Chapter 15

Circuit analysis in the s-domain

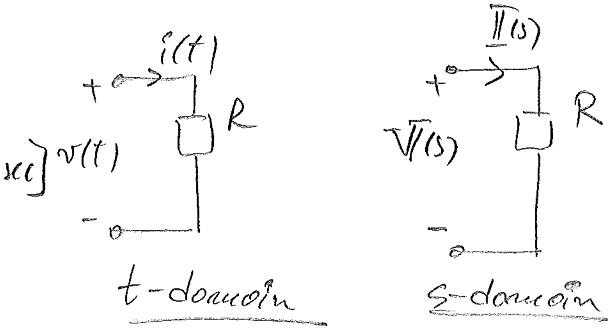
[1] Circuit element models

(a) Resistors

$v(t) = R \cdot i(t)$ take Laplace transform of both sides:

$$[\text{Volts} \times \text{sec}] \quad \boxed{\mathbb{V}(s) = R \cdot \mathbb{I}(s)} \quad (1) \quad [\text{Amperes} \times \text{sec}]$$

$$\boxed{\mathbb{Z}(s) = \frac{\mathbb{V}(s)}{\mathbb{I}(s)} = R}$$



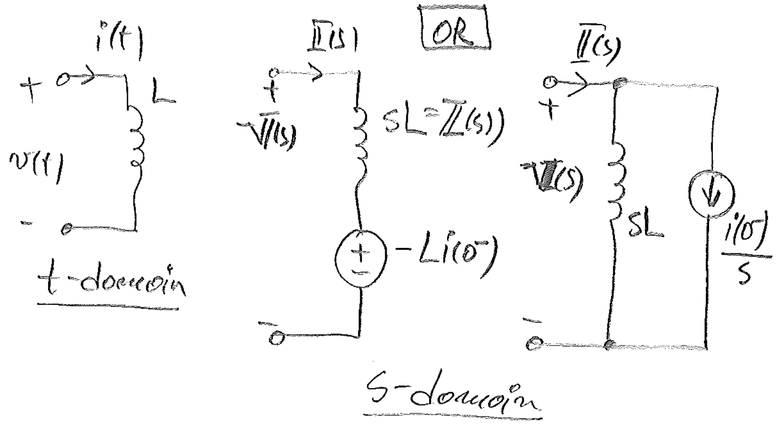
(b) inductors

$$v(t) = L \frac{di}{dt}$$

$$\boxed{\mathbb{V}(s) = sL\,\mathbb{I}(s) - L\,i(0^-)} \quad (2)$$

$$\boxed{\mathbb{Z}(s) = sL}$$

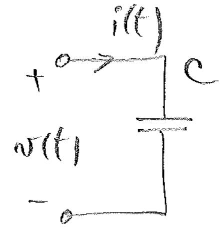
initial condition is the inductance current at time t=0!



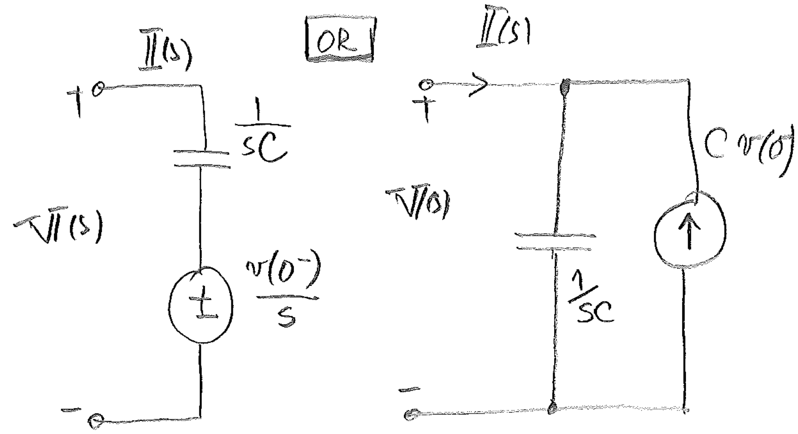
Note: if we had discussed first s-domain analysis without having introduced Laplace transform, we would have talked about "generalized impedance and admittance". ← These do not consider initial conditions. As a consequence, the unknown coefficients appearing in the transient component would have to be found on the basis of the initial conditions in the circuit. This could be a tedious and time-consuming process. In conclusion, using/studying directly Laplace transform is better!!!

(c) Capacitors

$$i(t) = C\frac{dv}{dt}$$



t-domain



s-domain

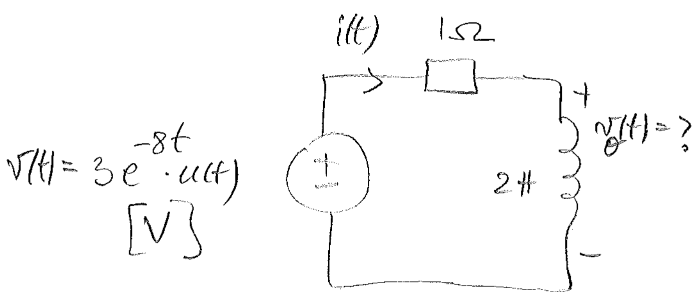
$$I(s) = sC\,V(s) - C\,v(0^-)$$

$$\boxed{V(s) = \frac{1}{sC}\cdot I(s) + \frac{v(0^-)}{s}} \quad (3)$$

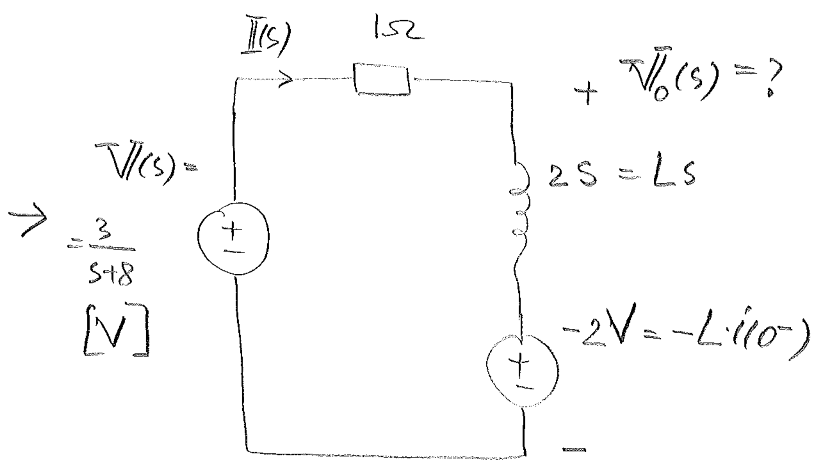
$$\boxed{Z(s) = \frac{1}{sC}}$$

initial condition is the voltage across capacitance.

Example: Find voltage $v_o(t)$ given the initial current $i(0^-) = 1A$



time-domain



s-domain

Solution: Transform whole thing to s-domain, find $V_o(s)$, then transform back to time-domain to find $v_o(t)$!

- KVL around loop:

$$V(s) = 1 \cdot I(s) + 2s \cdot I(s) + (-2)$$

$$\frac{3}{s+8} + 2 = (1+2s) \cdot I(s) \Rightarrow I(s) = \frac{\frac{3}{s+8} + 2}{1+2s} = \frac{s+9.5}{(s+8)(s+0.5)}$$

- Hence:

$$V_0(s) = 2s \cdot I(s) - 2 = \frac{2s(s+9.5)}{(s+8)(s+0.5)} - 2 = \frac{2s-8}{(s+8)(s+0.5)}$$

$$\left.\begin{aligned} V_0(s) &= \frac{A_1}{(s+8)} + \frac{A_2}{(s+0.5)} \\ A_1 &= (s+8) \cdot V_0(s)\Big|_{s=-8} = \frac{2s-8}{s+0.5}\Big|_{s=-8} = 3.2 \\ A_2 &= (s+0.5) \cdot V_0(s)\Big|_{s=0.5} = \frac{2s-8}{s+8}\Big|_{s=0.5} = -1.2 \end{aligned}\right\} \Rightarrow$$

$$\Rightarrow V_0(s) = \frac{3.2}{s+8} - \frac{1.2}{s+0.5}$$

- Finally:

$$v_0(t) = \mathcal{L}^{-1}\{V_0(s)\} = \left[3.2 \cdot e^{-8t} - 1.2 \cdot e^{-0.5t}\right] \cdot u(t)$$