4 The a parameters

- Also called ABCD parameters, or transmission parameters or t parameters:

$$\begin{cases} V_1 = a_{11} V_2 - a_{12} I_2 \\ I_1 = a_{21} V_2 - a_{22} I_2 \end{cases} \Rightarrow \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = [a] \cdot \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

- They can be found as:

$$a_{11} = \frac{V_1}{V_2}\Big|_{I_2=0} \qquad a_{21} = \frac{I_1}{V_2}\Big|_{I_2=0}$$

transfer admittance

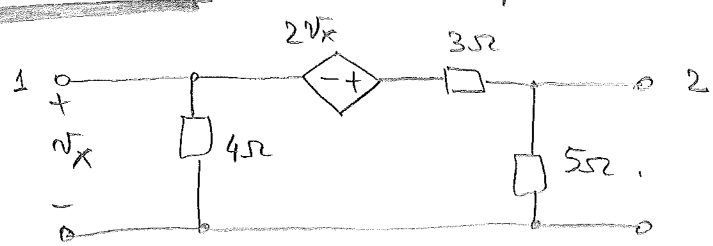
transfer impedance

$$a_{12} = -\frac{V_1}{I_2}\Big|_{V_2=0} \qquad a_{22} = -\frac{I_1}{I_2}\Big|_{V_2=0}$$

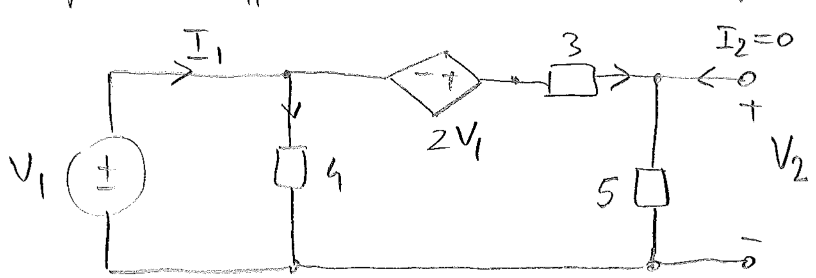
- Note that all parameters are of the <u>transfer</u> type!

gain functions

Example Find a parameters.



- To find a_{11} and a_{21} leave output on open circuit:

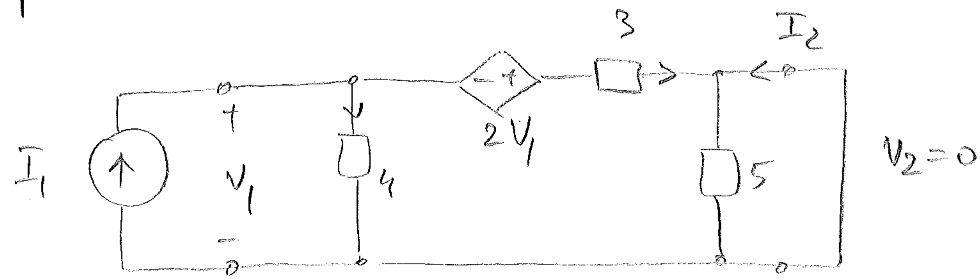


$$\boxed{V_2 = \frac{5}{5+3}(2V_1 + V_1) = \frac{15}{8} V_1} \Rightarrow a_{11} = \frac{V_1}{V_2} = \frac{8}{15}$$

$$I_1 = \frac{V_1}{4} + \frac{(V_1 + 2V_1) - V_2}{3} = \frac{5}{8} V_1$$

$$a_{21} = \frac{I_1}{V_2} = \frac{I_1}{V_1} \cdot \frac{V_1}{V_2} = \frac{5}{8} \cdot \frac{8}{15} = \frac{1}{3} \, [\Omega^{-1}]$$

- To find a_{12} and a_{22} use circuit with output port short-circuited!



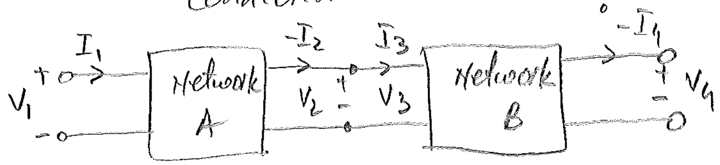
$$I_2 = -\frac{V_1 + 2V_1}{3} = -V_1 \implies a_{12} = -\frac{V_1}{I_2} = 1 \, [\Omega]$$

$$I_1 = \frac{V_1}{4} + \frac{V_1 + 2V_1}{3} = \frac{5}{4} V_1$$

$$a_{22} = -\frac{I_1}{I_2} = -\frac{I_1}{V_1} \cdot \frac{V_1}{I_2} = \left(-\frac{5}{4}\right) \cdot (-1) = \frac{5}{4}$$

- In summary, we have: $[a] = \begin{bmatrix} \frac{8}{15} & 1\,\Omega \\ \frac{1}{3}\,\Omega^{-1} & \frac{5}{4} \end{bmatrix}$

$\boxed{\text{NOTE}}$ a parameters are useful when two two-ports are connected in cascade!



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = [a_A] \cdot \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} = [a_A] \begin{bmatrix} V_3 \\ I_3 \end{bmatrix}$$

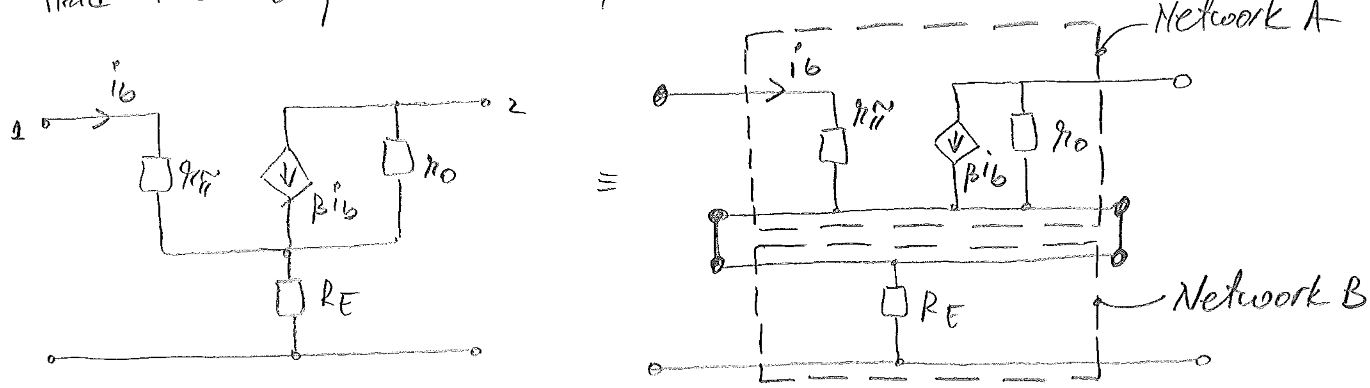
$$\begin{bmatrix} V_3 \\ I_3 \end{bmatrix} = [a_B] \begin{bmatrix} V_4 \\ -I_4 \end{bmatrix}$$

$$\implies \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \underbrace{[a_A][a_B]} \cdot \begin{bmatrix} V_4 \\ -I_4 \end{bmatrix}$$

$$\boxed{[a] \triangleq [a_A] \cdot [a_B]}$$

Example

Find the z parameters of the CE BJT amplifier.



series connection of two two-port networks!

- First find individual z parameters of the two-port networks:

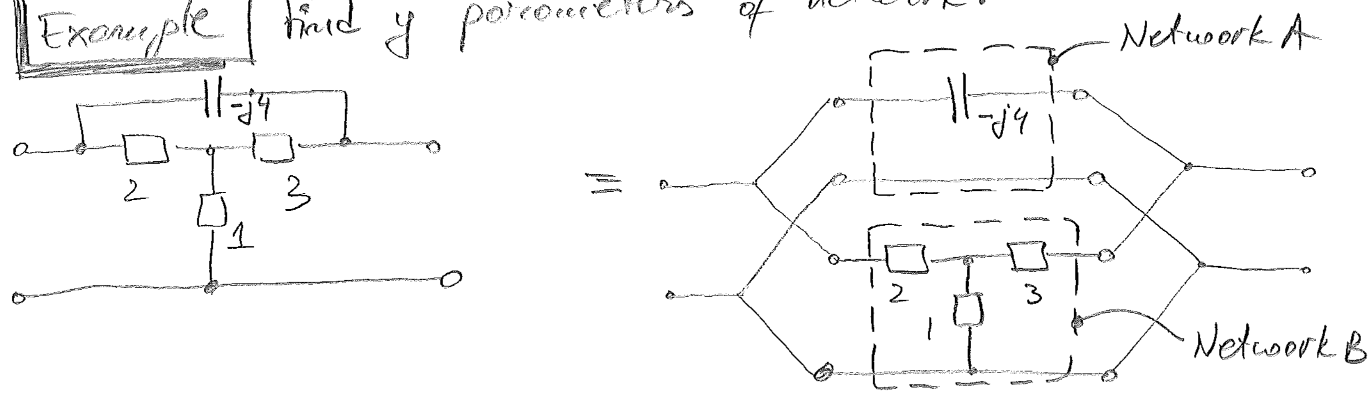
$$[Z_A] = \begin{bmatrix} r_\pi & 0 \\ -\beta r_o & r_o \end{bmatrix}$$

$$[Z_B] = \begin{bmatrix} R_E & R_E \\ R_E & R_E \end{bmatrix}$$

- Then, add them up as studied in sub-section $\boxed{2}$ of these lecture notes:

$$[Z] = [Z_A] + [Z_B] = \begin{bmatrix} r_\pi + R_E & R_E \\ -\beta r_o + R_E & r_o + R_E \end{bmatrix}$$

Example Find y parameters of network:



Parallel connection of two two-port Networks

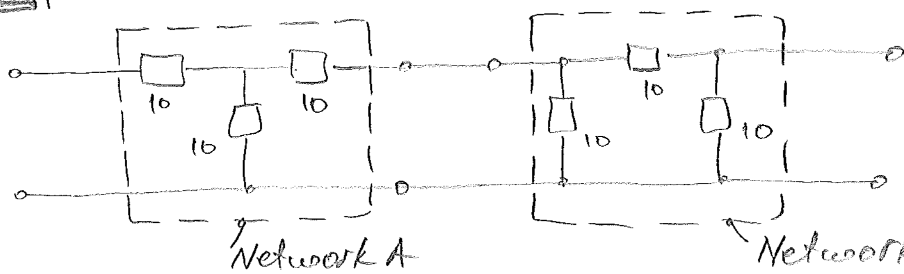
First find individual y parameters of networks A, B and then add them up as discussed in subsection $\boxed{1}$ of these lecture notes:

$$[y_A] = \begin{bmatrix} j\frac{1}{4} & -j\frac{1}{4} \\ -j\frac{1}{4} & j\frac{1}{4} \end{bmatrix}$$

$$[y_B] = \begin{bmatrix} \frac{4}{11} & -\frac{1}{11} \\ -\frac{1}{11} & \frac{3}{11} \end{bmatrix}$$

$$\Rightarrow [y] = [y_A] + [y_B] = \begin{bmatrix} \frac{16+j11}{44} & -\frac{4+j11}{44} \\ -\frac{4+j11}{44} & \frac{12+j11}{44} \end{bmatrix} \Omega^{-1}$$

$\boxed{\text{Example}}$ Find a parameters.



cascode connection of two two-port networks!

First, find individual a parameters, then multiply them as discussed in sub-section $\boxed{4}$ of these lecture notes:

$$[a] = \underbrace{\begin{bmatrix} 2 & 30 \\ 0.1 & 2 \end{bmatrix}}_{=[a_A]} \times \underbrace{\begin{bmatrix} 2 & 10 \\ 0.3 & 2 \end{bmatrix}}_{=[a_B]} = \begin{bmatrix} 13 & 80\,\Omega \\ 0.8\,\Omega^{-1} & 5 \end{bmatrix}$$