4 BODE DIAGRAMS

- The manner in which the ac response varies with frequency is called frequency response!

- Frequency response can be predicted mathematically using the transfer function $H(s)$. First, we derive $H(s)$ using s-domain techniques; then, we calculate it on the $j\omega$ axis by letting $s \to j\omega$.

- Frequency response can be visualized graphically by plotting:
  - the magnitude $|H(j\omega)|$
  - the phase $\measuredangle H(j\omega)$ versus ω.

- NOTE
  - ac response is the response to an ac signal after all transients have died out. This means that all poles must lie in the left half of the s-plane!

- We saw that location of critical frequencies has a profound impact on the natural response. So, we have good reasons to expect an impact upon the frequency response too!

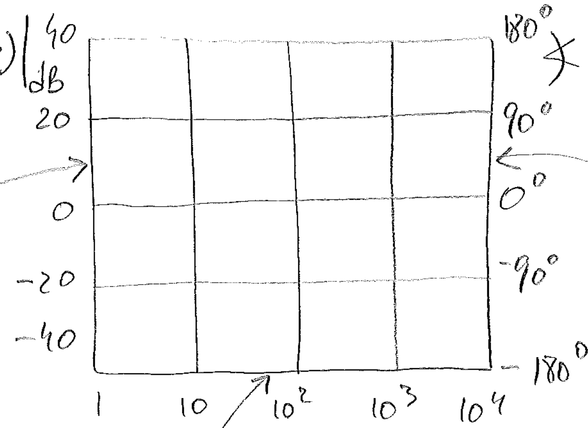
- We can illustrate the use of $H(s)$ to:
  - locate the critical frequencies in the s-plane
  - visualize the frequency response via suitable plots known as Bode plots!

- Note We'll restrict ourselves to network functions of the gain type.

Semilogarithmic and decibel (dB) scales

$G(\omega) \triangleq |H(j\omega)|_{dB}$ (dB) called "gain"

$\measuredangle H(j\omega) \triangleq \phi(\omega) = \arg H(\omega)$ (deg)



ω (rad/s)

linear scale calibrated in decibels

linear scale calibrated in degrees.

logarithmic scale calibrated in frequency decades.

- Semilogarithmic plots are called $\boxed{\text{Bode plots}}$ / diagrams (after American Hendrick W. Bode, pronounced bodee)
- A frequency decade is called a cycle.
- At times, it is convenient to use the normalized frequency $\frac{\omega}{\omega_0}$ where ω_0 is the characteristic frequency of the circuit.

- The decibel value (or dB) of a gain function H is defined as:

$$\boxed{|H|_{dB} \triangleq 20 \log_{10} |H|} = \boxed{10 \log_{10} |H|^2}$$

due to energy considerations

Example: $20 \log |-400| = 52\,dB$

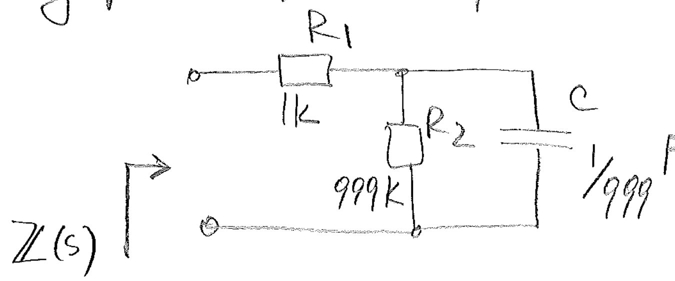
NOTE
- unity gain corresponds to zero dB
- gain magnitudes $> 1 \Rightarrow$ positive dB's
- gain magnitude $< 1 \Rightarrow$ negative dB's.

- Properties:

$$|H_1 \cdot H_2|_{dB} = |H_1|_{dB} + |H_2|_{dB}$$
$$\measuredangle (H_1 \cdot H_2) = \measuredangle H_1 + \measuredangle H_2$$
$$|H_1 / H_2|_{dB} = |H_1|_{dB} - |H_2|_{dB}$$
$$\measuredangle (H_1 \cdot H_2) = \measuredangle H_1 - \measuredangle H_2$$

$\boxed{\text{Example}}$

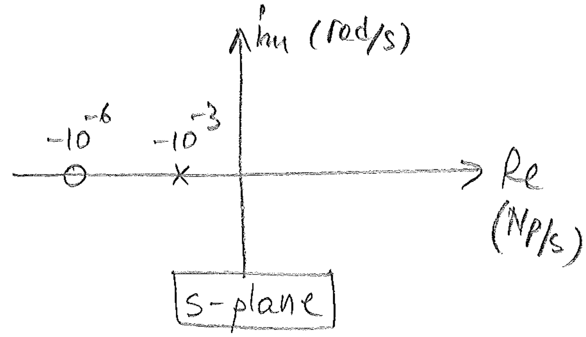
Sketch the pole-zero plot and the Bode plots for the driving-point impedance of the one-port:



$$Z(s) = R_1 + R_2 \parallel \frac{1}{sC} = (R_1 + R_2) \cdot \underbrace{\frac{(R_1 \parallel R_2) C s + 1}{R_2 C s + 1}}_{\text{frequency dependent part of } Z(s)} = (R_1 + R_2) \cdot H(s)$$

$$\boxed{H(s) = \frac{\frac{s}{10^6} + 1}{\frac{s}{10^3} + 1}}$$

- $H(s)$ has a zero at $s = z_1 = -10^6$ Np/s and a pole at $s = p_2 = -10^3$ Np/s



- To find frequency-response, we let $s \to j\omega$:

$$H(j\omega) = \frac{1 + j\frac{\omega}{10^6}}{1 + j\frac{\omega}{10^3}}$$

$$|H(j\omega)| = \sqrt{\frac{1 + \left(\frac{\omega}{10^6}\right)^2}{1 + \left(\frac{\omega}{10^3}\right)^2}}$$

$$\boxed{|H(j\omega)|_{dB} = 20 \log |H(j\omega)| = 10 \log \frac{1 + \left(\frac{\omega}{10^6}\right)^2}{1 + \left(\frac{\omega}{10^3}\right)^2} = 10 \log\left[1 + \left(\frac{\omega}{10^6}\right)^2\right] - 10 \log\left[1 + \left(\frac{\omega}{10^3}\right)^2\right]}$$

$$\boxed{\measuredangle H(j\omega) = \tan^{-1}\left(\frac{\omega}{10^6}\right) - \tan^{-1}\left(\frac{\omega}{10^3}\right)}$$

Evaluate these 2 expressions for different values of ω to get Bode plots: