3 Hybrid parameters (the "h parameters")

(1) $\begin{cases} V_1 = h_{11} I_1 + h_{12} V_2 \\ I_2 = h_{21} I_1 + h_{22} V_2 \end{cases} \Rightarrow \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = [h] \cdot \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$

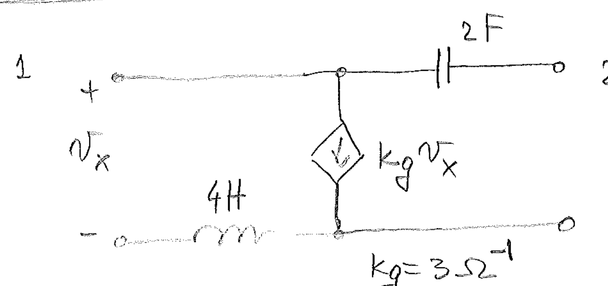
(2) $h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$: short-circuit input impedance $[\Omega]$

(3) $h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$: short-circuit forward current gain

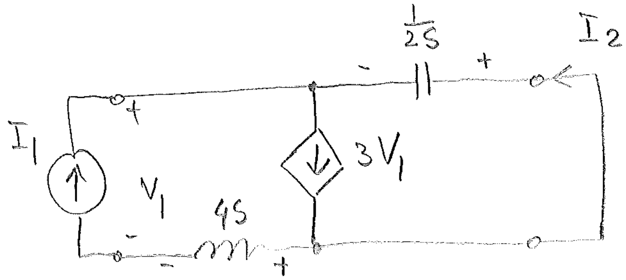
(4) $h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$: open-circuit reverse voltage gain

(5) $h_{22} = \left.\frac{I_2}{V_1}\right|_{I_1=0}$: open-circuit output admittance $[\Omega^{-1}]$

Example Find h parameters.



- To find h_{11} and h_{21} we use the circuit with the output short-circuited:



KCL: $I_1 + I_2 = 3V_1$

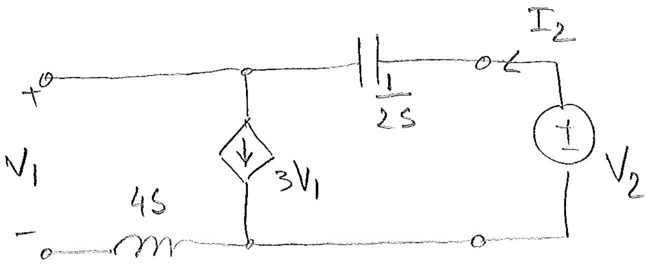
KVL: $V_1 + \frac{1}{2s} I_2 = 4s I_1$

$\Rightarrow$ eliminate I_2 or V_1

$\overset{I_2}{\Rightarrow}$ $\boxed{V_1 = \frac{8s^2+1}{2s+3} I_1}$

$\overset{V_1}{\Rightarrow}$ $\boxed{I_2 = \frac{2s(12s-1)}{2s+3} I_1}$

– To find h_{12} and h_{22} we use the circuit with the input port as open-circuit:



$$\begin{cases} I_2 = 3V_1 \\ V_2 - V_1 = \frac{1}{2s} \cdot I_2 \end{cases} \Rightarrow \boxed{V_1 = \frac{2s}{2s+3} V_2} \text{ by } I_2 \text{ elimination}$$

$$\Rightarrow \boxed{I_2 = \frac{6s}{2s+3} V_2} \text{ by } V_1 \text{ elimination.}$$

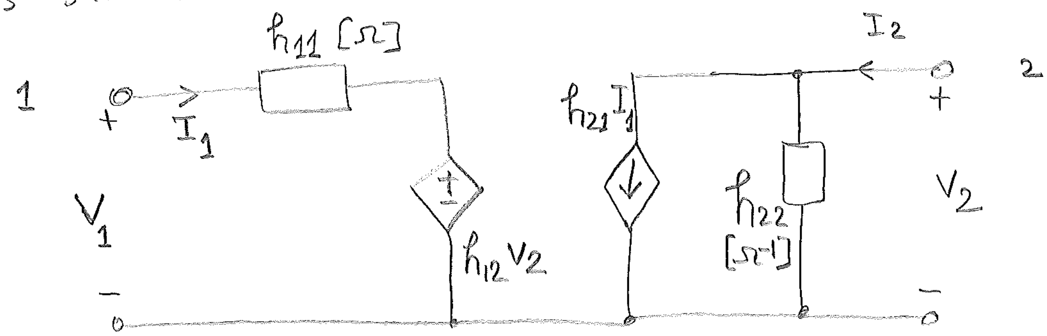
– Finally:

$$[h] = \begin{bmatrix} \frac{8s^2+1}{2s+3}\,\Omega & \frac{2s}{2s+3} \\ \frac{2s(12s-1)}{2s+3} & \frac{6s}{2s+3}\,\Omega^{-1} \end{bmatrix}$$

NOTE: h parameters can be added directly when 2 ports are connected in series at the input and in parallel at the output; not used often!!

Models of h parameters

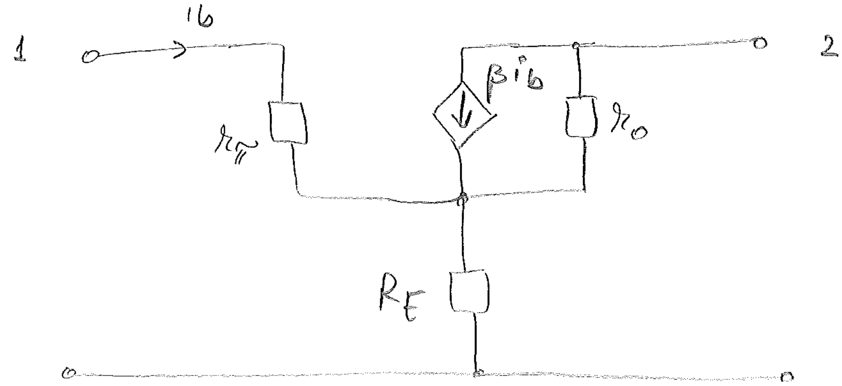
– Given a set of h parameters, a circuit that realizes them is shown below:



Note: This circuit is used to model the <u>small-signal</u> behavior of BJT's!

Example

For the small-signal model of a common-emitter (CE) BJT amplifier, find its h parameters.

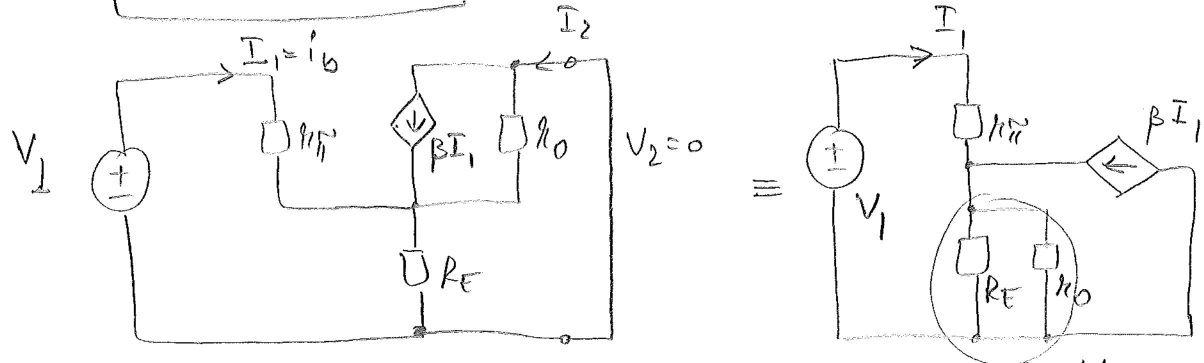


Solution: Utilize techniques similar to the previous example to derive:

$$[h_e] = \begin{bmatrix} h_{ie} & h_{re} \\ h_{fe} & h_{oe} \end{bmatrix} = \begin{bmatrix} r_\pi + (\beta+1)(R_E \| r_o) & \dfrac{R_E}{R_E + r_o} \\ \beta - (\beta+1)\dfrac{R_E}{R_E + r_o} & \dfrac{1}{R_E + r_o} \end{bmatrix}$$

$\boxed{e}$ stands for common emitter

$$\boxed{h_{ie} = \left.\frac{V_1}{I_1}\right|_{V_2=0}}$$



$= R_E \| r_o$

$$V_1 = r_\pi \cdot I_1 + (R_E \| r_o)(I_1 + \beta I_1)$$

$$V_1 = \left[r_\pi + (\beta+1)(R_E \| r_o)\right] I_1$$

$$\Rightarrow \boxed{h_{ie} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = r_\pi + (\beta+1)(R_E \| r_o)}$$

DIY: h_{re}, h_{fe}, h_{oe} !