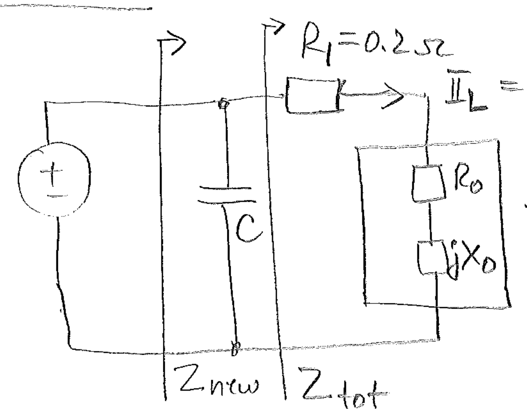
Problem 46 Ch. 11

Given:



$I_L = |I_{L(rms)}| \angle 0^\circ$ ← assume for convenience

Load:

$Z_0 = R_0 + jX_0$

Apparent power: $S_0 = |\underline{S}_0| = 10^3$ [VA]

$pf = 0.8$ lagging

Find: $C = ?$ such that the source power-factor $pf_{new} = 0.9$ lagging.

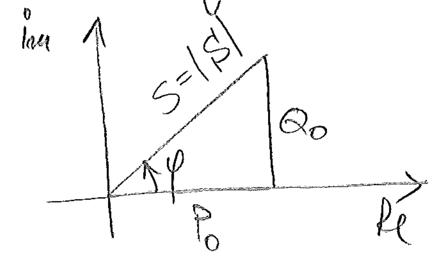
↑ Very important
not of the load!

Solution:

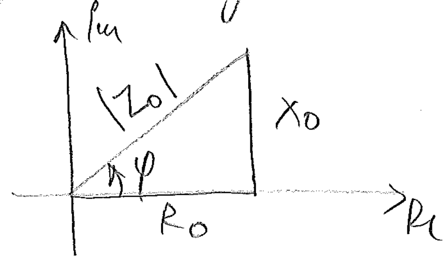
- From first lecture on "ac power" recall:

power-factor $\quad pf = \cos\varphi = \dfrac{\text{average power}}{\text{apparent power}} = \dfrac{P_0}{S_0}$

- power triangle of the load: — impedance triangle of the load.

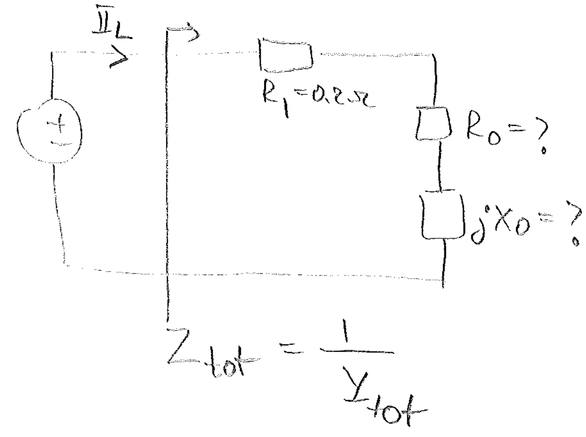


Recall:
⟷
$P_0 = R_0 \cdot |I_{L(rms)}|^2$
$Q_0 = X_0 \cdot |I_{L(rms)}|^2$



$P_0 = S_0 \times pf = 10^3 \times 0.8 = 8000$ [W]

- Our goal is to find Z_{tot} or Y_{tot}:



$Z_{tot} = \dfrac{1}{Y_{tot}}$

$P_{tot} = P_1 + P_0 = R_1 |I_{L(rms)}|^2 + P_0$

$= 0.2 \times 40^2 + 8000 = 320 + 8000$

$= 8320$ [W]

- Use relationship between power-triangle and impedance-triangle:

$$R_o = \frac{P_o}{|I_{L(rms)}|^2} = \frac{8000}{40^2} = 5\,\Omega.$$

$$X_o = R_o \tan\varphi = R_o \cdot \tan(\underset{\text{lagging}}{+}\cos^{-1} 0.8) = 5\cdot\tan(36.87^\circ) = 3.75\,\Omega$$

$\varphi = \cos^{-1} pf$ because $pf \triangleq \cos\varphi$

$$\Rightarrow \boxed{Z_o = 5 + j3.75\,\Omega}$$

- Hence, $Z_{tot} = (R_1 + R_o) + jX_o = 5.2 + j3.75 \Rightarrow pf_{old} = \cos(35.79) = 0.811$ ← of source V_s !!! not of Load.

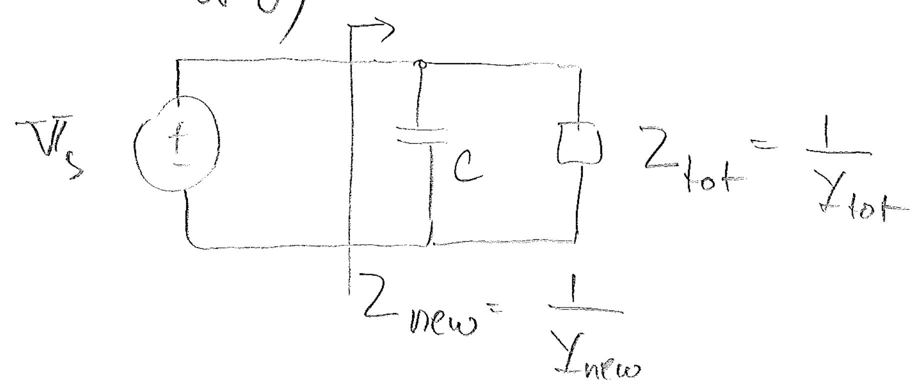
$$\Rightarrow Y_{tot} = \frac{1}{Z_{tot}} = \frac{1}{5.2 + j3.75} = 0.126 - j0.091$$

- Now, we can find V_s which creates I_L thru Z_{tot}!

$$V_s = Z_{tot} \times I_L = Z_{tot} \times \left(|I_{L(rms)}|\angle 0^\circ\right)$$

$$= (5.2 + j3.75) \times 40 = 256.4\angle 35.80^\circ$$

- With C in place the source (recall: whose pf now must become 0.9 lagging) sees:

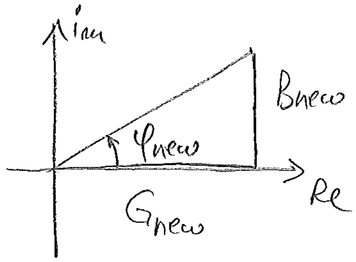


$$Y_{new} = Y_{tot} + jB_p = (0.126 - j0.091) + j(2\pi \overset{60\,Hz}{f} C)$$

$$= \underbrace{0.126}_{G_{new}} + j\underbrace{(120\pi C - 0.091)}_{B_{new}}\ \Omega.$$

1 → One way to do it:

$$pf_{new} = \cos \varphi_{new} \Rightarrow \varphi_{new} = \cos^{-1} pf_{new} = \cos^{-1} 0.9 = 25.84°$$



$$\tan \varphi_{new} = \frac{B_{new}}{G_{new}}$$

$$\tan 25.84° = \frac{120\pi C - 0.091}{0.126} \Rightarrow \boxed{C = 79.48\,\mu F}$$

2 → Another way to to it:
use formula:

$$X_p = \frac{|\bar{V}_{s(rms)}|^2 / P_{tot}}{\tan(\cos^{-1} pf_{new}) - \tan(\cos^{-1} pf_{old})}$$

of the source!

$$X_p = \frac{256.4^2/8320}{\tan(\cos^{-1} 0.9) - \tan(\cos^{-1} 0.811)} = \frac{7.901}{-0.237} = -33.34$$

$$X_p = -\frac{1}{\omega C_p} \Rightarrow C_p = -\frac{1}{\omega X_p} = \frac{1}{2\pi f \times 33.34} = \frac{1}{120\pi \times 33.34} \cong 79.48\,\mu F$$

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