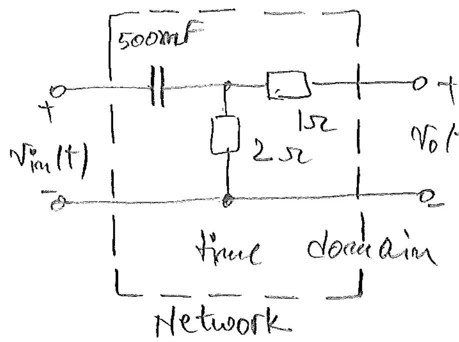
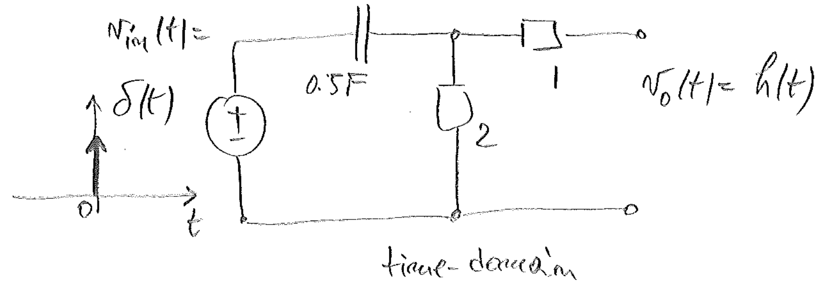
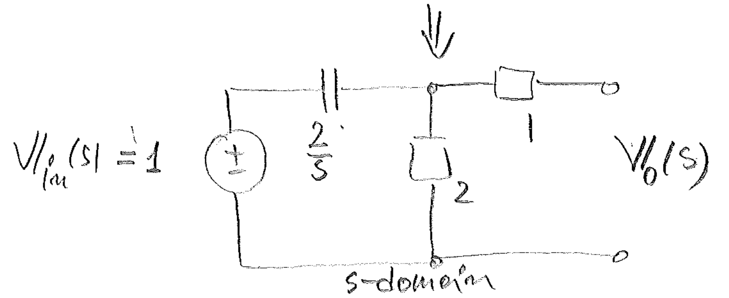
$\boxed{\text{Example:}}$ Find the impulse-response and then compute the forced response for $v_{in}(t) = 6e^{-t} \cdot u(t)$ [V]



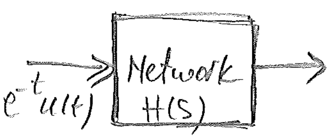




- Use voltage division: $\boxed{H(s) = V_0(s)/V_{in}(s) = \frac{2}{2+\frac{2}{s}} = \frac{s}{s+1}} = 1 - \frac{1}{s+1}$

- Therefore, $h(t) = \mathcal{L}^{-1}\{H(s)\} = \mathcal{L}^{-1}\left\{\frac{s}{s+1}\right\}$

- Now find the forced response of Network to forcing function:



$v_{in}(t) = 6e^{-t}u(t)$ $\updownarrow$ $V_{in}(s) = \frac{6}{s+1}$

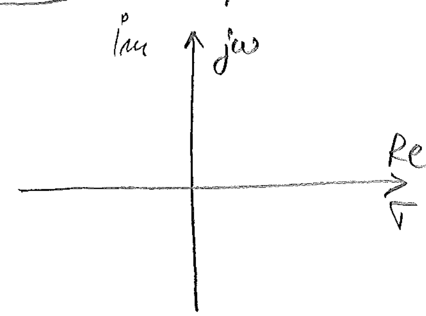
$v_0(t) = ?$ $\updownarrow$ $V_0(s) = H(s) \cdot V_{in}(s) = \frac{s}{s+1} \cdot \frac{6}{s+1}$

$$V_0(s) = \frac{6}{s+1} - \frac{6}{(s+1)^2}$$

- Finally: $\boxed{v_0(t) = \mathcal{L}^{-1}\{V_0(s)\} = 6e^{-t}(1-t) \cdot u(t)}$

4 The complex-frequency plane

- Complex frequency $\boxed{s = \sigma + j\omega}$ allows us to visulize it with the presence of the $\boxed{j}$ operator in a two-dimensional plane called $\boxed{\text{complex plane}}$ or $\boxed{\text{s-plane}}$.

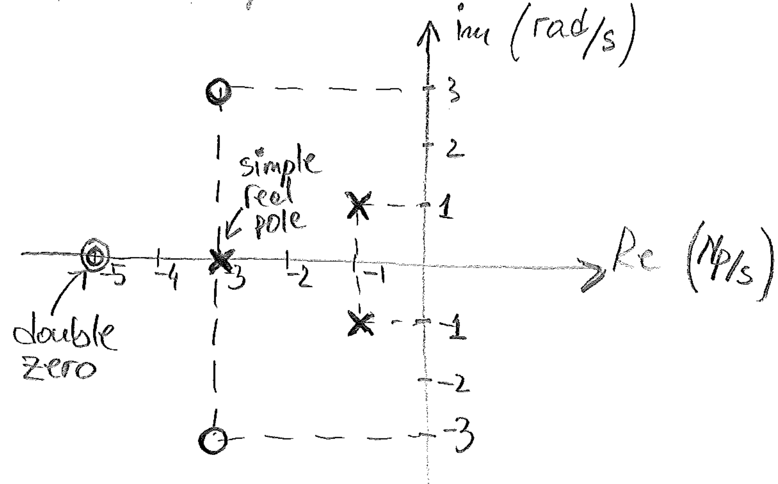


$$H(s) = K \frac{(s-z_1)(s-z_2)\cdots(s-z_m)}{(s-p_1)(s-p_2)\cdots(s-p_n)} \qquad K = \frac{a_m}{b_n} \text{ : scaling factor}$$

- Zeros and poles are collectively referred to as the $\boxed{\text{roots}}$ or the $\boxed{\text{critical frequencies}}$.
- They are special complex-frequency values that depend solely on the circuit, regardless of the applied of the applied signals or the energy conditions of its reactive elements!!
  They represent the "personality" of the circuit!!
- They are conviniently represented as points in the complex plane. This useful because essential properties can be identified by just looking at the pole-zero pattern in the s-plane!

$\boxed{\text{Example}}$

$$H(s) = 10 \frac{(s+5)^2 (s^2+4s+13)}{s^2 (s+3)(s^2+2s+2)}$$



pole-zero plot

In the <u>analysis</u> or <u>synthesis</u> of circuits it is very useful to plot the response as functions of σ and ω.

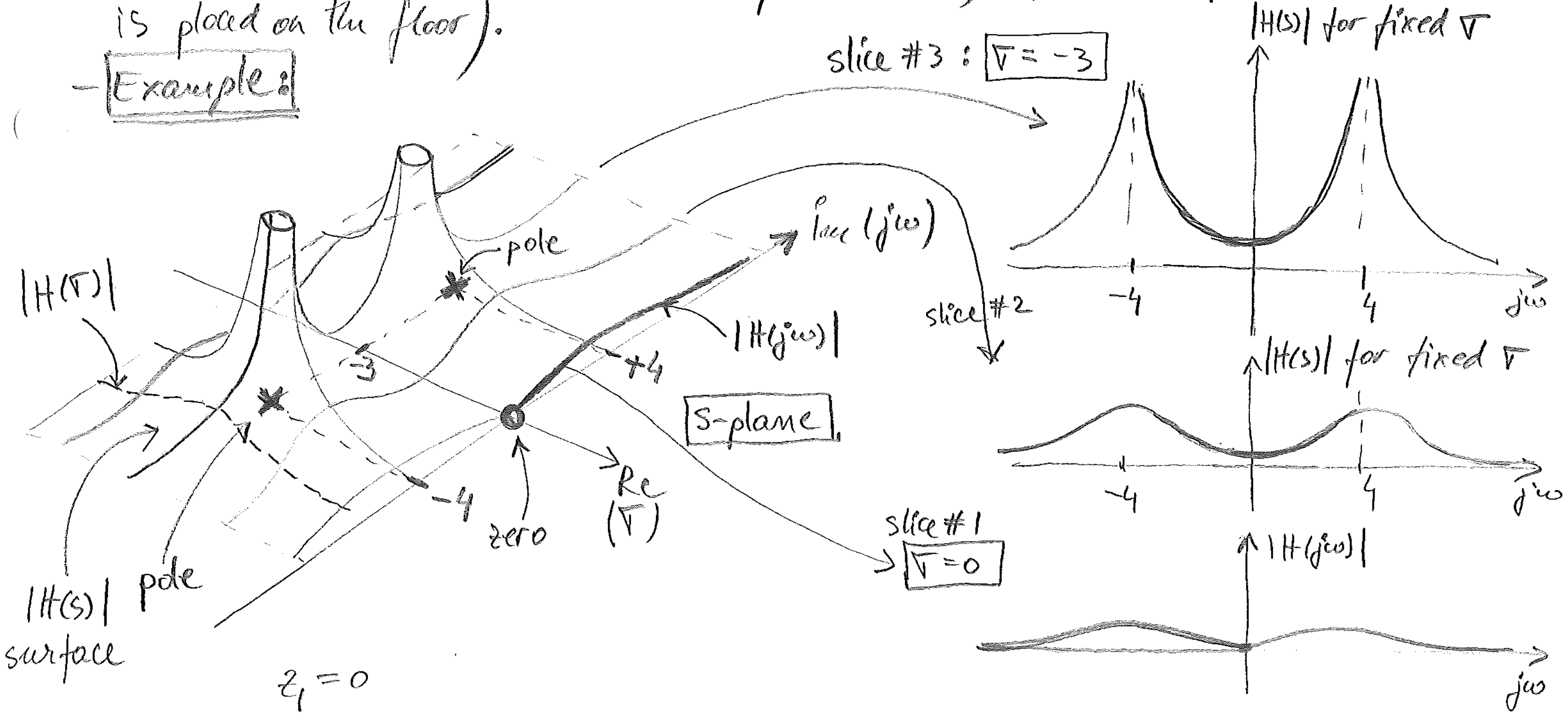
$$\mathbb{Y} = H(s) \cdot \mathbb{X}$$ which we want to visualize for each specific point $\boxed{s}$

- This splits into:

$$|\mathbb{Y}| = |H(s)| \times |\mathbb{X}|$$
$$\measuredangle \mathbb{Y} = \measuredangle H(s) + \measuredangle \mathbb{X}$$

both of which can be plotted point by point versus $\boxed{s}$ to obtain <u>magnitude</u> and <u>phase</u> plots.

- Because $\boxed{s}$ is two dimensional we can plot $|H(s)|$ and $\measuredangle H(s)$ as vertical distances (obtain this way surfaces) from the plane s (which is placed on the floor).
- **Example:**



$z_1 = 0$

$P_{1,2} = -3 \pm j4$

"A tent pitched on the s-plane"

- Height of the tent is infinite for s at poles: $s = P_K$
- Touches the s-plane at zeros: $s = Z_K$

Meaning of a zero $s = z_k$

complex

- Assume complex exponential signal $x(t) = X e^{z_k t}$ is applied to circuit whose transfer function is $H(s)$.

- Response is $y(t) = H(z_k) \cdot x(t) = 0 \cdot x(t) = 0$

  complex

- Conclusion: if we subject a circuit to a signal with complex frequency s equal to one of the zeros of its transfer/network function $H(s)$, then the result is a zero response!

Meaning of a pole $s = p_k$

- Apply signal (again complex exponential): $x(t) = X \cdot e^{st}$
  where $s \to p_k$
  "sufficiently close to p_k"
  to make $|H(s)|$ fairly large

- To sustain response $y(t)$ of given amplitude $|Y|$, the amplitude $|X|$ needs be vanishingly small

- in the limit $s \to p_k$, circuit will supply a non-zero response even with no applied signal! This is a source-free or natural response component, indicating that a pole p_k contributes a natural term of the type

  $y_k(t) = Y_k \cdot e^{p_k t}$

  complex.

$\boxed{\text{NOTE}}$ This is possible due to the ability of reactive elements in circuit to release previously stored energy!