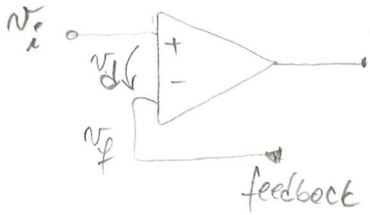
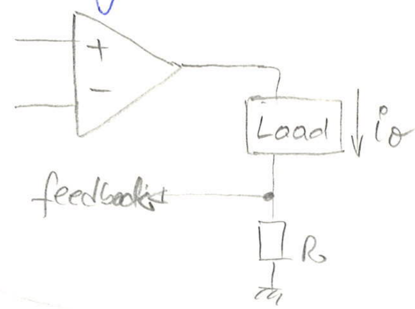
1.6. Negative Feedback in OpAmp circuits

Basic topologies

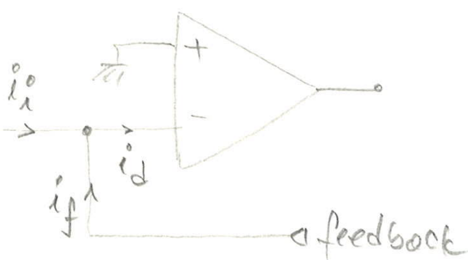
Series at input
"comparison of voltage" $\boxed{v}$



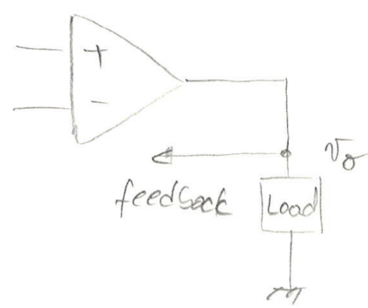
Series at Output
"Sampling current" $\boxed{i}$



Shunt at input
(aka parallel at input)
"comparison of current" $\boxed{i}$



Shunt / at output
parallel
"Sampling voltage" $\boxed{v}$



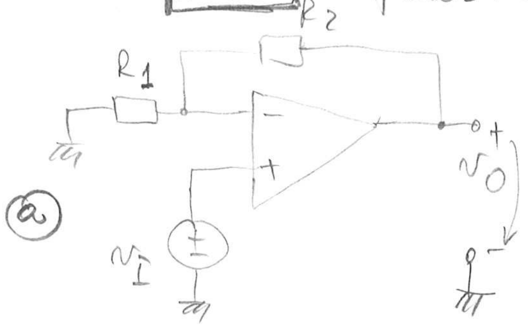
We can buid amplifiers in four possible configurations:

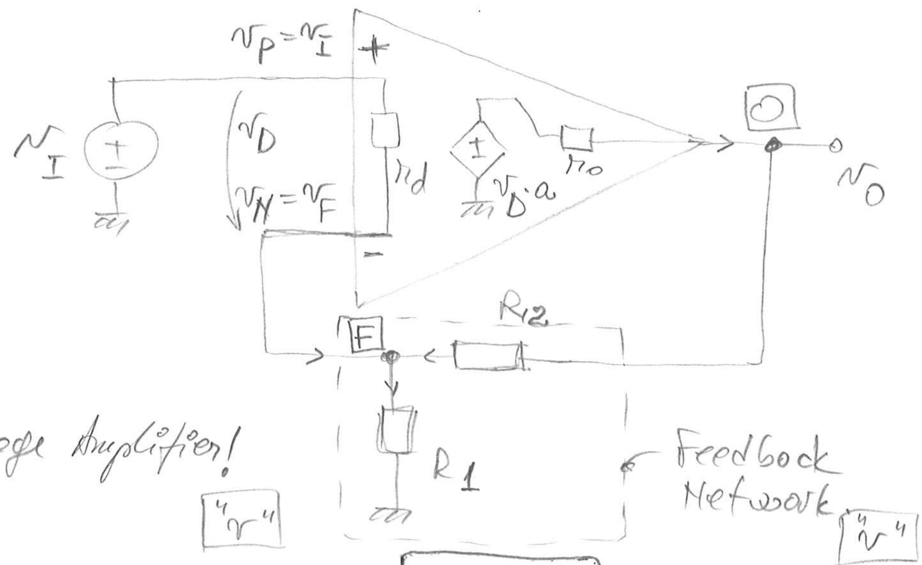
	Input compare	Output sample	Type of amplifier	
shunt - shunt	i	v	$A_r = \frac{v}{i}$	transimpedance
series - shunt	v	v	$A_v = \frac{v}{v}$	voltage
series - series	v	i	$A_g = \frac{i}{v}$	transconductance
shunt - series	i	i	$A_i = \frac{i}{i}$	current

I Example 1: Noninverting configuration.

Method 1 → follows the textbook

(a)





Feedback: series-shunt

$\left\{ \begin{array}{l} \text{Sampling: voltage} \\ \text{Comparing: voltage} \end{array} \right| \Rightarrow$ voltage Amplifier! "v"

$$\boxed{A = \frac{v_O}{v_I} = ?}$$

$$\left\{ \begin{array}{l} \dfrac{v_I - v_F}{r_d} - \dfrac{v_F}{R_1} + \dfrac{v_O - v_F}{R_2} = 0 \\[2ex] \dfrac{v_O - v_F}{R_2} = \dfrac{(v_I - v_F)a - v_O}{r_o} \end{array} \right.$$

Eliminate v_F and derive (see page 2') the expression for:

details on exercise

$$A_v = \frac{v_O}{v_I} = \frac{\left(1 + \frac{R_2}{R_1}\right)a + \underbrace{\frac{r_o}{r_d}}_{\cong 0}}{1 + a + \frac{R_2}{R_1} + \underbrace{\frac{(R_2 + r_o)}{r_d}}_{\cong 0} + \underbrace{\frac{r_o}{R_1}}_{\cong 0}}$$

$$(1) \quad \boxed{A \cong \left(1 + \frac{R_2}{R_1}\right) \cdot \frac{a}{\left(1 + \frac{R_2}{R_1}\right) + a} = \frac{1}{\beta} \cdot \frac{T}{1 + T}}$$

where $\beta = \dfrac{R_1}{R_1 + R_2}$

$$\frac{1}{\beta} = 1 + \frac{R_2}{R_1} = A_{ideal}.$$

$$\boxed{A \cong A_{ideal} \cdot \frac{T}{1 + T}}$$

$T = a\beta \triangleq$ loop gain.

Derivation of $A_v = \frac{v_O}{v_I}$ — ②'

$$\begin{cases} \dfrac{v_I - v_F}{r_d} - \dfrac{v_F}{R_1} + \dfrac{v_O - v_F}{R_2} = 0 \\[2ex] \dfrac{v_O - v_F}{R_2} = \dfrac{v_D \cdot a - v_O}{r_o} = \dfrac{(v_I - v_F)a - v_O}{r_o} \end{cases}$$

$$\begin{cases} g_d(v_I - v_F) - g_1 v_F + g_2(v_O - v_F) = 0 \iff \underbrace{(g_d + g_1 + g_2)}_{\triangleq g} v_F = g_d v_I + g_2 v_O \\ g_2 v_O - g_2 v_F = g_o a(v_I - v_F) - g_o v_O \end{cases}$$

$$v_F = \frac{g_d v_I + g_2 v_O}{g}$$

$$g_2 v_O + g_o v_O = g_o a v_I + (g_2 - a g_o) v_F$$

$$(g_2 + g_o) v_O = a g_o v_I + (g_2 - a g_o)\frac{g_d}{g} v_I + (g_2 - a g_o)\frac{g_2}{g} v_O$$

$$\left[(g_o + g_2) - (g_2 - a g_o)\frac{g_2}{g}\right] v_O = \left[a g_o + (g_2 - a g_o)\frac{g_d}{g}\right] v_I$$

$$\Rightarrow A_v = \frac{v_O}{v_I} = \frac{a g_o + (g_2 - a g_o)\cdot \frac{g_d}{g}}{g_o + g_2 - (g_2 - a g_o)\cdot\frac{g_2}{g}}$$

$$A_v = \frac{a g_o g + g_2 g_d - a g_o g_d}{g_o g + g_2 g - g_2^2 + a g_o g_2}$$

$$= \frac{a g_o(g_1 + g_2 + \cancel{g_d}) + g_2 g_d - \cancel{a g_o g_d}}{g_o(g_1 + g_2 + g_d) + g_2(g_1 + \cancel{g_2} + g_d) - \cancel{g_2^2} + a g_o g_2}$$

$$= \frac{a g_o(g_1 + g_2) + g_2 g_d}{g_o g_2(1 + a) + g_o(g_1 + g_d) + g_2(g_1 + g_d)}$$

$$\boxed{A_v = \frac{a g_o(g_1 + g_2) + g_2 g_d}{g_o g_2(1 + a) + (g_o + g_d)(g_1 + g_d)}}$$

The quantity:

$$\lim_{a\to 0} A = \frac{r_o/r_d}{1+\frac{R_2}{R_1}+(R_2+r_o)/r_d+\frac{r_o}{R_1}} \triangleq \text{feedthrough gain.}$$

- it refers to signal transmission from input → output thru the feedback network! Negligible to keep things simple.

b Input resistance:

Apply $\boxed{V}$ test source ⇒ $R_i = r_d\left(1+\frac{a}{1+(R_2+r_o)/R_1}\right) + \underbrace{R_1 \| (R_2+r_o)}_{\cong 0}$.

$$R_i \cong r_d\left(1+\frac{a}{\underbrace{1+\frac{R_2}{R_1}}_{1/\beta}}\right) = r_d(1+a\beta) = \boxed{r_d(1+T) = R_i} \quad (2)$$

c Output resistance

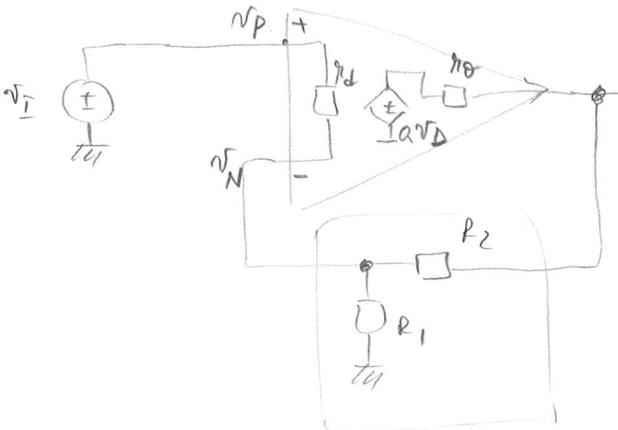
Apply $\boxed{V}$ test source ⇒

$$\Rightarrow R_o = \frac{r_o}{1+\left(a+\frac{r_o}{R_1}+\frac{r_o}{r_d}\right)/\left(1+\frac{R_2}{R_1}+\frac{R_2}{r_d}\right)}$$

$$\boxed{R_o \cong \frac{r_o}{1+T}} \quad (3)$$

Method 2: Uses the technique based on circuit "a" and circuit "β"

- series input
- comparison of voltage $\boxed{v}$

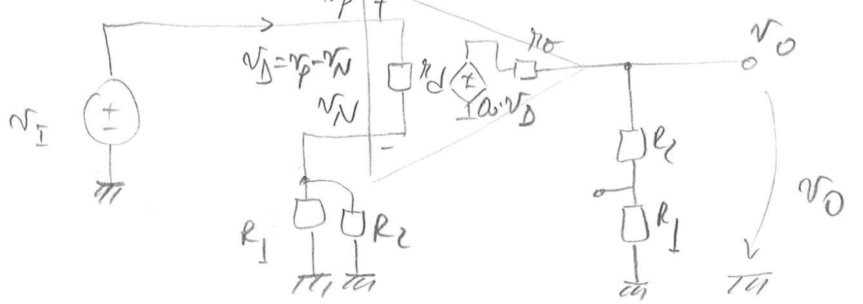


- parallel/shunt output
- sampling of voltage. $\boxed{v}$

Hence, this amplifier is best studied as a <u>voltage amplifier!</u>

$$A_v = \frac{a_v}{1+a_v \cdot \beta_v} = \frac{1}{\beta_v} \cdot \frac{T}{1+T} \quad ; \quad T = a_v \cdot \beta_v$$

(i) To find a_v we use circuit "a"



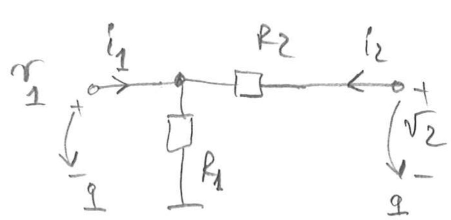
$$a_v = \frac{v_O}{v_I}$$

$$v_D = \frac{r_d \to \infty}{r_d + R_1 \| R_2} \cdot v_I \cong v_I$$

$$v_O = \frac{R_1+R_2}{R_1+R_2+r_o} \cdot (a v_D) \cong a v_D = a v_I \Rightarrow \boxed{a_v = \frac{v_O}{v_I} = a}$$

the open loop gain of the Op Amp!

(ii) To find β_v we use circuit "β"



$$\boxed{\beta_v = \left.\frac{v_1}{v_2}\right|_{i_1=0} = \frac{R_1}{R_1+R_2}}$$

Analyzed with "h" parameters

Therefore the final closed loop amplification is:

$$A_v = \frac{1}{\beta_v} \times \frac{Q_v \beta_v}{1 + Q_v \beta_v} = \frac{1}{\beta_v} \times \frac{T}{1+T} = \underbrace{\left(1 + \frac{R_2}{R_1}\right)}_{= A_{v,ideal}^{o}} \cdot \frac{Q \cdot \frac{R_1}{R_1 + R_2}}{1 + Q \cdot \frac{R_1}{R_1 + R_2}} \quad (4)$$

Note: → This is exactly the amplification we were interested in the first place!

→ Compare it to equation (1) from Method 1: here we found the result via a simpler and more general technique!

II Example 2 Inverting Configuration

Method 1 (Textbook): We could have worked directly with voltages (like in the case of non-inverting) and arrive to:

(a)

$$\boxed{A = \frac{v_O}{v_I} = -\frac{aR_2 - r_o}{(1+a)R_1 + (R_2+r_o)\left(1+\frac{R_1}{r_d}\right)}} \quad (5) \Rightarrow$$

Typically $r_o \ll R_2$, $\frac{R_1}{r_d} \ll 1$

$$\Rightarrow A \cong \left(-\frac{R_2}{R_1}\right) \cdot \frac{T}{1+T} \quad \text{with the loop gain } T = \frac{aR_1}{R_1+R_2}$$

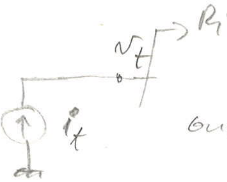
Observation: **Method 2** ← is better because fits the well established technique of studying circuits with feed back using the "a" and "β" (or "f") "circuits", and then just plug them into: $A = A_{ideal} \cdot \frac{T}{1+T} = \frac{1}{\beta} \cdot \frac{a\beta}{1+a\beta}$.

What we did here is we used the Op Amp model with $\{r_d, r_o, av_D\}$ and used Kirkoff's laws to derive $\frac{v_O}{v_I}$ or $\frac{v_O}{i_I}$. After that we took the expression so that to identify $A = A_{ideal} \cdot \frac{T}{1+T}$. The other (more correct way) is to treat the amplifier as A_v, A_i, A_r, A_g and use the technique based on "a" and "f" circuit to derive $A = \frac{1}{\beta} \cdot \frac{a\beta}{1+a\beta}$ directly!

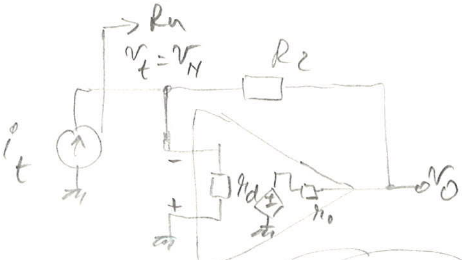
The feedthrough gain:

$$\lim_{a \to 0} A = \frac{r_o}{R_1 + (R_2+r_o)(1+R_1/r_d)}$$

(b) — Input resistance: Apply test current



i_t and measure v_t,



then $R_n = \frac{v_t}{i_t} = \frac{R_2 + r_o}{1 + a + \left(\frac{R_2+r_o}{r_d}\right)} \xrightarrow{} 0$, $r_o \to 0$ $\Rightarrow$

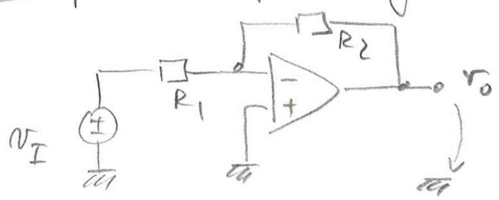
≜ Miller Effect

$$\Rightarrow R_n \cong \frac{R_2}{1+a} \Rightarrow \boxed{R_i = R_1 + R_n \cong R_1} \quad (6)$$

(c) Output resistance (similar to noninverting config):

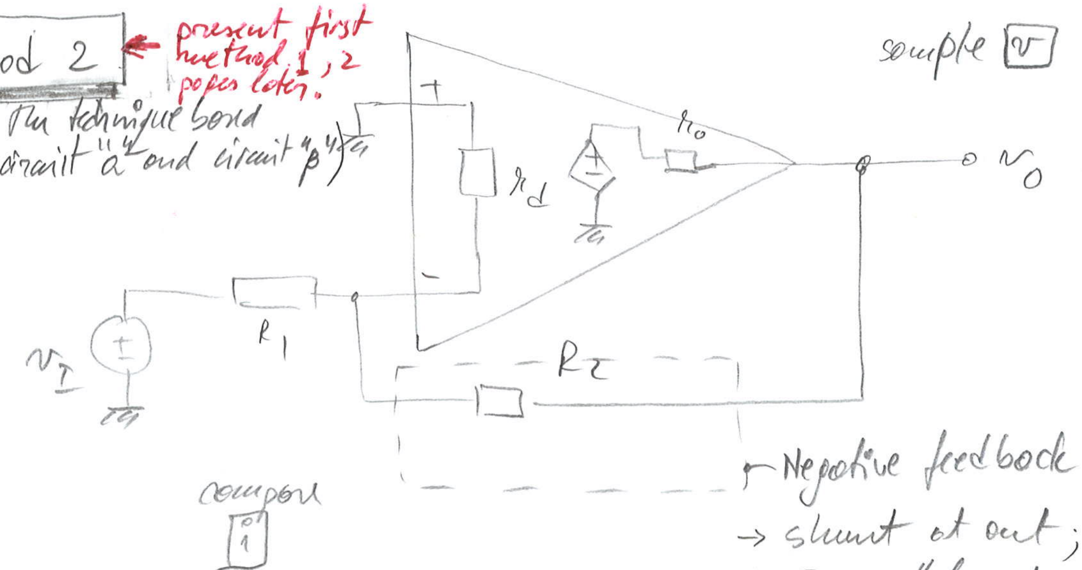
$$\boxed{R_o \cong \frac{r_o}{1+T}} \quad (7)$$

II Example 2 — Inverting configuration



Method 2 ← present first method 1, 2 pages later.

(using the technique based on circuit "a" and circuit "β")



→ Negative feedback network

→ shunt at out ; shunt at input

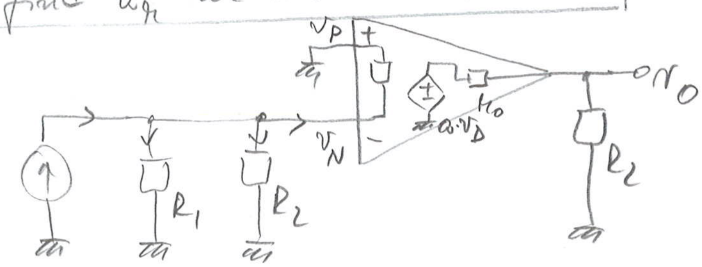
→ or parallel out ; parallel input

→ or sampling of <u>voltage</u> ; comparison of <u>current</u>

Hence, this amplifier is best studied as a <u>transimpedance</u> amplifier.

$$A_r = \frac{a_r}{1 + a_r \beta_g} = \frac{1}{\beta_g} \cdot \frac{T}{1+T} \; ; \quad \boxed{T = a_r \beta_g}$$

(i) To find a_r we use circuit "a"



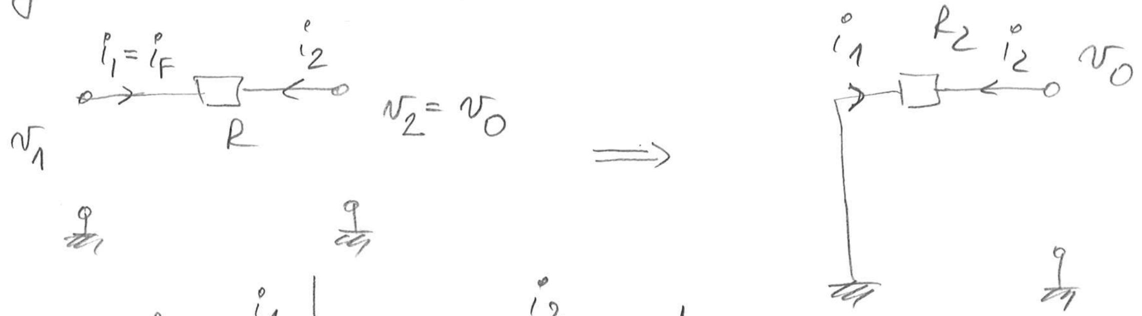
$$i_I = \frac{v_I}{R_1}$$

$$\left.\begin{aligned} v_D &= v_P - v_N = 0 - v_N = -v_N \\ i_I &= \frac{v_N}{R_1} + \frac{v_N}{R_2} + \frac{v_N}{r_d} = v_N \left(\frac{1}{R_1} + \frac{1}{R_2} + \underbrace{\frac{1}{r_d}}_{\simeq 0} \right) \simeq v_N \underbrace{\left(\frac{1}{R_1} + \frac{1}{R_2} \right)}_{= 1/R_1 \| R_2} \end{aligned}\right\} \Rightarrow v_D = -R_1 \| R_2 \cdot i_I$$

$$v_O = \frac{R_2}{R_2 + \underbrace{r_o}_{\simeq 0}} (a \cdot v_D) \simeq a \cdot v_D = -a R_1 \| R_2 \cdot i_I$$

$$\Rightarrow a_r = \frac{v_O}{i_I} = -a R_1 \| R_2 = \boxed{-a \frac{R_1 R_2}{R_1 + R_2} = a_r}$$

To find β_g we use circuit "β"



$$\beta_g = \left.\frac{i_1}{v_2}\right|_{v_1=0} = -\frac{i_2}{v_2} = -\frac{1}{R_2}$$

Analyzed with "y" parameters.

$$\boxed{\beta_g = -\frac{1}{R_2}}$$

Finally, the closed loop gain/amplification:

$$A_r = \frac{v_O}{i_I} = \frac{a_r}{1 + a_r \cdot \beta_g} = \frac{1}{\beta_g} \cdot \frac{T}{1+T} \quad ; \quad T = a_r \cdot \beta_g = -a(R_1 \| R_2) \cdot \left(-\frac{1}{R_2}\right) = a \cdot \frac{R_1}{R_1 + R_2}$$

$$\boxed{A_r = (-R_2) \times \frac{a \dfrac{R_1}{R_1+R_2}}{1 + a \dfrac{R_1}{R_1+R_2}}} \qquad (8)$$

if we want the voltage amplification:

$$\boxed{A = \frac{v_O}{v_I}} = \underbrace{\frac{v_O}{i_I}}_{=A_r} \cdot \underbrace{\frac{i_I}{v_I}}_{=1/R_1} = \underbrace{\left(-\frac{R_2}{R_1}\right)}_{=A_{ideal}} \times \frac{a \cdot \dfrac{R_1}{R_1+R_2}}{1 + a \dfrac{R_1}{R_1+R_2}} \qquad (9)$$

NOTE: Compare this to equation (1.62) from textbook; whose derivation is done like of the noninverting amplifier (using KCL, KVL)! Also, compare to eq. 15) from Method 1

Note: Compare this method 1 to page 36 from Book.

1.7 The Loop Gain

← Homework assignment or next time?

Generally, for Op Am circuits, the voltage gain is:

$$\boxed{A = A_{ideal} \cdot \frac{T}{1+T}}$$

found using the ideal op amp model (using the "virtual-short" technique)

$$\boxed{R \cong r \times (1+T)^{\pm 1}}$$

$r \equiv$ the open-loop resistance calculated in the limit $a \to 0$.