5 Power-factor correction

$p_X(t)$ does not contribute to average power (see figure 1)

- But it places an additional burden upon the source, which besides supplying the rated real power, must also contend with reactive power exchanges.
- This burden is especially undesired in generation & distribution of industrial power:

$$\dot{I}_{rms} = \frac{P}{V_{rms} \times pf} \quad (11)$$

see lecture #1 earlier (eq.(4))

reveals that the current needed to supply a specified ac power at the rated line voltage is inversely proportional to pf of the load.

(1) => lower pf's require greater current-carrying capacity be built into system.

(2) => larger currents => greater ohmic losses in the power line resistance => lower efficiency

Example: Assume $\varphi = \pm 45^\circ \Rightarrow pf = \cos 45^\circ = \frac{1}{\sqrt{2}} = 0.707$

$$(11) \Rightarrow \dot{I}_{rms\,(pf=0.707)} = \sqrt{2}\,\dot{I}_{rms\,(pf=1)} = 1.414\,\dot{I}_{rms\,(pf=1)}$$

i.e.: current capacity must be increased by 41.4% to accomodate the same reactive power exchanges as when $pf = 1$!

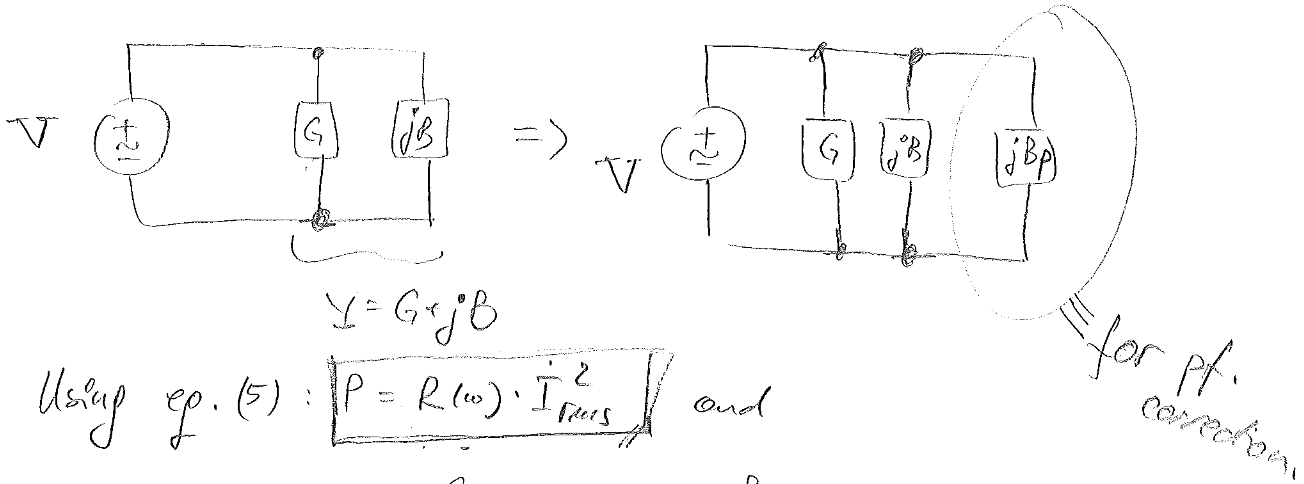
Also:

$$R_{line} \cdot \dot{I}^2_{rms\,(pf=0.707)} = 2 \times R_{line} \times \dot{I}^2_{rms\,(pf=1)}$$

indicating a doubling in line-power loss due to reactive exchanges!

OBS: To optimize power generation & distribution, it is desirable to have $pf \to 1$ or $\varphi \to 0$. In that case we have $Q \to 0$, $p_X(t) \to 0$, $p(t) \to p_R(t)$

- Such a condition is met by connecting a suitable reactive element at the load end so that reactive power is exchanged between load and this element, rather than between load and source! (parallel connection is preferred)
- This procedure is called "power-factor correction"!



- Using eq. (5): $\boxed{P = R(\omega) \cdot I_{rms}^2}$ and

Also knowing: $R = \frac{G}{G^2 + B^2}$ $X = \frac{-B}{G^2 + B^2}$

we get: $\begin{cases} P = G\, V_{rms}^2 \\ Q = -B\, V_{rms}^2 \end{cases}$

- Now, find an element with susceptance B_p, which connected in // with load will raise the power-factor of the composite admittance: $Y + jB_p = G + j(B + B_p)$ to unity. That is equivalent to imposing $j(B + B_p) = 0 \Rightarrow \boxed{B_p = -B}$ (12)

- Composite admittance becomes purely resistive: $Y + jB_p = G$
$\Rightarrow \boxed{Q = 0} \Rightarrow$ the source no longer has to contend with reactive exchanges, and supplies now only real power $p(t) = p_R(t)$.

OBS: Physically: combination of parallel equal but opposite reactances (& susceptances) acts as an open circuit! phenomenon called resonance.

Δ To raise pf to unity, the load must be made resonant!

- In practical situations a pf close to 1 is sufficient.
- Given a load $Z = R + jX$, find a reactance X_p connected in // with Z to raise the composite load $jX_p \| Z$ to a new value $pf_{new} = \cos[\measuredangle(jX_p \| Z)]$

$$X_p = \frac{R^2 + X^2}{R \cdot \tan\left[\pm\cos^{-1}(pf_{new})\right] - X} \quad (13)$$

+ sign if pf_{new} is lagging
− sign if pf_{new} is leading.

- An alternate form: multiply with $\dot{I}_{rms}^2$ num. and denom.:
observing that: $V_{rms}^2 = (R^2 + X^2) \cdot \dot{I}_{rms}^2$
& using: $P = R\,I_{rms}^2$; $Q = X\,I_{rms}^2$ $\quad \overset{(13)}{\Longrightarrow}$

$$X_p = \frac{V_{rms}^2 / P}{\tan(\pm\cos^{-1} pf_{new}) - \tan(\pm\cos^{-1} pf_{old})} \quad (14)$$

also Recall: $X_p = -\frac{1}{\omega C}$

Example 1

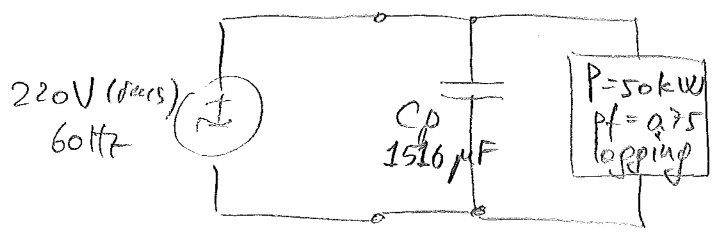
Source 220 V (rms), 60 Hz
industrial load consumes $P = 50$ kW ; $pf = 0.7$ lagging.

Find a suitable bank of parallel capacitors to ensure $pf = 0.95$ lagging
What current must these capacitor be able to withstand?

(14) $\Rightarrow$ $$X_p = \frac{\frac{220^2}{50\times 10^3}}{\tan(\cos^{-1}0.95) - \tan(\cos^{-1}0.75)} = -1.75\,\Omega$$

$\Rightarrow$ We need C_p such that: $$\boxed{-\frac{1}{\omega C_p} = X_p}$$

or $$C_p = -\frac{1}{\omega X_p} = -\frac{1}{2\pi 60\times(-1.75)} = 1516\ \mu F$$



By Ohm's law, current thru capacitor bank:

$$\boxed{I_{rms} = \frac{V_{rms}}{|Z_{C_p}|} = \frac{220}{1.75\Omega} = 125.7\ A}$$ ← must be kept in mind when specifying C_p!