# 5 Transfer function building blocks

## a

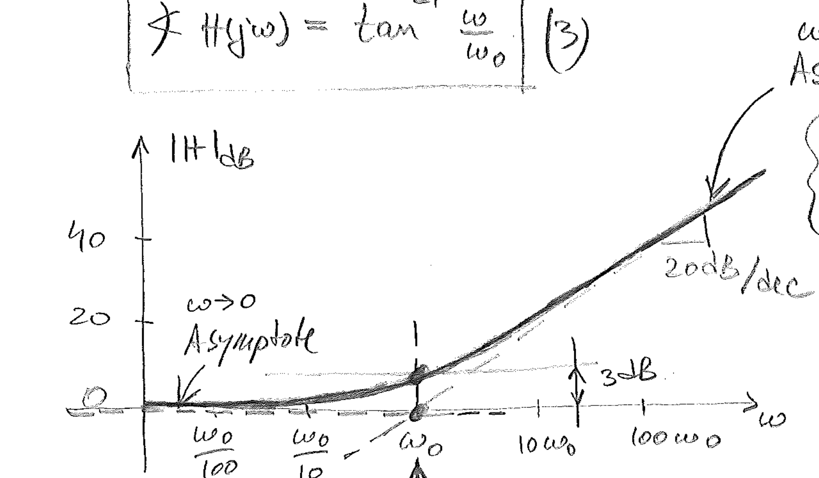
$$\boxed{H(s) = \frac{s}{\omega_0} + 1} \quad \boxed{\text{zero:}} \ \boxed{s = -\omega_0} \qquad \text{or} \quad \boxed{H(s) = \frac{s}{z_1} + 1}$$

Let $s \to j\omega \Rightarrow \boxed{H(j\omega) = 1 + j\frac{\omega}{\omega_0}}$ (1)

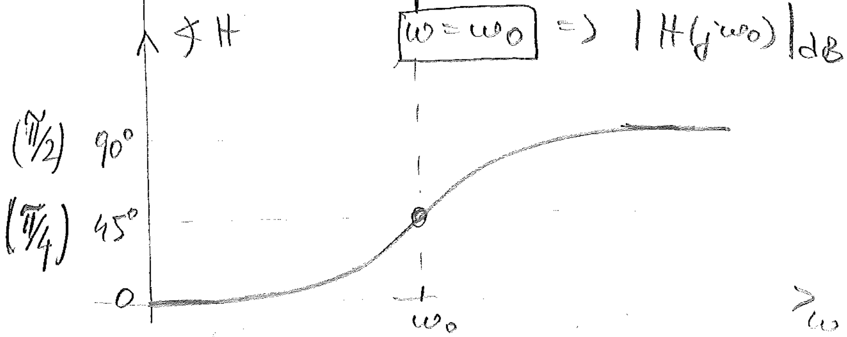
$\Rightarrow \boxed{|H(j\omega)|_{dB} = 10 \log\left[1 + \left(\frac{\omega}{\omega_0}\right)^2\right]}$ (2)

$\boxed{\measuredangle H(j\omega) = \tan^{-1}\frac{\omega}{\omega_0}}$ (3)

$\begin{cases} \omega \to 0 \Rightarrow |H|_{dB} = 10 \cdot \log(1) = 0 \\ \omega \to \infty \Rightarrow |H|_{dB} = 20 \cdot \log\frac{\omega}{\omega_0} \end{cases}$




$\boxed{\omega = \omega_0} \Rightarrow |H(j\omega_0)|_{dB} = 10 \log(2) = 3\text{ dB}$




$\begin{cases} \omega \to 0 \Rightarrow \measuredangle H = \tan^{-1}\left(\frac{0}{\omega_0}\right) = 0° \\ \omega \to \infty \Rightarrow \measuredangle H = \tan^{-1}(\infty) = 90° \end{cases}$

$\boxed{\omega = \omega_0} \Rightarrow \measuredangle H(j\omega) = \tan^{-1}(1) = 45°$

**Question**: what changes if $\boxed{H(s) = -\frac{s}{\omega_0} + 1}$ or $\boxed{H(s) = +\frac{s}{\omega_0} - 1}$ or $\boxed{H(s) = -\frac{s}{\omega_0} - 1}$

**Answer**:
- magnitude Bode plots remain the same!
- phase angle plots change!

(see verso)

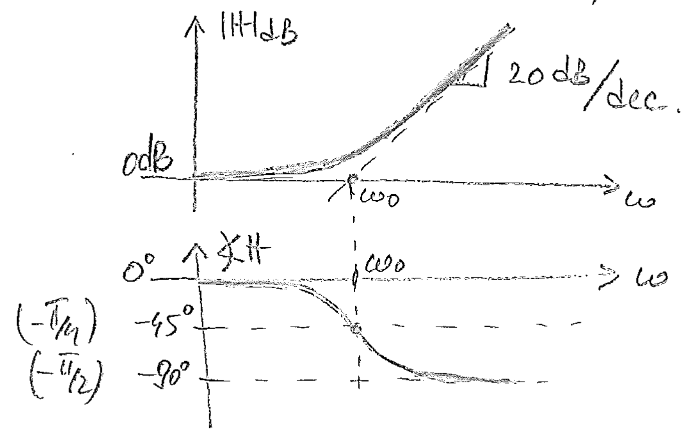
**Case:** $\boxed{H(s) = -\frac{s}{\omega_0} + 1}$ with $\omega_0 > 0$ $\xrightarrow{s \to j\omega}$ $H(j\omega) = 1 - j\frac{\omega}{\omega_0}$

**Gain:** $|H(j\omega)|_{dB} = 10 \cdot \log\left[1 + \left(-\frac{\omega}{\omega_0}\right)^2\right] = \underbrace{10 \cdot \log\left[1 + \left(\frac{\omega}{\omega_0}\right)^2\right]}_{\uparrow}$

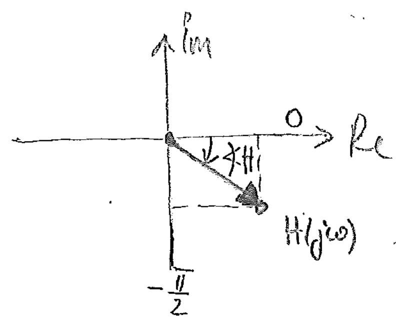
same as eq. (2)

**Phase:** $\measuredangle H(j\omega) = \tan^{-1}\left(-\frac{\omega}{\omega_0}\right) = -\tan^{-1}\left(\frac{\omega}{\omega_0}\right)$

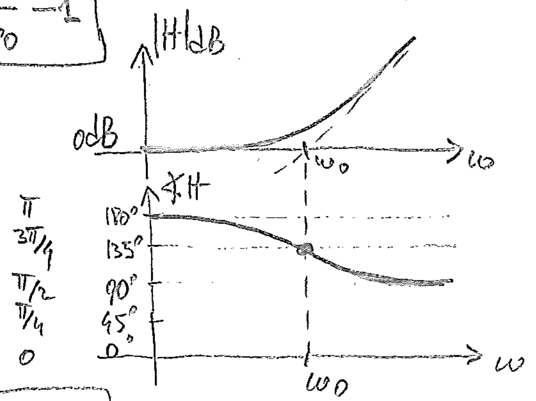
**Bode plots:**

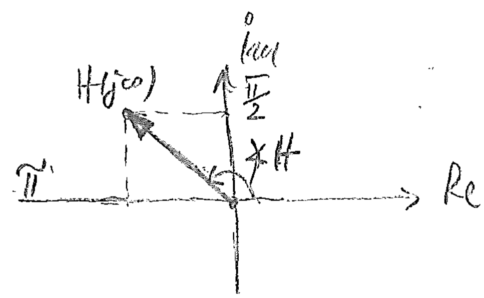



Helpful "**visual**":

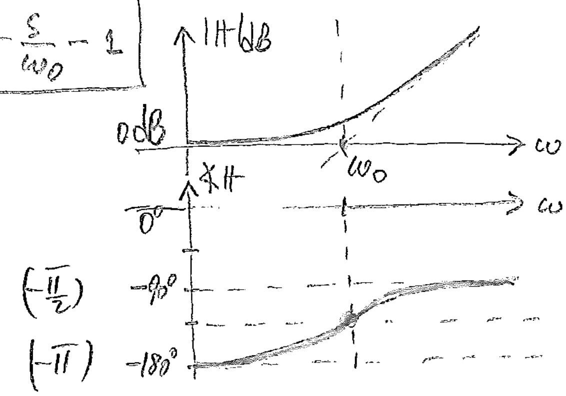


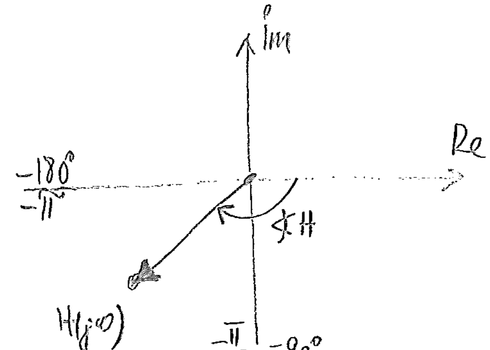

**Case:** $\boxed{H(s) = \frac{s}{\omega_0} - 1}$




**Case:** $\boxed{H(s) = -\frac{s}{\omega_0} - 1}$



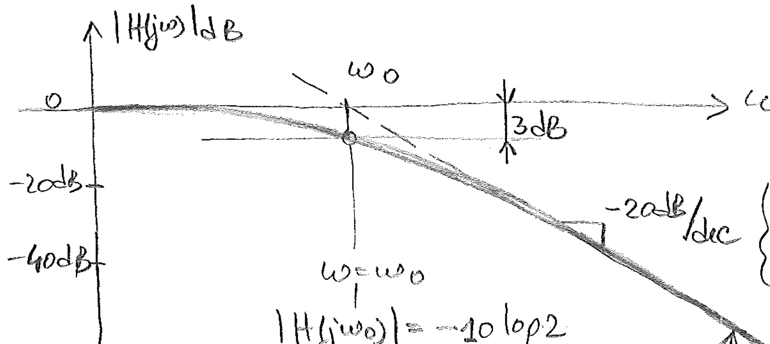



(b) $\boxed{H(s) = \dfrac{1}{\frac{s}{\omega_0} + 1}}$ $\boxed{\text{pole}}$ at $\boxed{s = -\omega_0}$

Let $s \to j\omega \Rightarrow H(j\omega) = \dfrac{1}{1 + j\left(\frac{\omega}{\omega_0}\right)}$

$$\Rightarrow \boxed{|H(j\omega)| = -10 \log\left[1 + \left(\frac{\omega}{\omega_0}\right)^2\right]}$$

$$\boxed{\measuredangle H(j\omega) = -\tan^{-1}\left(\frac{\omega}{\omega_0}\right)}$$

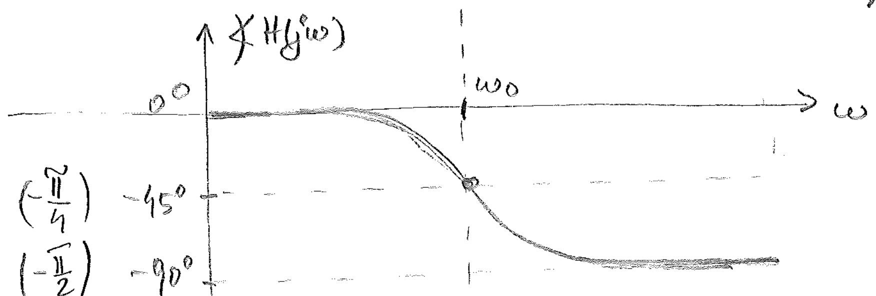



$\begin{cases} \omega \to 0 \Rightarrow |H|_{dB} = 0 \\ \omega \to \infty \Rightarrow |H|_{dB} = -20 \log\left(\frac{\omega}{\omega_0}\right) \end{cases}$

$\omega = \omega_0$

$|H(j\omega_0)| = -10 \log 2$
$= -3\,dB$

Asymptote as $\omega \to \infty$




$\begin{cases} \omega \to 0 \Rightarrow \measuredangle H = -\tan^{-1} 0 = 0^\circ \\ \omega \to \infty \Rightarrow \measuredangle H = -\tan^{-1}(\infty) = -90^\circ \end{cases}$

$\omega = \omega_0 \Rightarrow \measuredangle H = -\tan^{-1}(1) = -45^\circ$

$\boxed{\text{NOTE}}$ $\omega_0$ is called corner, break, ±3 dB, ±45° frequency

$\boxed{\text{Question}}$ what changes if $\boxed{H(s) = \dfrac{1}{-\frac{s}{\omega_0} + 1}}$ or $\boxed{H(s) = \dfrac{1}{\frac{s}{\omega_0} - 1}}$ or $\boxed{H(s) = \dfrac{1}{-\frac{s}{\omega_0} - 1}}$

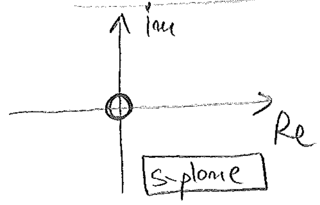
$\boxed{\text{Answer}}$ $\begin{cases} \text{– magnitude Bode plot stays the same} \\ \text{– phase Bode plot changes!!!} \end{cases}$

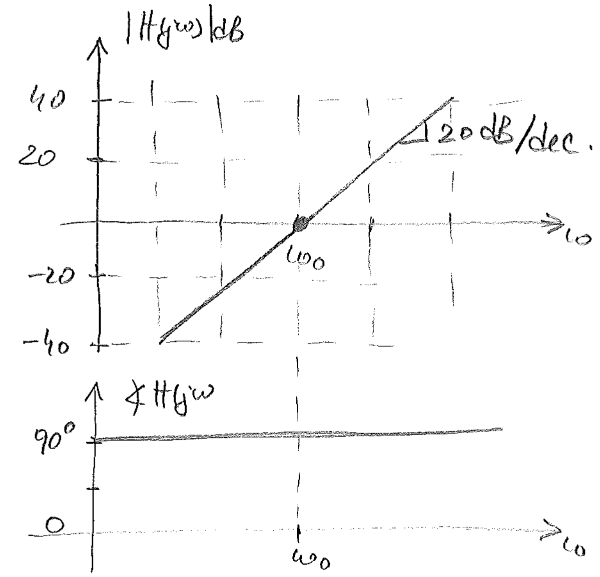
(c) $\boxed{H(s) = \frac{s}{\omega_0}}$ $\begin{cases} \text{– zero at the origin!} \\ \text{– it is the s-domain counterpart of } \underline{\text{differentiation}}\text{!} \end{cases}$

Let $s \to j\omega \Rightarrow H(j\omega) = j\frac{\omega}{\omega_0}$

$\Rightarrow \boxed{|H(j\omega)|_{dB} = 20\log\left(\frac{\omega}{\omega_0}\right)}$

$\boxed{\measuredangle H(j\omega) = 90^\circ}$



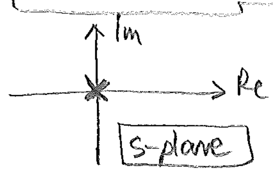



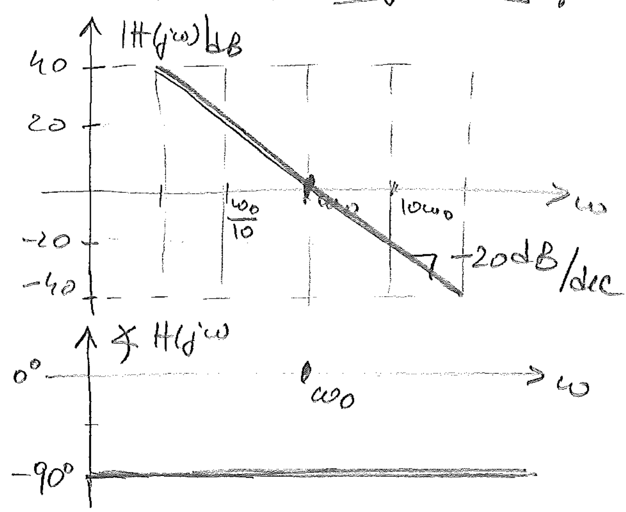

(d) $\boxed{H(s) = \frac{\omega_0}{s}}$ $\begin{cases} \text{– pole at the origin!} \\ \text{– it is the s-domain counterpart of } \underline{\text{integration}}\text{!} \end{cases}$

Let $s \to j\omega \Rightarrow H(j\omega) = \frac{1}{j\left(\frac{\omega}{\omega_0}\right)}$

$\Rightarrow \boxed{|H(j\omega)|_{dB} = -20\log\left(\frac{\omega}{\omega_0}\right)}$

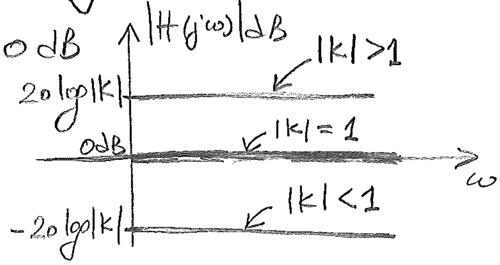
$\boxed{\measuredangle H(j\omega) = -90^\circ}$



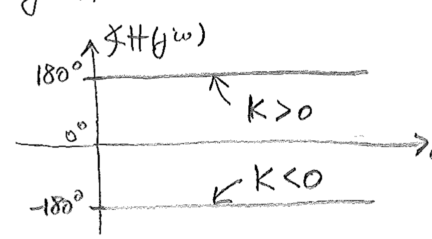




(e) $\boxed{H(s) = K}$ constant; independent of frequency!

Magnitude Bode plot $\begin{cases} \underline{\text{case}}\ |K| = 1 \Rightarrow |H(j\omega)|_{dB} = 0\,dB \\ \underline{\text{case}}\ |K| > 1 \\ \underline{\text{case}}\ |K| < 1 \end{cases}$




Phase Bode plot $\begin{cases} \underline{\text{case}}\ K > 0 \\ \underline{\text{case}}\ K < 0 \end{cases}$




Example: inverting/non-inverting amplifier