4 Complex Power

- From the viewpoint of power generation and transmission, average power alone is not enough to fully characterize the capacity requirements and efficiency of the system. There are additional parameters/variables that we must take into consideration!
- Consider general case:

$v(t) = V_m \cos(\omega t + \theta)$ – across a load
$i(t) = I_m \cos(\omega t + \phi)$ – thru that load.

- The instantaneous power (using $\cos(a+b) = \cos a \cos b - \sin a \sin b$) is:

$$p(t) = \frac{1}{2} V_m I_m \left[\cos(\underbrace{\theta - \phi}_{=\varphi}) + \cos 2\omega t \cdot \cos(\theta - \phi) - \sin 2\omega t \sin(\theta - \phi) \right]$$

$$p(t) = V_{rms} \cdot I_{rms} \left[\cos \varphi + \cos 2\omega t \cdot \cos \varphi - \sin 2\omega t \cdot \sin \varphi \right] \quad (1)$$

phase difference between voltage and current.

$$p(t) = \underbrace{\underbrace{V_{rms} I_{rms} \cdot \cos \varphi}_{\triangleq P = \text{real power}} \left[1 + \cos 2\omega t\right]}_{= p_R(t) = \text{instantaneous real power}} - \underbrace{\underbrace{V_{rms} I_{rms} \cdot \sin \varphi}_{\triangleq Q = \text{reactive power}} \cdot \sin 2\omega t}_{= p_X(t) = \text{instantaneous reactive power}}$$

$$\Rightarrow \boxed{p(t) = p_R(t) - p_X(t)} \quad (2) \qquad \text{where} \quad \boxed{P = V_{rms} \cdot I_{rms} \cdot \cos \varphi} \quad (3)$$

$$\boxed{Q = V_{rms} \cdot I_{rms} \cdot \sin \varphi} \quad (4)$$

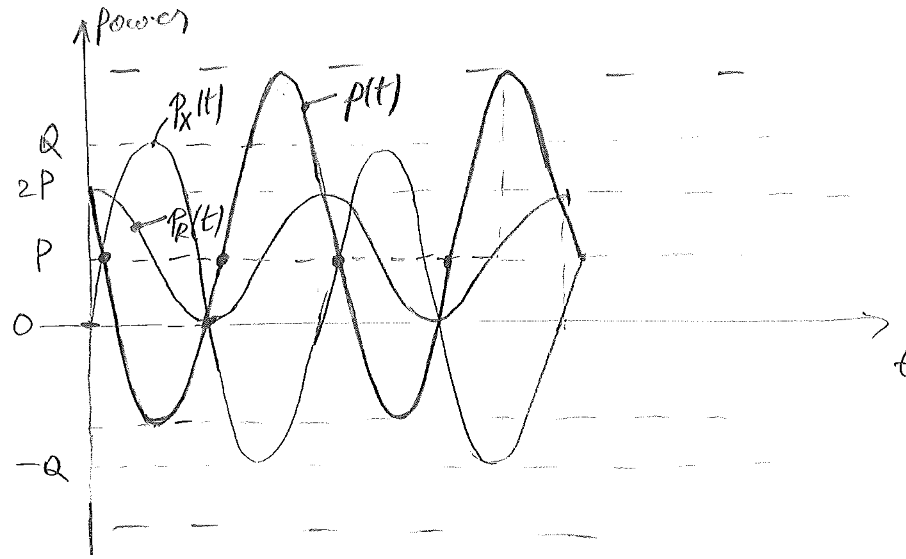
$p_X(t)$: alternates between $+Q$ and $-Q$ and averages to zero!

$p_R(t)$: alternates between 0 and $2P$ and averages to P!

$$P = V_{rms} \cdot I_{rms} \cdot \cos \varphi = \left\{ \frac{V_{rms}}{I_{rms}} \times \cos \varphi \right\} \cdot I_{rms}^2 = \left\{ Z_m \cdot \cos \varphi \right\} I_{rms}^2 = R(\omega) \cdot I_{rms}^2$$

(5) $\boxed{P = R(\omega) \cdot I_{rms}^2}$ <u>absorbed</u> by the resistive component $R(\omega)$ of the load!

- $p_R(t)$ is called the "<u>instantaneous real power</u>" and is irreversibly converted from electrical to nonelectrical form!
- P is referred to as "<u>real power</u>" [W].



$$\boxed{p(t) = p_R(t) - p_X(t)}$$

<u>Figure 1</u>

- Component $p_X(t)$ alternates between $+Q$ and $-Q$ and averages to zero! => does not intervene in the irreversible power-transfer process!

$$\boxed{Q = \left\{\frac{V_{rms}}{I_{rms}} \times \sin\varphi\right\} I_{rms}^2 = (Z_m \sin\varphi) I_{rms}^2 = X(\omega) \cdot I_{rms}^2} \quad (6)$$

accounts for the energy exchange between the source and the reactive component $X(\omega)$ of the load.

- During → the half-cycle when $p_X(t) > 0$ energy is stored in the electric or magnetic fields of the reactive elements.
  → the half-cycle whe $p_X(t) < 0$ energy is <u>returned</u> to the source!

-> Q is referred as "<u>reactive power</u>" [VAR] volt-ampere-reactive to distinguish it from P.

Reminder:

(1) inductive loads (voltage leads current) $\varphi > 0$ => $\boxed{Q > 0}$
(2) capacitive loads (current leads voltage) $\varphi < 0$ => $\boxed{Q < 0}$

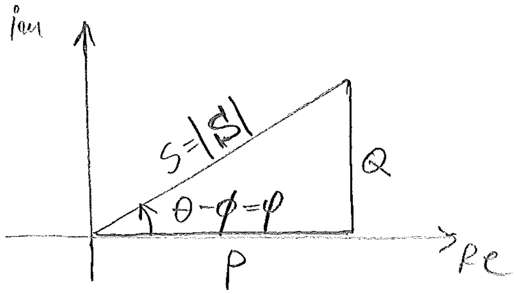
→ Power engineers recognize this difference by saying: $\left\{ \begin{array}{l} \text{– inductive loads \underline{consume}} \\ \text{– capacitive loads \underline{produce}} \end{array} \right\}$ reactive power

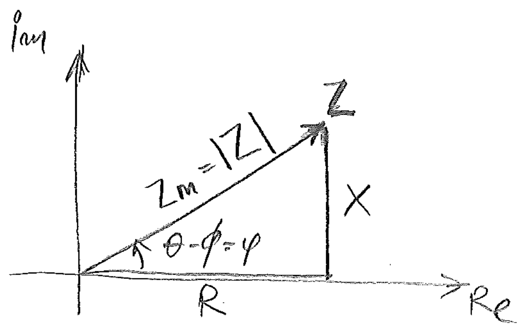
From eqs. (3) & (4) ⇒ $$\boxed{Q = P \tan\varphi = P \cdot \tan(\pm \cos^{-1} pf)} \quad (7)$$

{ + if load is inductive (lagging pf)
{ – if load is capacitive (leading pf)

$\boxed{\text{OBS}}$: P and Q can be regarded as the <u>real</u> and <u>imaginary</u> parts of a complex variable $\mathbb{S}$ called the <u>"complex power"</u> with length $|S|$ and phase angle $\varphi = \theta - \phi$.

$$(8) \quad \boxed{\mathbb{S} = P + jQ = |\mathbb{S}|\angle\varphi}$$





<u>"power triangle"</u> has the same shape as <u>"impedance triangle"</u>

Using: $\left\{ \begin{array}{l} P = R(\omega) \cdot I_{rms}^2 \\ Q = X(\omega) \cdot I_{rms}^2 \end{array} \right.$ $\overset{(8)}{\Rightarrow}$ $$\boxed{\mathbb{S} = (R + jX)\, I_{rms}^2 = Z \cdot I_{rms}^2} \quad (9)$$

⇒ power-triangle can be obtained from impedance-triangle by multiplying each of its sides with I_{rms}^2 !

<u>Summary</u>:

$$\mathbb{S} = P + jQ = S\angle\varphi$$

Real power [W] — Reactive power [VAR] — Apparent power [VA]

→ Yet another way to express complex power: NOT phasors

<u>Denote phasors:</u> phasors $\mathbb{V}_{rms} \triangleq V_{rms} \cdot \angle\theta$

$\mathbb{I}_{rms} \triangleq I_{rms} \cdot \angle\phi$

and <u>exploit:</u> $I_{rms}^2 = \mathbb{I}_{rms} \cdot \mathbb{I}_{rms}^*$

(9) ⇒ not phasor, phasors

$$S = Z \cdot I_{rms}^2 = Z \cdot \mathbb{I}_{rms} \cdot \mathbb{I}_{rms}^*$$

Ohm's law: $Z \cdot \mathbb{I}_{rms} = \mathbb{V}_{rms}$ ⇒

$$\Rightarrow \boxed{S = \mathbb{V}_{rms} \cdot \mathbb{I}_{rms}^*} \quad (10)$$

V. imp.! <u>Ac power conservation principle</u>: The complex, real, and reactive power of the source equals, respectively, the sums of the complex, real, and reactive powers of the individual impedances!

<u>OBS</u>: not true generally for apparent powers!

<u>Example 1:</u>

Household ac voltage is applied to a load $Z = 100 + j300\ \Omega$.
Find S, P, Q. Assume $\angle I = 0$ find $p(t), p_R(t), p_X(t)$

<u>Solution:</u>

$2\omega = 2 \times 2\pi 60 = 754$ rad/s.

$V_{rms} = 120\,V$

$$I_{rms} = \frac{V_{rms}}{\sqrt{R^2 + X^2}} = \frac{120}{\sqrt{100^2 + 300^2}} = 0.3795\ A$$

Hence:

$$|S| = S = V_{rms} \cdot I_{rms} = 120 \times 0.3795 = 45.54\ VA$$

$$P = R \cdot I_{rms}^2 = 100 \times 0.3795^2 = 14.40\ W$$

$$Q = X \cdot I_{rms}^2 = 300 \times 0.3795^2 = 43.20\ VAR$$

$$p(t) = p_R(t) - p_X(t) = 14.40[1 + \cos(754t)] - 43.20 \sin(754t)\ W$$