5 The natural response using H(s)

How to use H(s) to derive the natural response of a circuit? This is the case when there are no applied signals!

Note: Subtle obs. is that all this discussion can be done w/o having Laplace transform introduced!!! Talk in class about it!

$$H(s) = K\,\frac{(s-z_1)(s-z_2)\cdots(s-z_m)}{(s-p_1)(s-p_2)\cdots(s-p_n)} = \frac{N(s)}{D(s)}$$

For a circuit to yield a response $y(t) \neq 0$: $\boxed{D(s)=0}$

- The values of s satisfying this equation are precisely the poles $p_1, p_2, \dots, p_n$ of H(s), indicating that the transfer/network function contains all the information needed to predict the functional form of the natural response! This is the real function:

$$\boxed{y(t) = A_1 e^{p_1 t} + A_2 e^{p_2 t} + \dots + A_n e^{p_n t}}$$

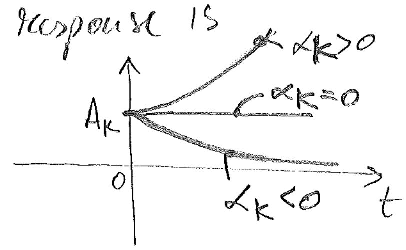
$A_1, A_2, \dots, A_n$: suitable time-independent coefficients reflecting the initial-conditions in the circuit.

case 1: Real poles: $\boxed{p_k = \alpha_k + j0}$

Its contribution to the natural response is

$$\boxed{y_k(t) = A_k \cdot e^{\alpha_k t}}$$

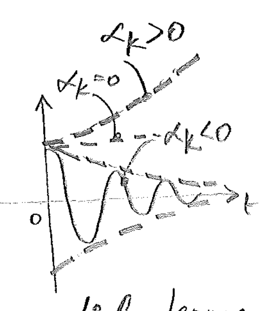
A_k real.



case 2: Complex pole pairs

$$\boxed{p_k = \alpha_k \pm j\omega_k}$$

Their combined contribution to the natural response is:



$$\boxed{y_k(t) = 2|A_k| \cdot e^{\alpha_k t} \cdot \cos(\omega_k t + \theta_k)}$$

where: $A_k = |A_k| \cdot \angle\theta_k$ gives the complex-exponential terms:

$\boxed{A_k \cdot e^{p_k t}}$ that corresponds to $p_k = \alpha_k + j\omega_k$

and

$\boxed{A_k^* \cdot e^{p_k^* t}}$ that corresponds to $p_k^* = \alpha_k - j\omega_k$

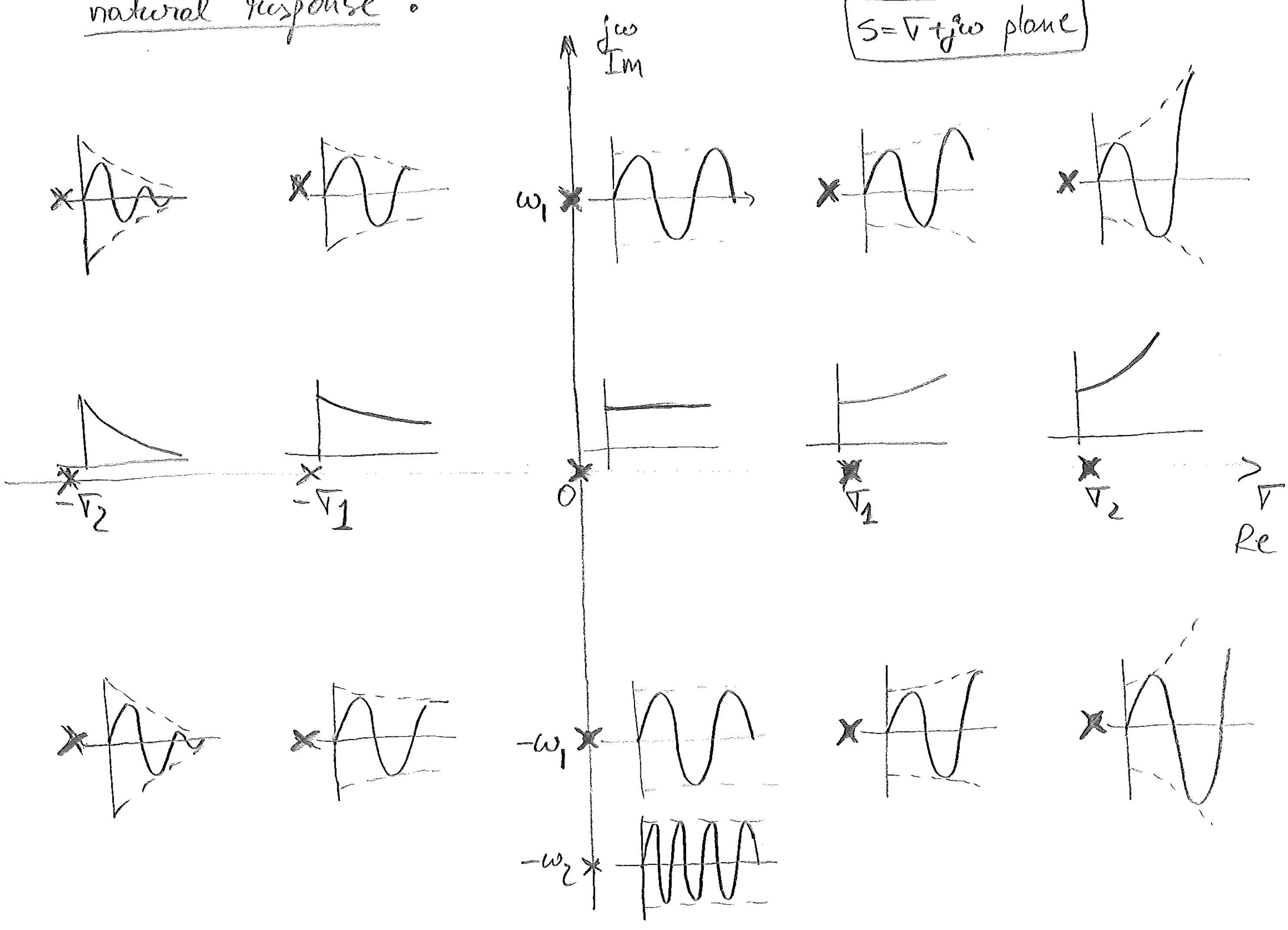
complex exponential signals!

$\boxed{\text{Case 3}}$ Repeated poles

If pole p_k has multiplicity r_k, then:

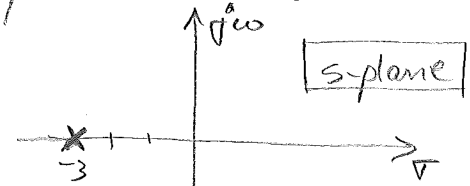
$$\boxed{y_k(t) = \left[A_{k,0} + A_{k,1} t + A_{k,2} \cdot t^2 + \dots + A_{k,r_k-1} \cdot t^{r_k - 1} \right] \cdot e^{p_k t}}$$ //

- A visual correspondence between the location of a pole in the s-plane and the functional form of its contribution to the natural response:



Example 1: Sketch the magnitude plot for admittance:

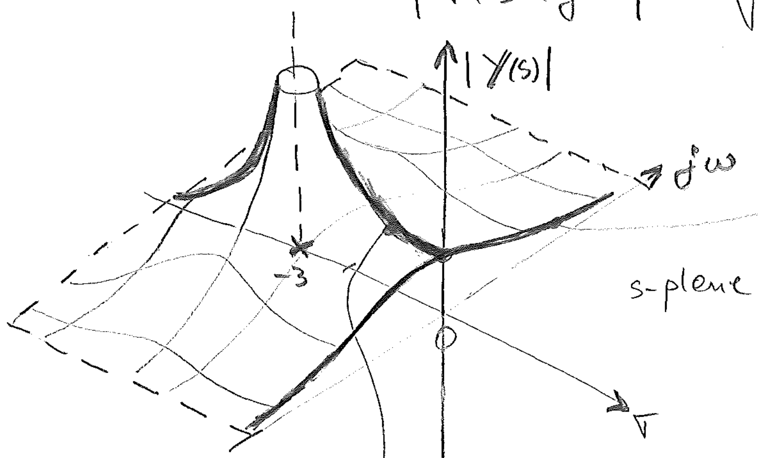
$$Y(s) = \frac{1}{s+3} \leftarrow \text{pole } p_1 = -3$$



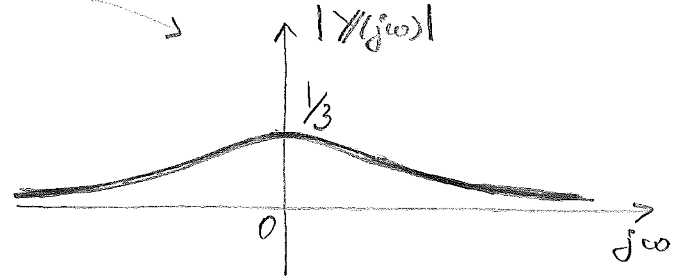
pole-zero pattern or constellation

$$|Y(s)| = \left| \frac{1}{\sigma + j\omega + 3} \right|$$

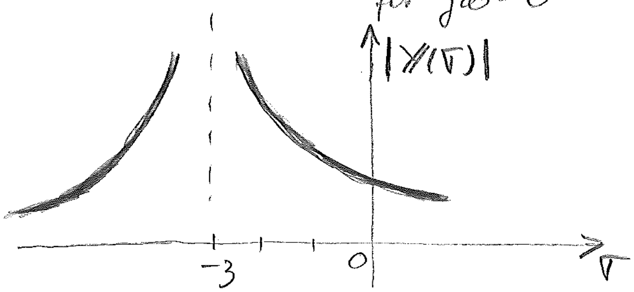
$$= \frac{1}{|\sigma + 3 + j\omega|} = \frac{1}{\sqrt{(\sigma+3)^2 + j\omega}}$$



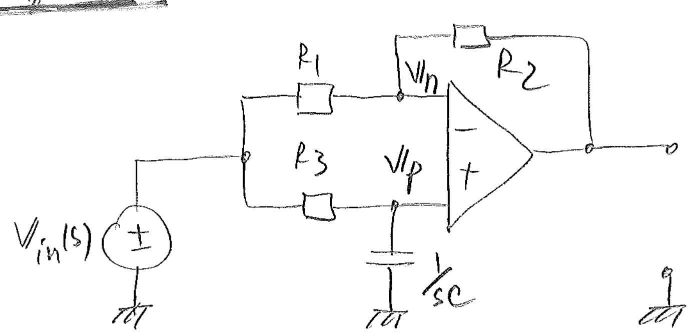
slice or cross-section for $\sigma = 0$



slice or cross-section for $j\omega = 0$



Example 2: Find the transfer function for the circuit:



$$H(s) = \frac{V_o(s)}{V_{in}(s)} = ?$$

Use superposition:

$$V_o = -\frac{R_2}{R_1} V_{in} + \left(1+\frac{R_2}{R_1}\right) V_p \qquad (1)$$

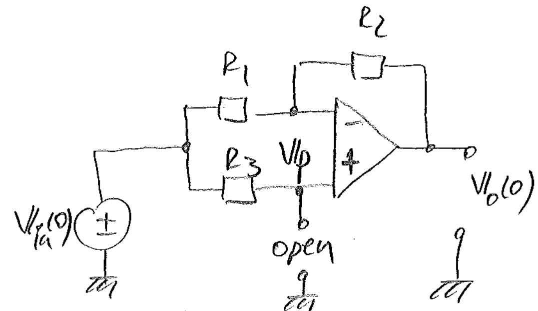
where: $$V_p = \frac{\frac{1}{sC}}{R_3 + \frac{1}{sC}} V_{in} = \frac{1}{R_3 C s + 1} V_{in} \qquad (2)$$ $\Rightarrow$

$$\Rightarrow \boxed{H(s) = \frac{V_o(s)}{V_{in}(s)} = \frac{-R_2 R_3 C \cdot s + R_1}{R_1 R_3 C \cdot s + R_1}}$$

$\boxed{\text{Asymptotic}}\boxed{\text{checks}}$:

Let $s \to 0 \Rightarrow H(0) = 1 \ \frac{V}{V}$

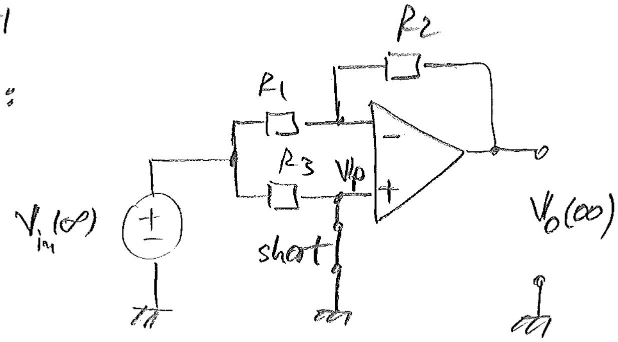
confirmation:



$$V_p = V_{in} \overset{(1)}{\Rightarrow} V_o = V_{in} \Rightarrow \frac{V_o}{V_{in}} = 1 \ (!)$$

Let $s \to \infty \Rightarrow H(\infty) = -\frac{R_2}{R_1}$

confirmation:



$$V_p = 0 \overset{(1)}{\Rightarrow} V_o = -\frac{R_2}{R_1} V_{in} \Rightarrow$$

$$\Rightarrow \frac{V_o}{V_{in}} = -\frac{R_2}{R_1} \ (!)$$