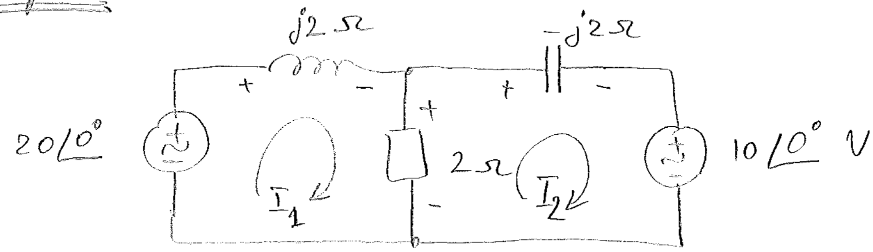
Example:



Find avg. power absorbed by each of the passive elements.

Solution:

- Avg. power absorbed by inductor & capacitor (or purely reactive elements) is zero: see case 2 in last lecture!
- Use mesh analysis to find current thru resistance 2Ω:

$$\begin{cases} j2 \cdot I_1 + 2(I_1 - I_2) = 20 \\ 2(I_1 - I_2) = -j2 I_2 + 10 \end{cases} \Rightarrow \begin{cases} (2+j2) I_1 - 2 I_2 = 20 \\ 2 I_1 + (-2+j2) I_2 = 10 \end{cases}$$

$$Ax = b$$

where: $A = \begin{bmatrix} 2+j2 & -2 \\ 2 & -2+j2 \end{bmatrix}$ $b = \begin{bmatrix} 20 \\ 10 \end{bmatrix}$ $X = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$

Solve system to get:

$$\begin{cases} I_1 = 5 - j10 = 11.18 \angle -63.43° \text{ A} \\ I_2 = 5 - j5 = 7.071 \angle -45° \text{ A} \end{cases}$$

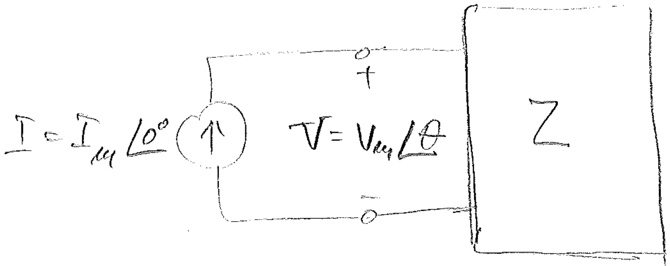
The net equivalent current thru 2Ω is:

$$\underbrace{I_1 - I_2 = -j5 = 5 \angle -90° \text{ A}}_{I_m}$$

Hence average power absorbed by resistor is:

$$\boxed{P_R = \frac{1}{2} V_m I_m = \frac{1}{2} R I_m^2 = \frac{V_m^2}{2R} = \frac{1}{2} \times 2 \times 25 = 25 \text{ [W]}}$$

2 Power and impedance



$$Z = \frac{V}{I} = |Z| \angle \theta$$

frequency-domain circ. representation.

$i(t) = I_m \cos(\omega t)$ $\qquad$ $v(t) = V_m \cos(\omega t + \theta)$

$\boxed{\phi = 0°}$

current used as reference.

$$\theta - \phi = \theta - 0° = \theta = \measuredangle Z$$

Therefore, equation (4) (see last lecture) can be written as:

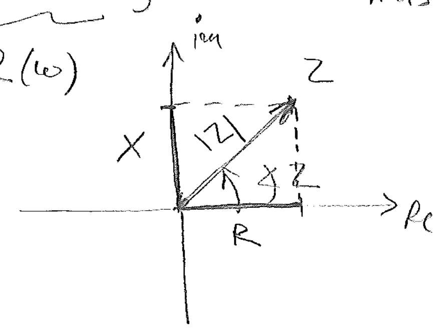
$$P = \frac{1}{2} V_m I_m \cdot \cos\theta = V_{rms} \cdot I_{rms} \cdot \cos\theta = V_{rms} \cdot I_{rms} \cos(\measuredangle Z)$$

with $V_{rms} = |Z| \cdot I_{rms}$ $\quad \Rightarrow$

$$\Rightarrow P = |Z| \cdot I_{rms}^2 \cdot \cos(\measuredangle Z) = I_{rms}^2 \cdot \left[\underbrace{|Z| \cdot \cos(\measuredangle Z)}_{= R(\omega)}\right] = R(\omega) \cdot I_{rms}^2$$

(1) $\boxed{P = R(\omega) I_{rms}^2} = \boxed{\dfrac{(V_{rms} \cdot \cos\theta)^2}{R(\omega)}}$

prove it on your own.



OBS: $R(\omega)$ (of an impedance) represents the ability of a circuit to dissipate power in response to ac current!

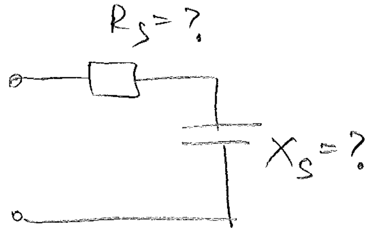
Example:

A circuit is connected to the 120 V (rms), 60 Hz household ac line and dissipates an average power of 1 kW with a power factor $pf = 0.75$. Find the simple series and parallel equivalents of the circuit, knowing that the pf is a "leading pf" $\equiv$ current i leads voltage v

series ↑ in class; parallel ↑ HW assignment.

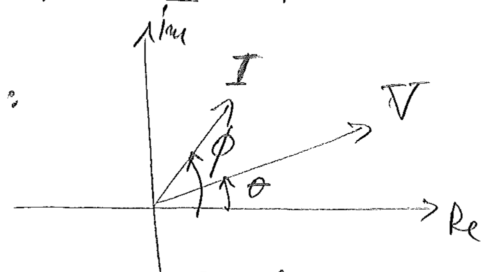
$$\varphi = \theta - \phi < 0$$

(a)



OBS: this is a capacitive impedance.

A "visual":



$\theta - \phi = \varphi < 0$

For simplicity assume current as reference with $\phi = 0°$. Then:

$$\varphi = \theta = -\cos^{-1} pf = -\cos^{-1} 0.75 = -41.41°$$

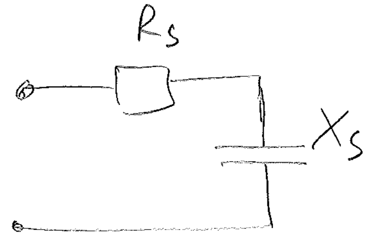
$Z = R + jX$ we have:

$$\text{Eq. (1)} \Rightarrow R = \frac{(V_{rms} \cdot \cos\theta)^2}{P} = \frac{(120 \times 0.75)^2}{10^3} = 8.1\,\Omega$$

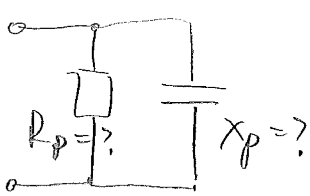
$$X = R \tan\theta = 8.1 \times \tan(-41.41°) = -7.14\,\Omega.$$

The equivalent series circuit:

is obviously $\begin{cases} R = R_s \\ X = X_s \end{cases}$



(b)



Hint: convert from above or work with admittance: $\underline{Y} = G + jB$

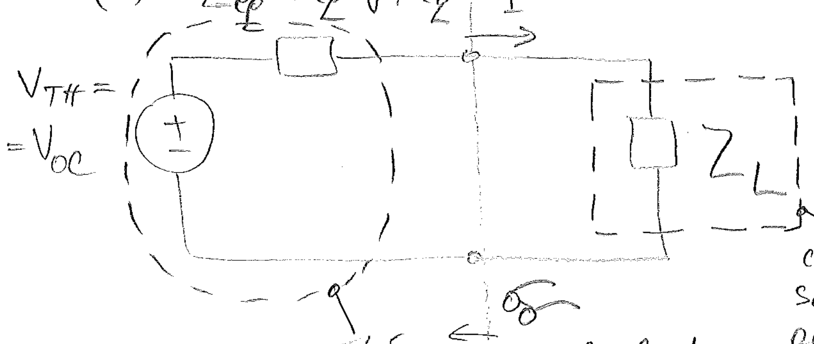
//

3 Maximum power transfer

Reminder: for resistive circuits we found that power transfer is maximized when the load is made equal to the source resistance. (condition called matching)

Now – what is the condition for an active ac port driving a load impedance:

(2) $Z_{eq} = R_{eq} + jX_{eq}$ $\qquad Z_L = R_L + jX_L$ (3)



can be any source-free ac network.

Thévenin equivalent of the active port.

$R_L = ?$, $X_L = ?$ that maximize power transfer to the load!

The load power is (see eq. (1)) $\quad P_L = R_L \cdot I_{rms}^2 = \frac{1}{2} R_L I_m^2$

where $\quad I_m = \left| \dfrac{V_{oc}}{Z_{eq} + Z_L} \right|$

$$\Rightarrow \quad P_L = \frac{1}{2} R_L \cdot \frac{|V_{oc}|^2}{(R_{eq}+R_L)^2 + \underbrace{(X_{eq}+X_L)^2}}$$

any non-zero value would only reduce P_L
Eliminate this term by imposing:

(4) $\boxed{X_L = -X_{eq}}$

so, we're left with:

$$\boxed{P_L = \frac{1}{2} R_L \cdot \frac{|V_{oc}|^2}{(R_{eq}+R_L)^2}}$$

← problem of finding the condition for R_L that maximizes P_L. We have shown in EE-206 that $\boxed{R_L = R_{eq}}$ (5)

$$\Rightarrow R_L + jX_L = R_{eq} - jX_{eq}$$

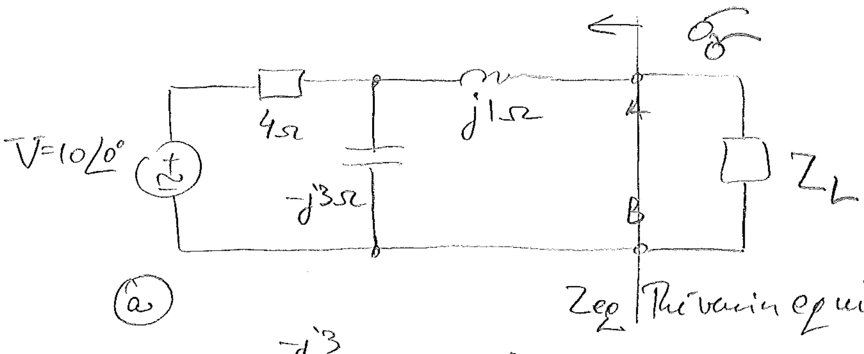
$$\boxed{Z_L = Z_{eq}^*} \quad (6)$$

- Power transfer from an active ac port (in particular an ac source in series w/ an impedance) to a load is maximized when the load impedance is made equal to the complex conjugate of the equivalent impedance of the port!
- When this condition is met, the load is said to be $\boxed{\text{matched}}$ to the ac port.
- Physically, the two reactances cancel each other, in effect forming a short-circuit. In this case the ac port and matched load form a $\boxed{\text{resonant}}$ circuit!

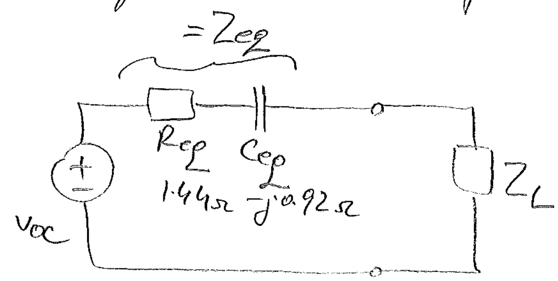
- Maximum average power:

$$\boxed{P_{L(max)} = \frac{1}{2}|V_{oc}|^2 \cdot \frac{R_{eq}}{(R_{eq}+R_{eq})^2} = \frac{1}{8}\frac{|V_{oc}|^2}{R_{eq}}} \quad (7)$$

Example: Find the load that will recive maxim power. Find that power.



$\equiv$



(a)

$$V_{oc} = \frac{-j3}{4-j3} \cdot 10\angle 0^\circ = 6\angle -53.13^\circ \text{ V}$$

$$Z_{eq} = j1 + [4 \,||\, (-j3)] = \dots = 1.44 - j0.92$$

$$\Rightarrow \boxed{Z_L = 1.44 + j0.92}$$

(b) $(7) \Rightarrow \boxed{P_{L(max)} = \frac{1}{8}\frac{|V_{oc}|^2}{R_{eq}} = \frac{1}{8} \times \frac{6^2}{1.44} = 3.125 \text{ W}}$