2 Nodal and mesh analysis in the s-domain

KCL: $$\sum_n I_{in}(s) = \sum_n I_{out}(s) \qquad (1)$$

KVL: $$\sum_\ell V_{rise}(s) = \sum_\ell V_{drop}(s) \qquad (2)$$

Circuit analysis via Laplace method involves steps: (formal statement)

1. – Use t-domain form of circuit to find $i_L(0^-)$ thru each inductance, and $v_C(0^-)$ across each capacitor.

2. – Redraw circuit in s-domain. Each voltage, current source replaced w/ its Laplace transform. Each element replaced by its s-domain equivalent.

3. – Analyze transformed circuit. The outcome is one or more algebraic equations relating the Laplace transform $Y(s)$ of the response to those of the applied sources!

– Once we know $Y(s)$ we find the time-domain:

$$y(t) = \mathcal{L}^{-1}\{Y(s)\}$$

– Particular interest: a single applied source $X(s)$:

$$\boxed{Y(s) = H(s)\cdot X(s) + Y_{natural}(s)} \qquad (3)$$

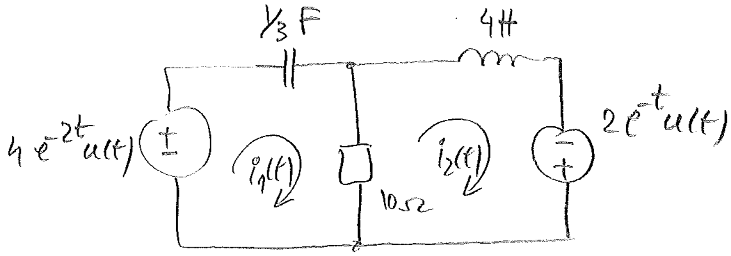
network function ($H(s)$)

function arising from initial stored energies in reactive elements ($Y_{natural}(s)$).

Complete response:

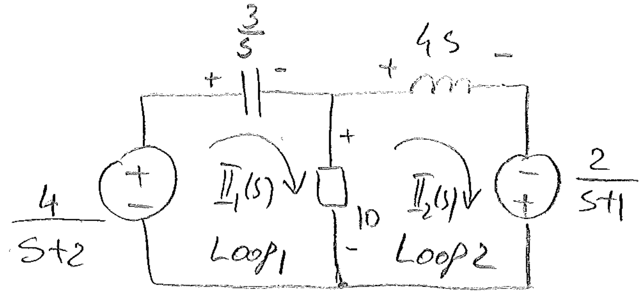
$$\boxed{y(t) = \underbrace{\mathcal{L}^{-1}\{H(s)\cdot X(s)\}}_{\triangleq\, y_{forced}(t)} + \underbrace{\mathcal{L}^{-1}\{Y_{natural}(s)\}}_{\triangleq\, y_{natural}(t)}} \qquad (4)$$

Example 1: Find mesh currents $i_1(t)$, $i_2(t)$.
Zero initial conditions!



t-domain

$\rightarrow$



$$\begin{cases} \text{Loop 1:} & \dfrac{4}{s+2} = \dfrac{3}{s}\cdot I_1(s) + 10\left[I_1(s) - I_2(s)\right] \\ \text{Loop 2:} & 10(I_1 - I_2) + \dfrac{2}{s+1} = 4s\cdot I_2 \end{cases}$$

$$\Rightarrow \begin{cases} \left(\dfrac{3}{s}+10\right)I_1 - 10\, I_2 = \dfrac{4}{s+2} \\ -10\, I_1 + (4s+10)\, I_2 = \dfrac{2}{s+1} \end{cases}$$

2 equations
2 unknowns.

$Ax = B$

where:

$$A = \begin{bmatrix} \frac{3}{s}+10 & -10 \\ -10 & 4s+10 \end{bmatrix} \quad B = \begin{bmatrix} \frac{4}{s+2} \\ \frac{2}{s+1} \end{bmatrix} \quad X = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

- Solve to find:

$$I_1 = \frac{2s(4s^2+19s+20)}{20s^4+66s^3+73s^2+57s+30} \text{ A}$$

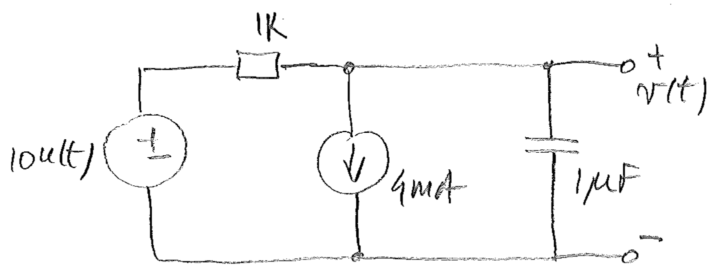
$$I_2 = \frac{30s^2+43s+6}{(s+2)(20s^3+26s^2+21s+15)} \text{ A}$$

- Take inverse Laplace transform:

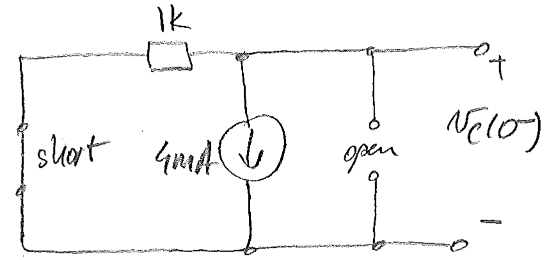
$$i_1(t) = -96.39\,e^{-2t} - 344.8\,e^{-t} + 841.2\,e^{0.15t}\cdot\cos(0.853t) + 197.7\,e^{-0.15}\sin(0.853t) \quad [\text{mA}]$$

$$i_2(t) = -481.9\,e^{-2t} - 241.4\,e^{-t} + 723.3\,e^{-0.15t}\cdot\cos(0.853t) + 472.8\,e^{-15t}\cdot\sin(0.853t)$$

Example 2: Find $v(t) = ?$



$\rightarrow$ (1)

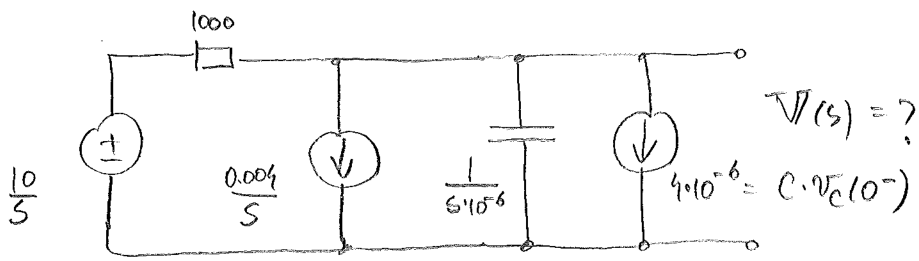


t-domain

at time $t = 0^-$

$$v_C(0^-) = -1k \times 4mA = 4V$$

(2) Convert to s-domain



- Apply KCL:

$$\frac{\frac{10}{s} - V(s)}{100} = \frac{4 \cdot 10^{-3}}{s} + \frac{V(s)}{1/s \cdot 10^{-6}} + 4 \times 10^{-6}$$

$$\Rightarrow V(s) = \frac{6 \times 10^3 - 4s}{s(s+10^3)} = \frac{6}{s} - \frac{10}{s+10^3}$$

- Taking inverse Laplace:

$$v(t) = \mathcal{L}^{-1}\{V(s)\} = \left(6 - 10 \cdot e^{-10^3 t}\right) \cdot u(t)$$