}{2}(R_1 + R_2) \Rightarrow \boxed{\frac{v_t}{i_t} = \left(\frac{R_1 + R_2}{2}\right) = R_{ic}}$$

Effect of resistance mismatches

$$\frac{R_4}{R_3} = \frac{R_2}{R_1}(1 - \varepsilon) \qquad \varepsilon \triangleq \text{imbalance factor.}$$



Use superposition:

$$v_O = -\frac{R_2(1-\varepsilon)}{R_1}\left(v_{CM} - \frac{v_{DM}}{2}\right) + \frac{R_1 + R_2(1-\varepsilon)}{R_1} \times \frac{R_2}{R_1+R_2}\left(v_{CM} + \frac{v_{DM}}{2}\right)$$

$$\Rightarrow \boxed{v_O = A_{dm} \cdot v_{DM} + A_{cm} \cdot v_{CM}}$$

$$A_{dm} = \frac{R_2}{R_1}\left(1 - \frac{R_1 + 2R_2}{R_1+R_2} \cdot \frac{\varepsilon}{2}\right) \xrightarrow[\varepsilon \to 0]{} \frac{R_2}{R_1}$$

$$A_{cm} = \frac{R_2}{R_1+R_2} \cdot \varepsilon \xrightarrow[\varepsilon \to 0]{} 0$$

$\frac{A_{dm}}{A_{cm}} \triangleq CMRR$ Common mode rejection ratio, expressed in dB:

$$CMRR_{dB} = 20 \cdot \log\left|\frac{A_{dm}}{A_{cm}}\right|$$

True difference amplifier $\Rightarrow CMRR_{dB} \to \infty$

$$CMRR_{dB} \cong 20 \cdot \log\left|\frac{1 + \frac{R_2}{R_1}}{\varepsilon}\right|$$

- Variable gain
- Ground-loop interference elimination

} home exercise for students!