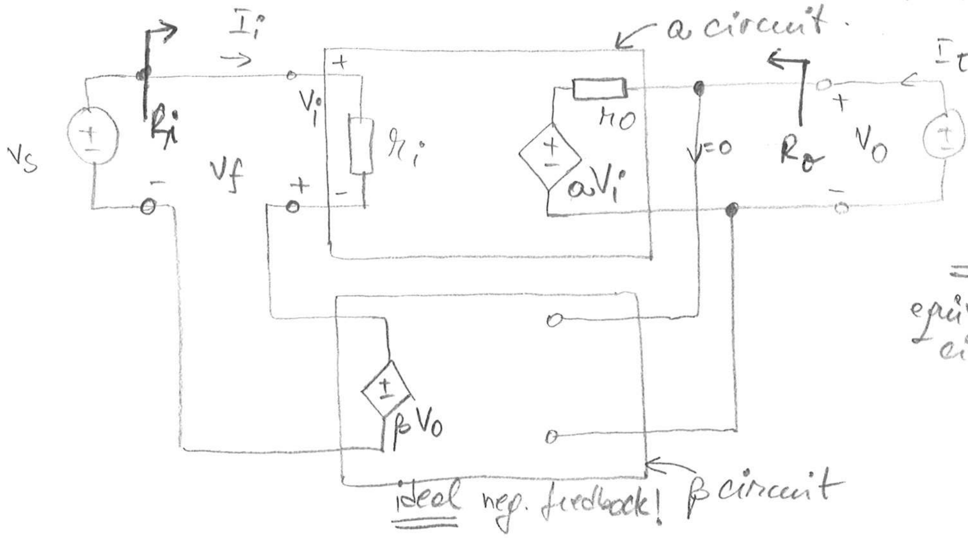
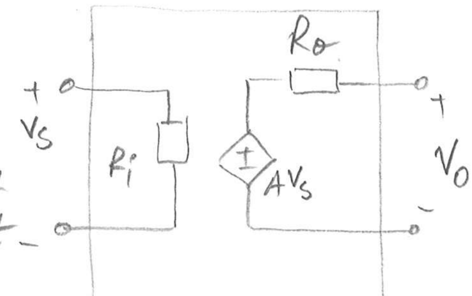
General detailed analysis of in/out resistances

1 Input/output resistances in closed-loop (negative-feedback) amplifiers → General case.

Series-in, shunt-out feedback amplifier (compare voltage [V], sample voltage [V])



⇒ equivalent circuit



$$A = \frac{V_o}{V_s} = \frac{a}{1+a\beta}$$

a, β have reciprocal units!

INPUT

$$R_i = \frac{V_s}{I_i} = \frac{V_s}{V_i/r_i} = r_i \frac{V_s}{I_i} = r_i \frac{V_i + \beta a V_i}{V_i}$$

$$(1) \quad \boxed{R_i = r_i(1+a\beta)}$$

Because the feedback voltage V_f substract from V_s, the voltage that appears across r_i becomes small ⇒ current I_i is small, which makes for R_i to "appear" large!

OUTPUT

$$I_t = \frac{V_t - aV_i}{r_o} = \frac{V_t + a\beta V_t}{r_o} \Rightarrow R_o = \frac{V_t}{I_t} = \boxed{\frac{r_o}{1+a\beta} = R_o} \quad (2)$$

$$\underset{V_s=0}{V_i} = -V_f = -\beta V_t$$

Note: (1) and (2) can be generalized to impedances: (1') $Z_i(jf) = z_i(jf)\left[1 + a(jf)\cdot\beta(jf)\right]$

$$(2') \quad Z_o(jf) = \frac{z_o(jf)}{1 + a(jf)\cdot\beta(jf)}$$

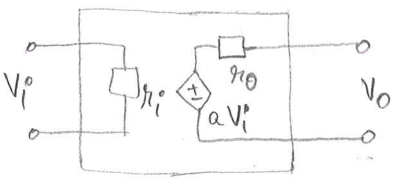
Crucial observation : - when the negative feedback network is not ideal, it will load the open-loop amplifier both at input and output!

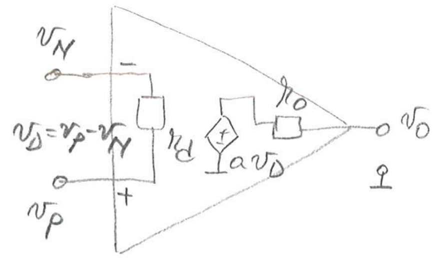
- These loadings can be included into the base amplifier by incorporating the parameters with wich we study the feedback network (example "h" parameters for series-in, shunt-out) into r_i and r_o. What's left from the feedback network is ideal!

very important!

→ Loading can be found by "looking" into feedback network with the other end shorted or left open (for shunt or series)

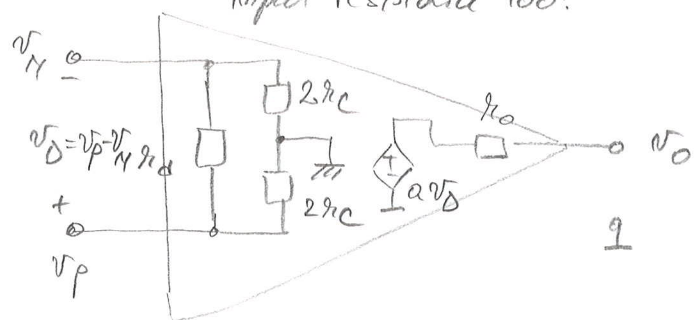
2 The particular case of the OpAmp as the base-amplifier (or the error amplifier)





model 1

or include also the common-mode input resistance too:

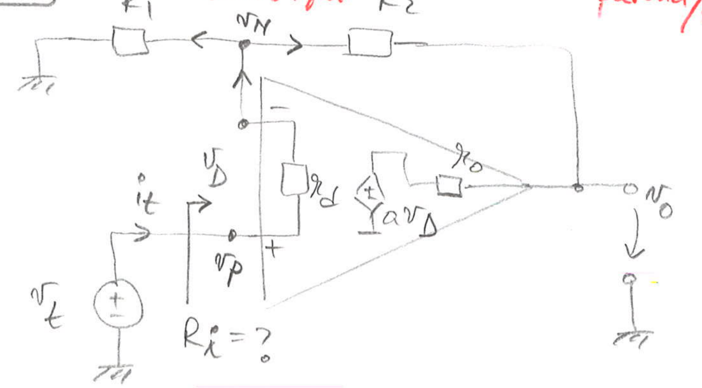


(a) Non-inverting configuration | Method 1 | "Brute-force" v_t/i_t

INPUT

-series input

- parallel/shunt out



Because series at the input is the neg. feedback topology we know and expect:

$$\boxed{R_i = r_i(1+T)}$$

where $r_i \cong r_d$

Verification: KCL at node v_N:

$$\frac{v_P - v_N}{r_d} = \frac{v_N}{R_1} + \frac{v_N - a v_D}{r_o + R_2}$$

$$\frac{v_t - v_N}{r_d} - \frac{v_N}{R_1} - \frac{v_N - a(r_d \cdot i_t)}{r_o + R_2} = 0 \;; \qquad v_N = v_t - r_d \cdot i_t$$

$$\Rightarrow \quad v_t\left(-\frac{1}{R_1} - \frac{1}{r_o + R_2}\right) + i_t + \frac{r_d}{R_1} i_t + \frac{r_d}{r_o + R_2} i_t + \frac{a r_d}{r_o + R_2} i_t = 0$$

$$v_t \underbrace{\left(\frac{1}{R_1} + \frac{1}{r_o + R_2}\right)}_{\triangleq \frac{1}{R_P}} = i_t\left[1 + r_d\left(\underbrace{\frac{1}{R_1} + \frac{1}{r_o + R_2}}_{\triangleq \frac{1}{R_P}} + \frac{a}{r_o + R_2}\right)\right]$$

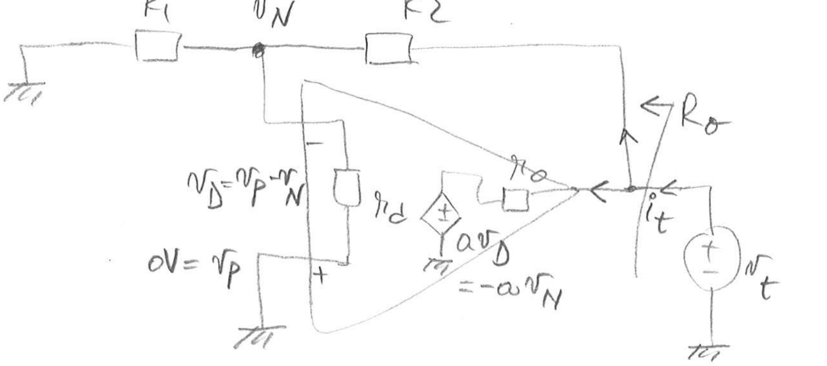
$$\frac{v_t}{i_t} = R_i = R_p + r_d R_p \left(\frac{1}{R_p} + \frac{a}{r_o + R_2} \right)$$

$$= \underbrace{\cancel{R_1 \| (r_o + R_2)}}_{\ll r_d} + r_d \left(1 + \frac{a}{\cancel{r_o + R_2}} \cdot \frac{R_1 \cdot \cancel{(r_o + R_2)}}{R_1 + r_o + R_2} \right)$$

$$\cong r_d \left(1 + \frac{a}{1 + \frac{r_o + R_2}{R_1}} \right) \cong r_d \left(1 + \frac{a}{\underbrace{1 + \frac{R_2}{R_1}}_{= \frac{1}{\beta}}} \right) = r_d (1 + a\beta)$$

$$\boxed{R_i = r_d (1 + T)}$$ exactly what we expected!

OUTPUT



Because the neg. feedback is shunt at output, we expect that:

$$\boxed{R_o = \frac{r_o}{1 + T}}$$

where $r_o \cong r_o$

<u>Verification:</u>

$$v_N = \frac{R_1 \| r_d}{R_1 \| r_d + R_2} \cdot v_t \quad ; \text{ Let } c \triangleq \frac{R_1 \| r_d}{R_1 \| r_d + R_2}$$

KCL at output node: $$i_t = \frac{v_t - (-a v_N)}{r_o} + \frac{v_t - v_N}{R_2}$$

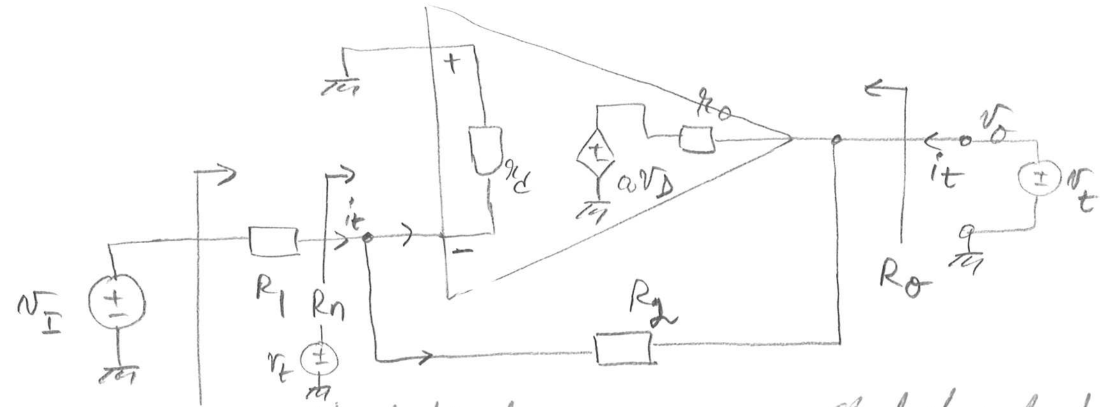
$$\Rightarrow i_t = \left(\frac{1 + ac}{r_o} + \frac{1 - c}{R_2} \right) v_t = \frac{(1 + ac) R_2 + (1 - c) r_o}{r_o R_2} v_t$$

$$\Rightarrow \frac{v_t}{i_t} = R_o = \frac{r_o R_2}{(1 + ac) R_2 + (1 - c) r_o} = \frac{r_o}{1 + ac + (1 - c) \frac{r_o}{R_2}} = \ldots =$$

$$= \frac{r_o}{1 + \left(a + \cancel{\frac{r_o}{R_1}} + \cancel{\frac{r_o}{r_d}} \right) / \left(1 + \frac{R_2}{R_1} + \cancel{\frac{R_2}{r_d}} \right)} \cong \frac{r_o}{1 + \frac{a}{\underbrace{1 + \frac{R_2}{R_1}}_{= \frac{1}{\beta}}}} = \frac{r_o}{1 + a\beta} =$$

$$\boxed{R_o = \frac{r_o}{1 + T}}$$ what we expected!

(b) inverting configuration



– short at input — short at output.

Because of neg. feedback, we expect:

$\boxed{R_i = R_1 + R_n}$ and: $\boxed{R_o = \frac{r_o}{1+T}}$; where $r_o = r_o$

where $R_n = \frac{r_i}{1+T}$; $T = a r_i \beta$!!!

INPUT

$$R_n = \frac{v_t}{i_t} = \frac{R_2 + \cancel{r_o}}{1 + a + \cancel{\frac{R_2 + r_o}{r_d}}} \cong \frac{R_2}{1+a}$$

$$\boxed{R_i = R_1 + R_n = R_1 + \frac{R_2}{1+a}} \cong R_1$$

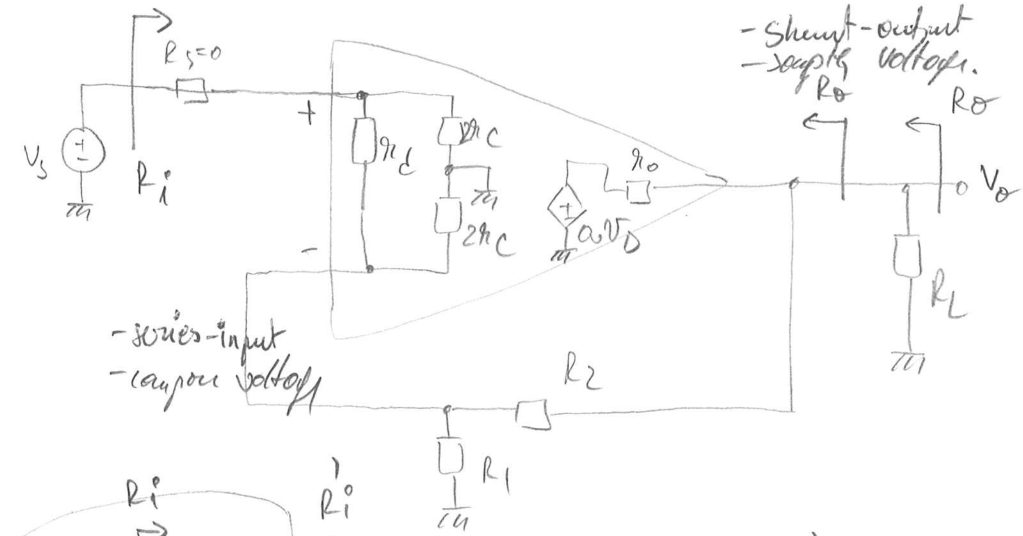
OUTPUT

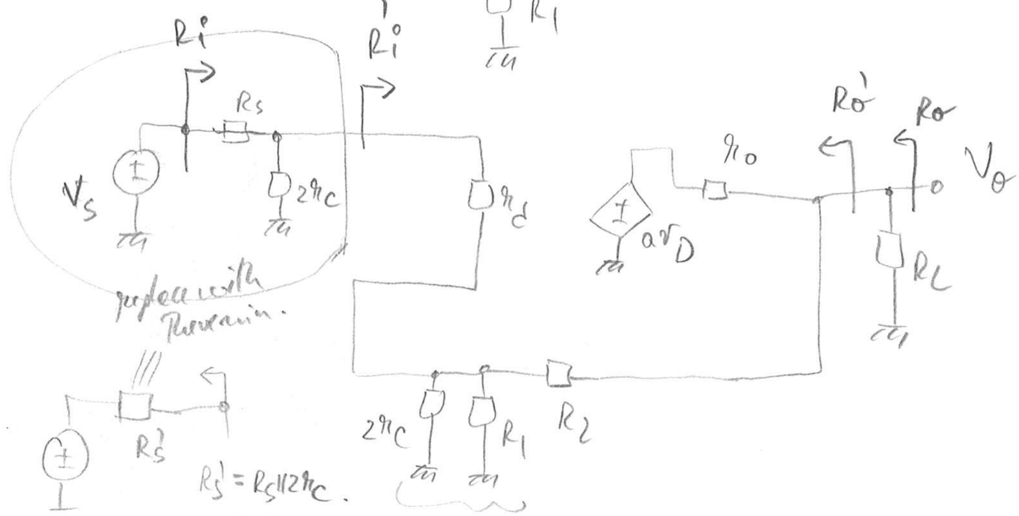
The same derivation as in the case of the non-inverting config:

$$\boxed{R_o \cong \frac{r_o}{1+T}}$$ ✓

(a) Non-inverting configuration "reloaded" → analysis as negative feedback amplifier.

Method 2





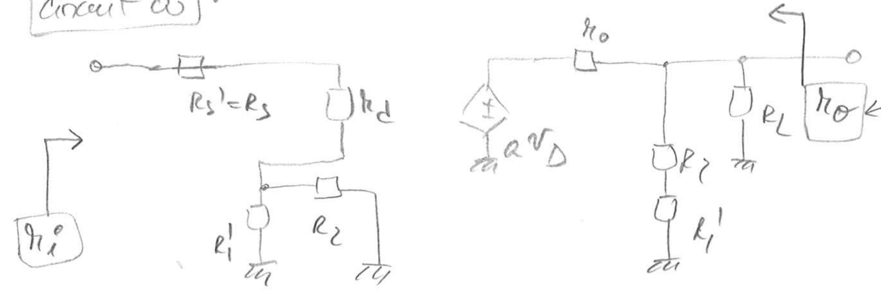
$R_S' = R_S \| 2r_C = R_S$ if $r_C=\infty$

$V_S \cdot \dfrac{2r_C}{2r_C+R_S}$

$R_1' = R_1 \| 2r_C = R_1$ if $r_C=\infty$

- input resistance is $\boxed{R_i = (2r_C)\|R_i'}$, where: $\boxed{R_i' = r_i(1+a_v\beta_v)}$ with $\boxed{r_i}$ being the input into circuit a, which has loading due to neg. feedback network included!

$\boxed{\text{circuit a}}$:



$aV_D=0$ because nothing is applied at the input! when computing output resistance!

- $r_i = R_S' + r_d + R_1'\|R_2 \underset{\uparrow}{=} r_d + \underbrace{R_1\|R_2}_{\text{loading}} \cong r_d.$

  for ideal voltage source V_S, $R_S=0$ and with $r_C=\infty$

- $\boxed{R_i = (2r_C)\,\|\,\left[r_d(1+a_v\beta_v)\right]}$

– Output resistance:

$$\boxed{R_o = \frac{r_o}{1+a_r\beta_g}}$$

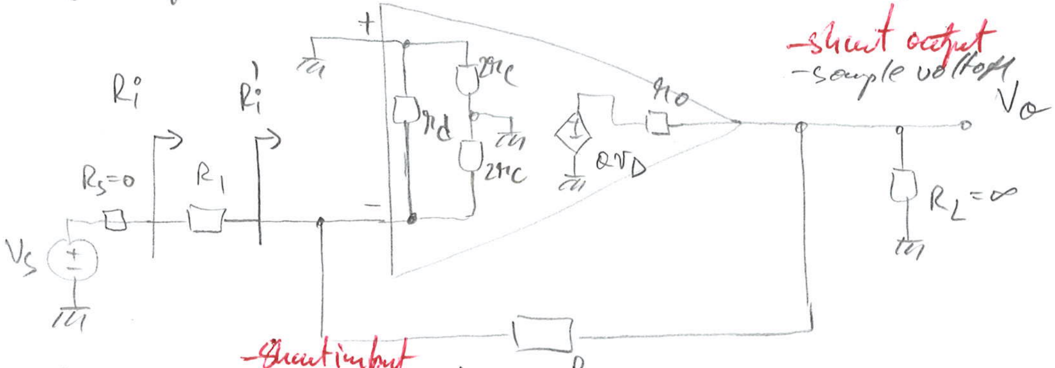
where $\boxed{r_o}$ being the output resistance of circuit a, with loading included!

Assume $R_L = \infty$ (no load); then $r_o = R_L \parallel (R_2 + R_1') \parallel r_o = (R_2+R_1') \parallel r_o \cong r_o$

Hence: $\boxed{R_o = \frac{r_o}{1+a_r\beta_g}} = \frac{r_o}{1+a\beta}$ ← happens to be exactly $a\cdot\beta$, with a being the open-loop gain of the OpAmp!!! & $1/\beta = 1+\frac{R_2}{R_1}$

(b) Inverting configuration → $\boxed{\text{Method 2}}$ analysis as negative feedback.

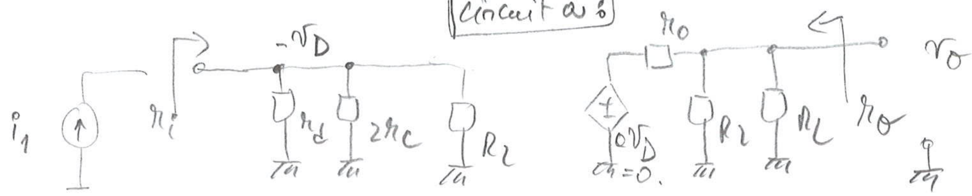
– shunt output
– sample voltage



Input resistance: – shunt input – compare current

$\boxed{R_i = R_1 + R_i'}$; $\boxed{R_i' = \frac{r_i}{1+a_r\beta_g}}$ with $\boxed{r_i}$ being input resistance in circuit a.

$\boxed{\text{circuit a:}}$



$r_i = r_d \parallel (2r_c) \parallel R_2 \cong R_2$

$$\boxed{R_i = R_1 + \frac{R_2}{1+a_r\beta_g}} = \boxed{R_1 + \frac{R_2}{1+a}}$$

same as that by method 1!?!?

Output resistance:

$$\boxed{R_o = \frac{r_o}{1+a_r\beta_g}}$$

$r_o = \underbrace{R_L}_{=\infty} \parallel \underbrace{R_2}_{>r_o} \parallel r_o \cong r_o \Rightarrow \boxed{R_o \cong \frac{r_o}{1+a_r\beta_g}}$

$\boxed{a_r}$ is found with circuit "a": $a_r = \frac{v_o}{i_1} = \frac{R_2 \parallel R_L}{R_2 \parallel R_L + r_o} \cdot a v_D \cdot \frac{1}{i_1} \cong \frac{R_2}{R_2 + r_o} \cdot a(-\cancel{i_1}) \frac{1}{\cancel{i_1}} (r_d \parallel R_2 \parallel 2r_c)$

$= -a \frac{R_2}{R_2+r_o} \cdot R_2 \cong -a R_2 \Big\} \Rightarrow \boxed{a_r \cdot \beta_g = a}$ of the OpAmp!

$\boxed{\beta_g}$ can be found with circuit "β":

$\Rightarrow \beta_g = -\frac{1}{\ }$