ECE-311 week #13 Fri (1)

(f) Functions with complex conjugate pair (notch filters)

(1) $$\boxed{H(s) = \left(\frac{s}{\omega_0}\right)^2 + 2\xi\left(\frac{s}{\omega_0}\right) + 1}$$ a complex-conjugate **zeros** pair.

ξ called "damping ratio"

ω_0 called "undamped natural frequency"

- We are interested in the case when $\xi > 1$, for then the roots are complex conjugate:

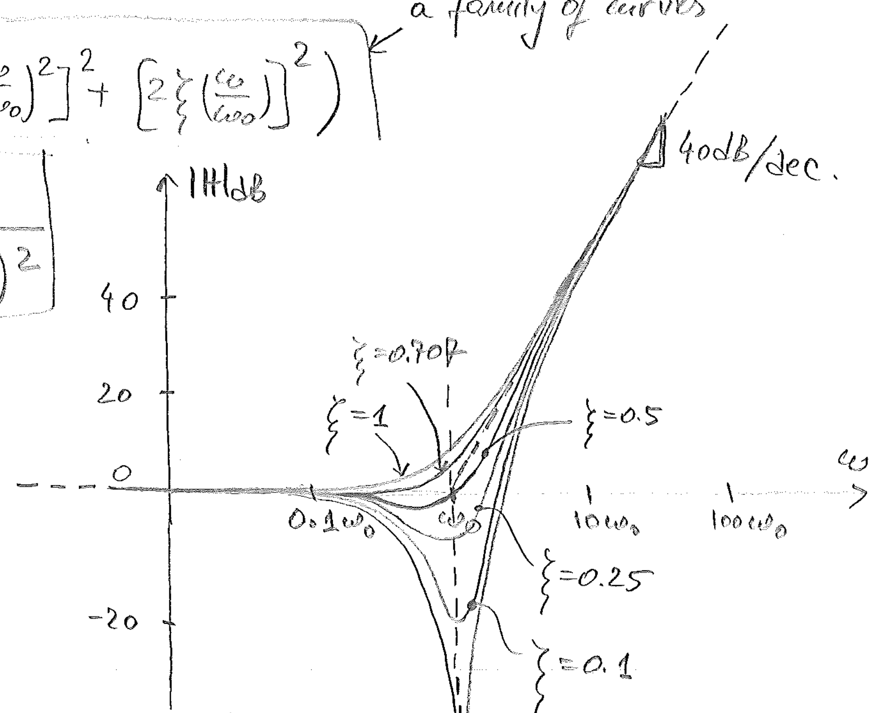
$$\frac{s}{\omega_0} = -\xi \pm j\sqrt{1-\xi^2}$$

- Let $s \to j\omega$ to get:

$$\boxed{H(j\omega) = 1 - \left(\frac{\omega}{\omega_0}\right)^2 + j\cdot 2\xi \cdot \frac{\omega}{\omega_0}}$$

$$\Rightarrow \boxed{|H(j\omega)|_{dB} = 10 \cdot \log\left(\left[1-\left(\frac{\omega}{\omega_0}\right)^2\right]^2 + \left[2\xi\left(\frac{\omega}{\omega_0}\right)\right]^2\right)}$$ a family of curves

$$\text{ang } H(j\omega) = \boxed{\measuredangle H(j\omega) = \tan^{-1} \frac{2\xi \cdot \frac{\omega}{\omega_0}}{1-\left(\frac{\omega}{\omega_0}\right)^2}}$$

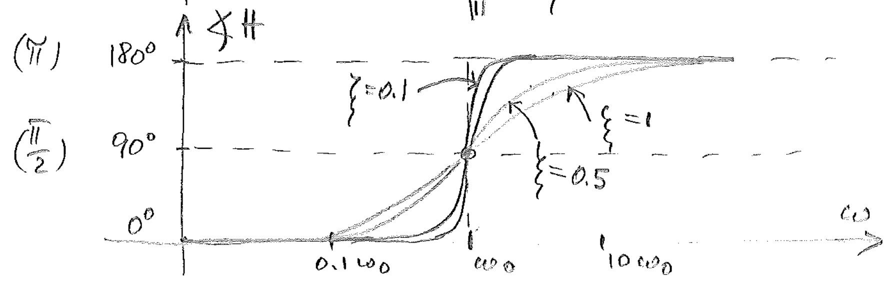


$$\begin{cases} \omega \to 0 \\ \omega \ll \omega_0 \end{cases} \Rightarrow |H|_{dB} = 10 \log 1 = 0$$

$$\begin{cases} \omega \to \infty \\ \omega \gg \omega_0 \end{cases} \Rightarrow |H|_{dB} = 40 \log \frac{\omega}{\omega_0}$$

$$\begin{cases} \omega \to 0,\ \omega \ll \omega_0 \Rightarrow \measuredangle H(j\omega) = \tan^{-1}(0) = 0 \\ \omega = \omega_0 \Rightarrow \measuredangle H(j\omega_0) = \tan^{-1}(\infty) = \frac{\pi}{2} \\ \omega \to \infty,\ \omega \gg \omega_0 \Rightarrow \measuredangle H(j\omega) = \pi \end{cases}$$

Explain!



- ω_0 is the corner frequency
- For $\omega = \omega_0 \Rightarrow$ $\boxed{|H(j\omega_0)|_{dB} = 20 \cdot \log(2\zeta)}$
- As long as $\boxed{\zeta \geq \frac{1}{\sqrt{2}}} = 0.707 \Rightarrow$ magnitude Bode plot is above 0dB.

  The curve corresponding to $\zeta = \frac{1}{\sqrt{2}}$ is said to be "maximally flat"
- For $\boxed{\zeta < \frac{1}{\sqrt{2}}}$ - there is a frequency band where the magnitude is less than 0dB

  - This phenomenon is called "peaking"
  - The smaller ζ the more pronounced the amount of peaking
- In the limit: $\zeta \to 0 \Rightarrow |H| \to 0$

Question what changes if $\boxed{H(s) = \left(\frac{s}{\omega_0}\right)^2 - 2\zeta\left(\frac{s}{\omega_0}\right) + 1}$?

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2 $\boxed{H(s) = \dfrac{1}{\left(\frac{s}{\omega_0}\right)^2 + 2\zeta\left(\frac{s}{\omega_0}\right) + 1}}$ a complex conjugate poles pair.

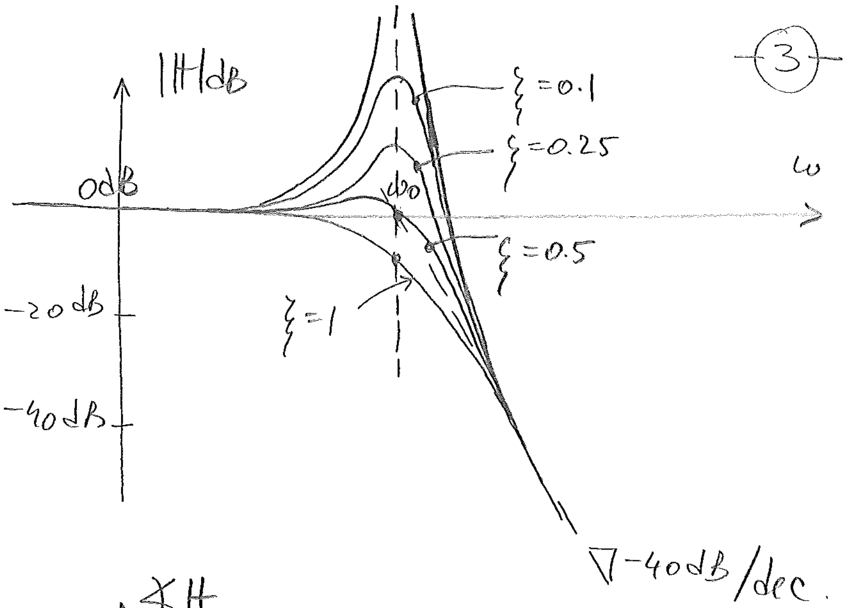
Let $s \to j\omega$ to get $\boxed{H(j\omega) = \dfrac{1}{1 - \left(\frac{\omega}{\omega_0}\right)^2 + j\,2\zeta\left(\frac{\omega}{\omega_0}\right)}}$

$$\Rightarrow |H(j\omega)|_{dB} = -10\log\left(\left[1 - \left(\frac{\omega}{\omega_0}\right)^2\right]^2 + \left[2\zeta\left(\frac{\omega}{\omega_0}\right)\right]^2\right)$$

$$\measuredangle H(j\omega) = -\tan^{-1}\frac{2\zeta\frac{\omega}{\omega_0}}{1 - \left(\frac{\omega}{\omega_0}\right)^2}$$

$$\begin{cases} \omega \to 0 \\ \omega \ll \omega_0 \end{cases} \Rightarrow |H|_{dB} = 0$$

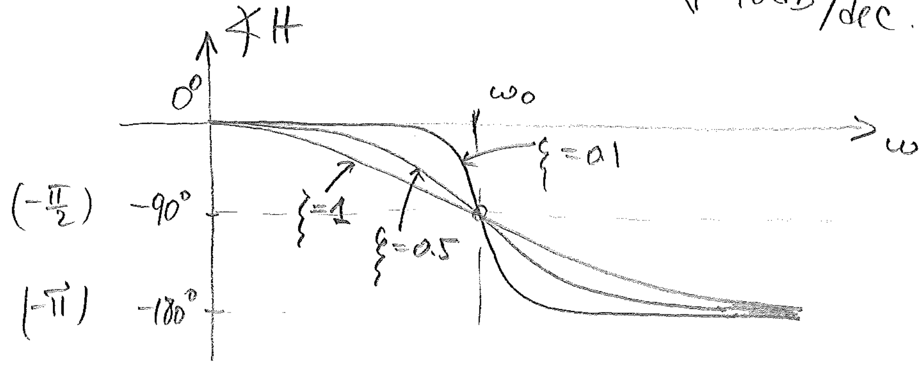
$$\begin{cases} \omega \to \infty \\ \omega \gg \omega_0 \end{cases} \Rightarrow |H|_{dB} = -40 \log \frac{\omega}{\omega_0}$$



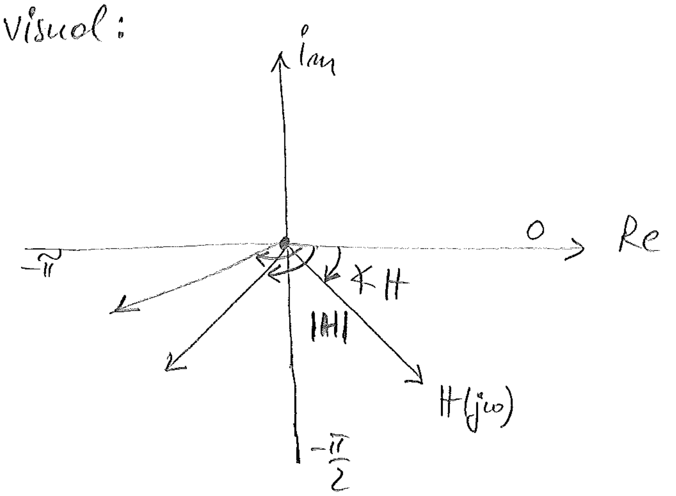
$$\begin{cases} \omega \to 0 \\ \omega \ll \omega_0 \end{cases} \Rightarrow \measuredangle H(j\omega) = 0$$

$$\omega = \omega_0 \Rightarrow \measuredangle H(j\omega) = -\frac{\pi}{2}$$

$$\begin{cases} \omega \to \infty \\ \omega \gg \omega_0 \end{cases} \Rightarrow \measuredangle H(j\omega) = -\pi$$



Helping visual:



When $\xi \to 0 \Rightarrow |H| \to \infty$ $\Rightarrow$ circuit capable of providing $\boxed{ac}$ response even in the absense of any input signal! $\Rightarrow$ sustained oscillation; poles are on imaginary axis $\frac{s}{\omega_0} = \pm j$

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