Example

$$H(s) = \frac{10 \cdot s}{\left(1+\frac{s}{10}\right)\left(1+\frac{s}{10000}\right)}$$

Draw the Bode plots (magnitude and phase).

Zero at: $s=0$
Poles at: $s=-10$, $s=-10^4$

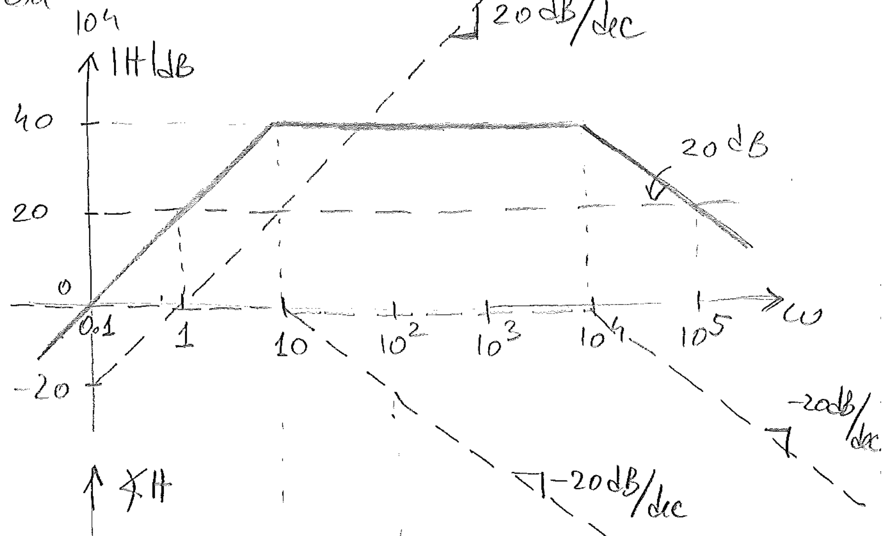
Let $s=j\omega$ => $H(j\omega) = 10 \cdot \frac{j\omega}{\left(1+j\frac{\omega}{10}\right)\left(1+j\frac{\omega}{10^4}\right)}$

$$|H(j\omega)|_{dB} = 20\log(10) + 20\log(\omega) - 20\log\sqrt{1+\left(\frac{\omega}{10}\right)^2} - 20\log\sqrt{1+\left(\frac{\omega}{10^4}\right)^2}$$

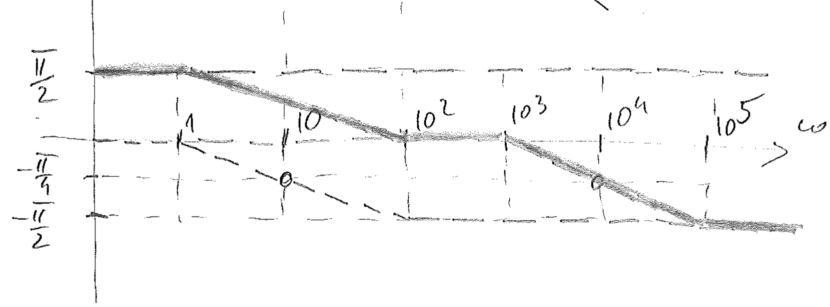
$$\measuredangle H(j\omega) = \underbrace{\tan^{-1}\frac{0}{10}}_{=0} + \underbrace{\tan^{-1}\frac{\omega}{0}}_{=90^\circ\ (\pi/2)} - \tan^{-1}\frac{\omega}{10} - \tan^{-1}\frac{\omega}{10^4}$$

$$\measuredangle H(j\omega) = \frac{\pi}{2} - \tan^{-1}\frac{\omega}{10} - \tan^{-1}\frac{\omega}{10^4}$$

Magnitude Bode plot



Phase Bode plot



Example

constant — simple real zero

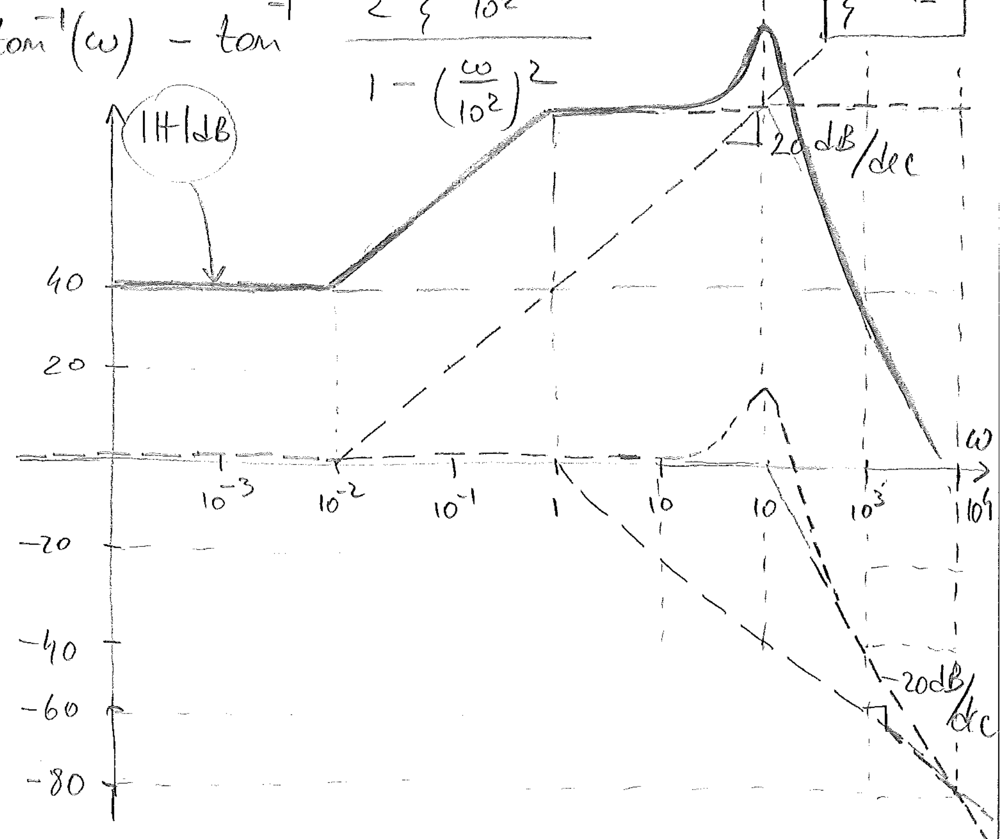
$$H(s) = \frac{100\left(1 + \frac{s}{10^{-2}}\right)}{(1+s)\left[1 + 0.2 \cdot \frac{s}{10^2} + \left(\frac{s}{10^2}\right)^2\right]}$$

simple real pole — complex-conjugate poles pair

$$s \to j\omega \Rightarrow \quad H(j\omega) = 100 \cdot \frac{\left(1 + j\frac{\omega}{10^{-2}}\right)}{(1+j\omega)\left[1 - \left(\frac{\omega}{10^2}\right)^2 + j \cdot 2 \underbrace{(0.1)}_{=\xi}\left(\frac{\omega}{10^2}\right)\right]}$$

$$\Rightarrow \begin{cases} |H(j\omega)|_{dB} = 40 + 20 \cdot \log\sqrt{1 + \left(\frac{\omega}{10^{-2}}\right)^2} - 20 \cdot \log\sqrt{1+\omega^2} - 20\log\sqrt{\left[1 - \left(\frac{\omega}{10^2}\right)^2\right]^2 + \left[2 (0.1) \left(\frac{\omega}{10^2}\right)\right]^2} \\ \measuredangle H(j\omega) = \tan^{-1}\left(\frac{\omega}{10^{-2}}\right) - \tan^{-1}(\omega) - \tan^{-1}\frac{2\xi\frac{\omega}{10^2}}{1 - \left(\frac{\omega}{10^2}\right)^2} \end{cases}$$

$$\boxed{\xi = 0.1}$$

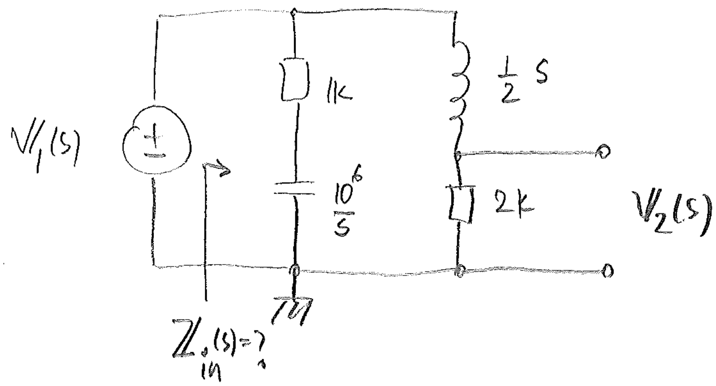


Example

(a) $Z_{in}(s) = ?$

(b) $H(s) = \frac{V_2(s)}{V_1(s)} = ?$

(c) Plot Bode diagrams of $H(s)$



(a)

$$Z_{in} = \left(1k + \frac{10^6}{s}\right) \parallel \left(\frac{1}{2}s + 2k\right) = \frac{(s+4k)(1000s+10^6)}{s^2 + 6\times10^3 s + 2\times10^6} =$$

$$\boxed{Z_{in} = \frac{1000(s+4000)(s+1000)}{(s+354)(s+5646)}} \ \Omega$$

(b)

$$V_2 = \frac{2000}{\frac{1}{2}s + 2000} \cdot V_1 \Rightarrow \boxed{H(s) = \frac{V_2(s)}{V_1(s)} = \frac{4000}{s+4000}}$$

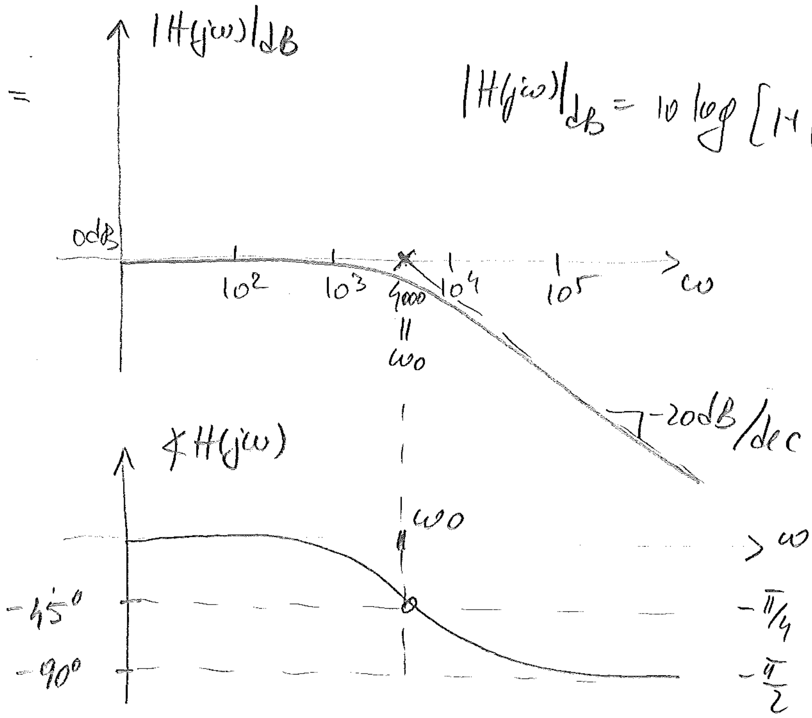
Single pole at $s = -4000$

(c) $s \to j\omega$

$$H(j\omega) = \frac{4000}{j\omega + 4000} = \frac{1}{1 + j\frac{\omega}{4000}}$$

$$\boxed{\omega_0 = 4000}$$

$$|H(j\omega)|_{dB} = 10 \log\left[1 + \left(\frac{\omega}{4000}\right)^2\right]$$



$$\measuredangle H(j\omega) = -\tan^{-1}\frac{\omega}{4000}$$

Example

Find the poles and zeros of the Laplace transform of:

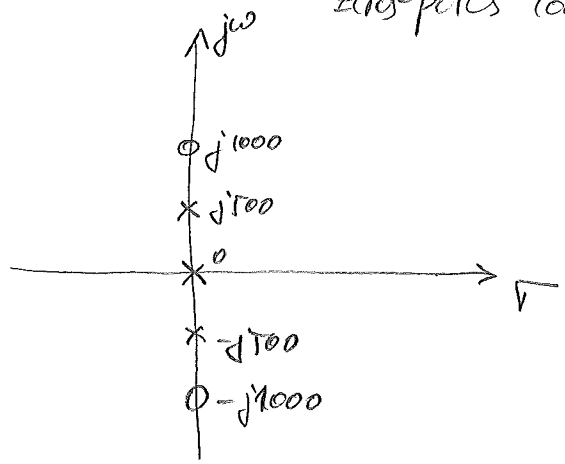
$$f(t) = [4 - 3\cos(500t)] \cdot u(t)$$

$$F(s) = \frac{4}{s} - \frac{3s}{s^2+(500)^2} = \frac{s^2 + 4\times 500^2}{s\,[s^2+500^2]} =$$

$$= \frac{(s-j1000)(s+j1000)}{s\,(s+j500)(s-j500)}$$

- two complex conjugate zeros at $\pm j1000$
- two complex conjugate poles at $\pm j500$
- Simple pole at $s=0$

zeros-poles constellation



Example

Use one capacitor C and one resistor R to design a low-pass filter with a corner frequency of $\omega_0 = 1\,kHz$. Specify values for R, C and plot Bode diagrams.

Example

The step response of a linear circuit is $g(t) = 5\left[1 - e^{-200t}\right]u(t)$

Find the output waveform when the input is $x(t) = \left[12\, e^{-200t}\right]u(t)$.

Step response is for when input is $u(t)$ and in the Laplace (s-domain) we have:

$$\boxed{G(s) = H(s) \cdot \frac{1}{s}} \quad (1)$$

But we know that: $g(t) \Leftrightarrow G(s) = \mathcal{L}\left\{5\left[1 - e^{-200t}\right] \cdot u(t)\right\}$

$$g(t) \Leftrightarrow \boxed{G(s) = \frac{5}{s} - \frac{5}{s+200} = \frac{1000}{s(s+200)}} \quad (2)$$

$$(1) + (2) \Rightarrow H(s) \cdot \frac{1}{s} = \frac{1000}{s(s+200)}$$

$$\Rightarrow \boxed{H(s) = \frac{1000}{s+200}} \quad (3)$$

The output response when input is $x(t)$ is:

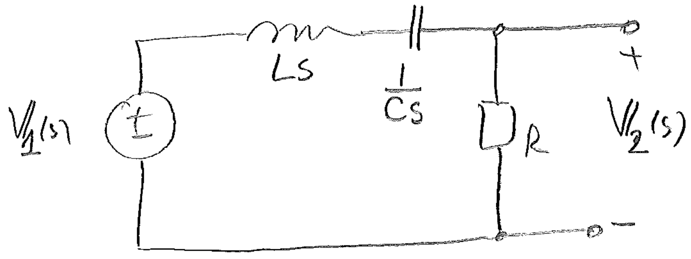
$$Y(s) = H(s) \cdot X(s) = \frac{1000}{s+200} \cdot \frac{12}{s+200} = \frac{12\,000}{(s+200)^2}$$

$$\Rightarrow \boxed{y(t) = \mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{\frac{12000}{(s+200)^2}\right\} = 12000 \cdot t\, e^{-200t} \cdot u(t)}$$

Example

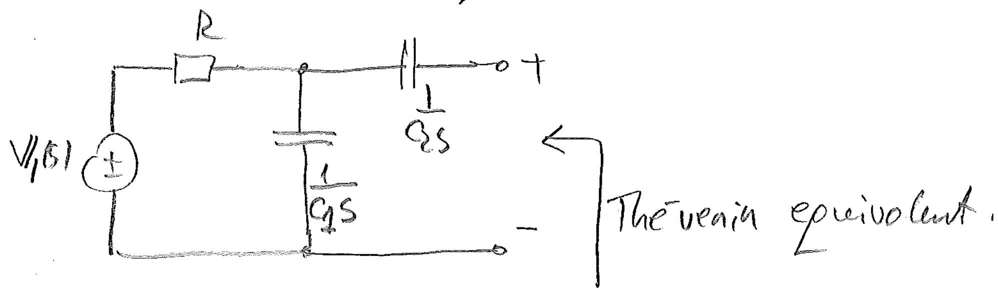
Find the voltage transfer function $H(s) = \dfrac{V_2(s)}{V_1(s)}$

and draw the zeros-poles constellation.



Example

Find the Thévenin equivalent:



Example

Find $v(t)$. Switch has been open for a long time and is closed at $t=0$. Use Laplace based techniques.