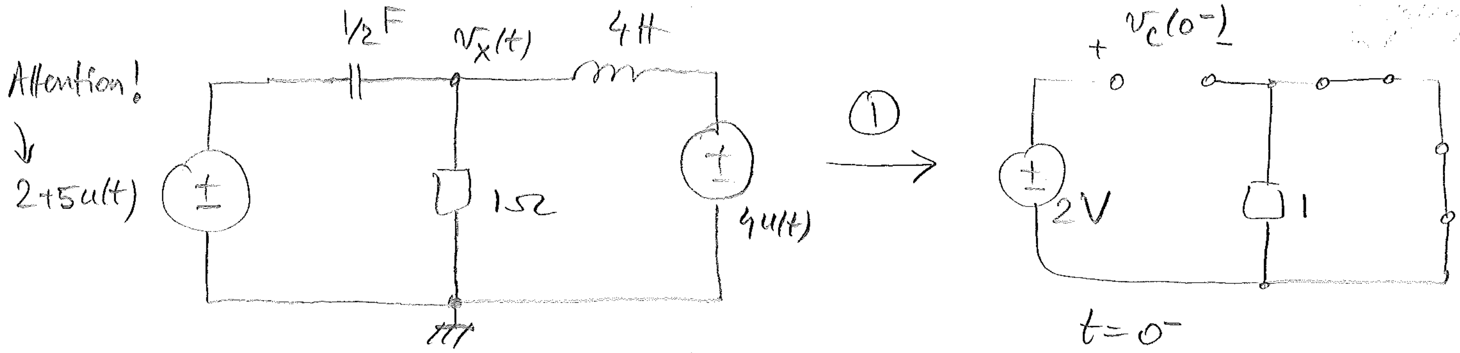
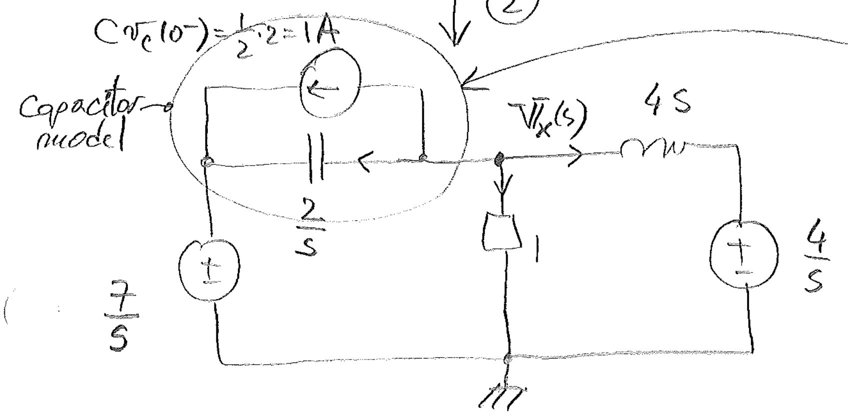
$\boxed{\text{Example 1:}}$ Find voltage $v_x(t)$. Use nodal analysis!



t-domain

$t = 0^-$

$v_C(0^-) = 2V$



s-domain

Convenient to use this model because we're asked to use nodal analysis!

KCL for node $V_x(s)$:

$$1 + \frac{V_x - \frac{7}{s}}{\frac{2}{s}} + \frac{V_x}{1} + \frac{V_x - \frac{4}{s}}{4s} = 0$$

$$1 + \frac{sV_x - 7}{2} + V_x + \frac{sV_x - 4}{4s^2} = 0 \qquad 4s^2),\ 2s^2),\ 4s^2)$$

$$4s^2 + \underline{2s^3V_x} - 14s^2 + \underline{4s^2 \cdot V_x} + \underline{sV_x} - 4 = 0$$

$$s(2s^2 + 4s + 1) \cdot V_x = 10s^2 + 4$$

$$V_x(s) = \frac{10s^2 + 4}{s(2s^2 + 4s + 1)} = \frac{5s^2 + 2}{s\left(s + 1 + \frac{\sqrt{2}}{2}\right)\left(s + 1 - \frac{\sqrt{2}}{2}\right)}$$

- See lecture notes of week #5, Mon:

Poles: $\begin{cases} p_1 = 0 \\ p_2 = -1 - \frac{\sqrt{2}}{2} \\ p_3 = -1 + \frac{\sqrt{2}}{2} \end{cases}$ all real and simple!

Hence:

$$V_x(s) = \frac{A_1}{s} + \frac{A_2}{s+1+\frac{\sqrt{2}}{2}} + \frac{A_3}{s+1-\frac{\sqrt{2}}{2}}$$

$$A_1 = s \cdot V_x(s)\Big|_{s=0} = \frac{10s^2+4}{2s^2+4s+1}\Big|_{s=0} = 4$$

$$A_2 = \left(s+1+\frac{\sqrt{2}}{2}\right) \cdot V_x(s)\Big|_{s=-1-\frac{\sqrt{2}}{2}} = \frac{5s^2+2}{s\left(s+1-\frac{\sqrt{2}}{2}\right)}\Bigg|_{s=-1-\frac{\sqrt{2}}{2}} = 6.84$$

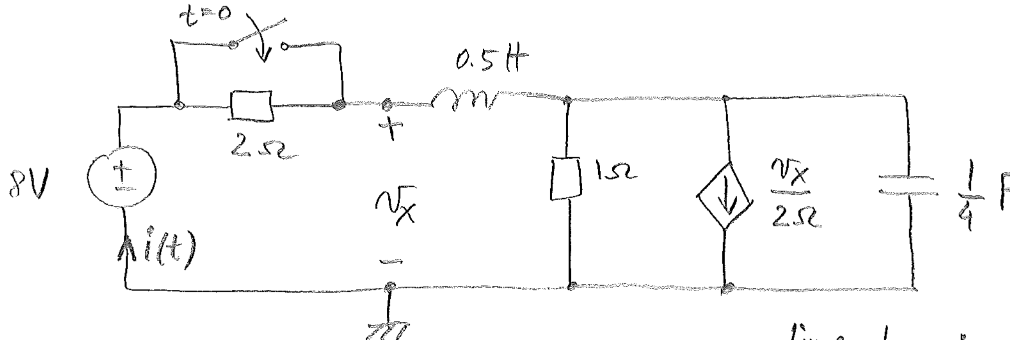
$$A_3 = \left(s+1-\frac{\sqrt{2}}{2}\right) \cdot V_x(s)\Big|_{s=-1+\frac{\sqrt{2}}{2}} = \frac{5s^2+2}{s\left(s+1+\frac{\sqrt{2}}{2}\right)}\Bigg|_{s=-1+\frac{\sqrt{2}}{2}} = -5.84$$

$$V_x(s) = \frac{4}{s} + \frac{6.84}{s+\underbrace{1+\frac{\sqrt{2}}{2}}_{1.707}} - \frac{5.84}{s+\underbrace{1-\frac{\sqrt{2}}{2}}_{0.293}}$$

- Finally:

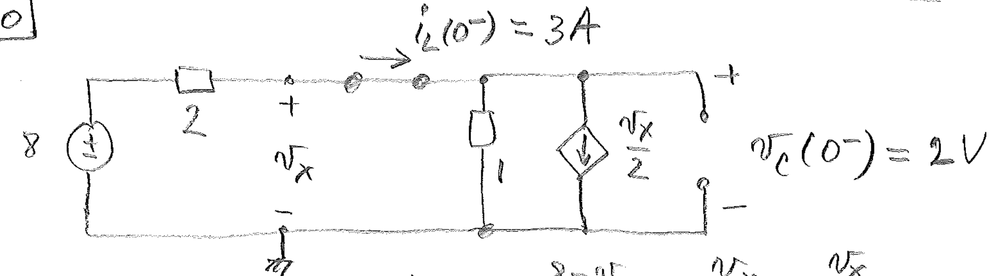
$$\boxed{v_x(t) = \mathcal{L}^{-1}\{V_x(s)\} = \left[4 + 6.84 \cdot e^{-1.707t} - 5.84 \cdot e^{-0.293t}\right] \cdot u(t)}$$

Example 2: Assume switch has been open for a looong time.
Find $i(t) = ?$ $t > 0$.



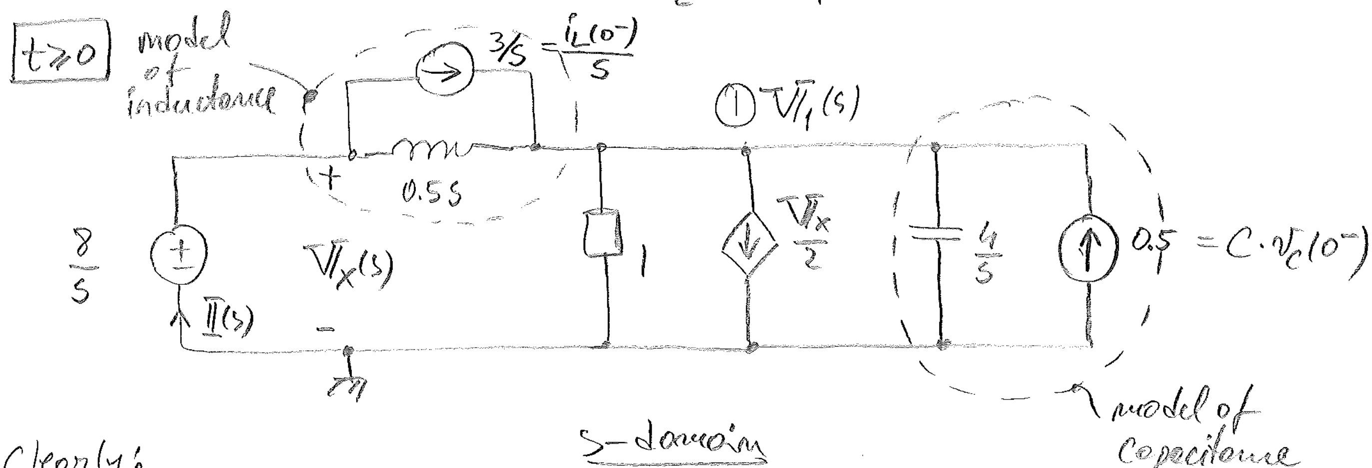
time-domain

t<0



KCL: $\frac{8 - v_x}{2} = \frac{v_x}{1} + \frac{v_x}{2} \Rightarrow v_x = 2\ V$

t≥0



s-domain

- Clearly:

$$V_x = \frac{8}{s}$$

- Use KCL: $\frac{3}{s} + \frac{\frac{8}{s} - V_1}{0.5s} + 0.5 = \frac{V_1}{1} + \frac{8/s}{2} + \frac{V_1}{4/s}$

- Ohm's law: $I = \frac{8/s - V_1}{0.5s} + \frac{3}{s}$

2 eq. 2 unknowns.

- Get V_1 and substitute to get:

$$I(s) = \frac{3s^2 + 24s + 96}{s(s^2 + 4s + 8)} = \frac{12}{s} + \frac{1.5\sqrt{10}\angle 161.56^\circ}{s + 2 - j2} + \frac{1.5\sqrt{10}\angle -161.56^\circ}{s + 2 + j2}$$

- See notes of week #5, Wednesday:

$$i(t) = \mathcal{L}^{-1}\{I(s)\} = \underline{3 \cdot \left[4 + \sqrt{10}\, e^{-2t} \cos(2t + 161.56^\circ)\right]\ A}$$