Bode characteristics diagrams

$$H(s) = \frac{k\cdot(s-z_1)(s-z_2)\cdots}{(s-p_1)(s-p_2)\cdots} \qquad \begin{matrix} :\ \text{zeros} \\ :\ \text{poles.} \end{matrix}$$

$\boxed{\omega = 2\pi f}$

$$\begin{cases} |H(j\omega)| = \dfrac{|k|\cdot\sqrt{\omega^2+z_1^2}\cdot\sqrt{\omega^2+z_2^2}\cdots}{\sqrt{\omega^2+p_1^2}\cdot\sqrt{\omega^2+p_2^2}\cdots} \\[2ex] \phi(\omega) = \arg H(j\omega) = \arg(k) + \arg(j\omega - z_1) + \arg(j\omega - z_2) + \dots \\ \qquad\qquad - \arg(j\omega - p_1) - \arg(j\omega - p_2) - \dots \end{cases}$$

Gain

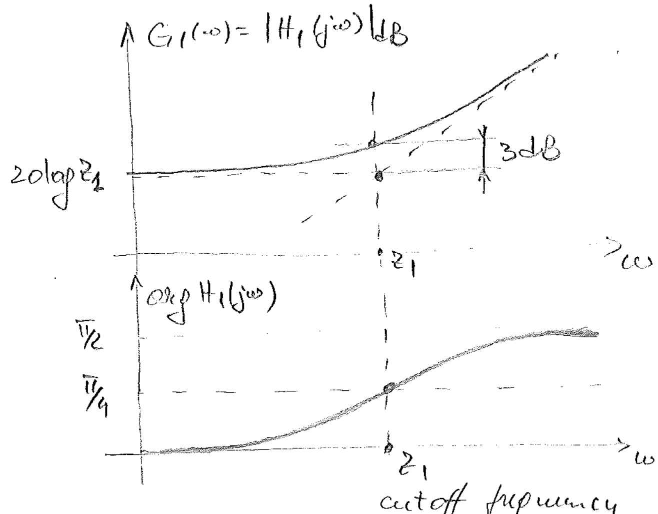
$$G(\omega) \overset{\Delta}{=} 10\cdot\log|H(j\omega)|^{\overset{\text{due to energetic considerations.}}{2}} = 20\cdot\log|H(j\omega)| \quad [dB]$$

$$G(\omega) = 20\log|k| + 20\log\sqrt{\omega^2+z_1^2} + 20\log\sqrt{\omega^2+z_2^2} + \dots - 20\log\sqrt{\omega^2+p_1^2} - 20\log\sqrt{\omega^2+p_2^2} - \dots$$

Examples:

(•) $\boxed{H_1(s) = s + z_1}$ $\Rightarrow$ $G_1(\omega) = 20\cdot\log\sqrt{\omega^2+z_1^2} = \begin{cases} 20\cdot\log z_1, & \omega \ll z_1 \\ 20\cdot\log\omega, & \omega \gg z_1 \end{cases}$

Asymptotes to which the actual $G_1(\omega)$ converges at extreme freq's.



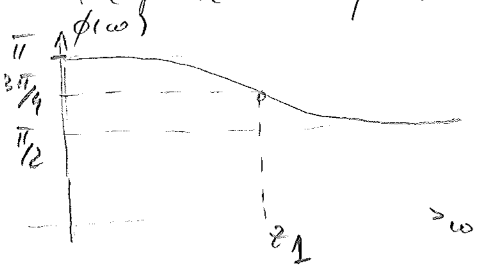
$$G_1(z_1) = 20\cdot\log\sqrt{2z_1^2} = 20\log z_1 + \underbrace{20\log\sqrt{2}}_{3dB}.$$

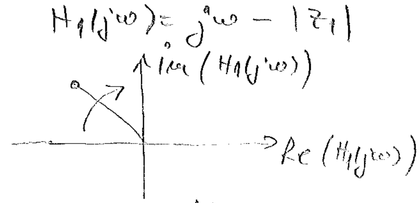
$$\phi(\omega) = \arg H_1(j\omega) = \operatorname{arctg}\frac{\omega}{z_1}$$

$$\begin{cases} \omega \to 0 \Rightarrow \phi(\omega) \to 0 \\ \omega \to \infty \Rightarrow \phi(\omega) \to \pi/2. \end{cases}$$

if $\boxed{z_1 < 0}$ $\Rightarrow$ $G_1(\omega) = 20\cdot\log\sqrt{\omega^2+|z_1|^2}$ remains the same but the phase changes:

$H_1(j\omega) = j\omega - |z_1|$



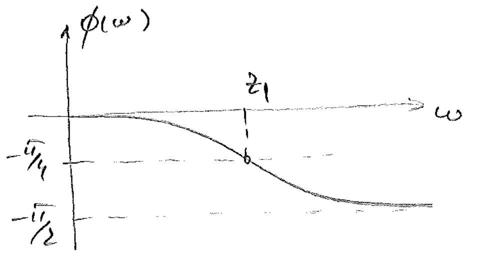


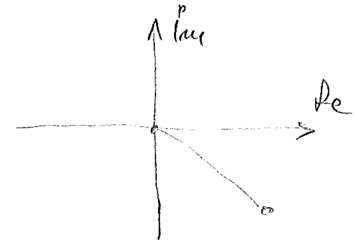
polar diagram or Nyquist diagram

If: $\boxed{H_1(s) = -s + z_1}$ $(z_1 > 0)$ $\Rightarrow$ $\boxed{H_1(j\omega) = -j\omega + z_1}$

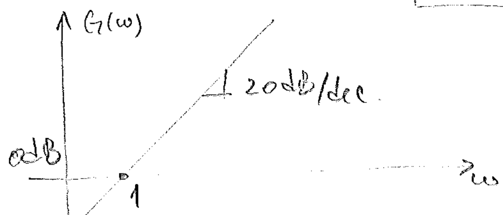
the gain remains still the same!

the phase $\phi(\omega) = \arg\left(H_1(j\omega)\right) = -\operatorname{arctg} \dfrac{\omega}{z_1}$

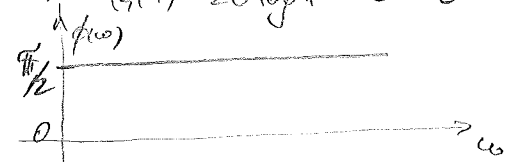




(b) Derivator: $H(s) = s$ $\Rightarrow$ $\boxed{H(j\omega) = j\omega}$ $\Rightarrow$ $\phi(\omega) = \arg(H(j\omega)) = \dfrac{\pi}{2}$



$G(1) = 20 \log 1 = 0$ dB

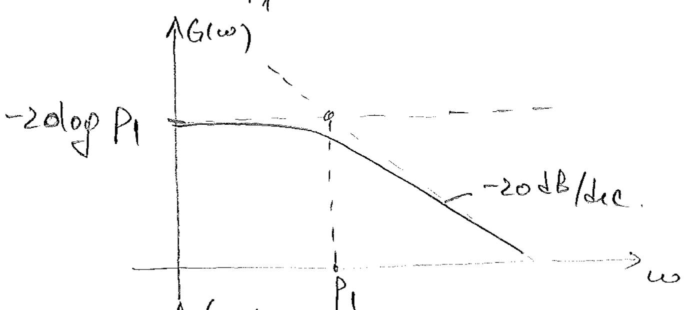


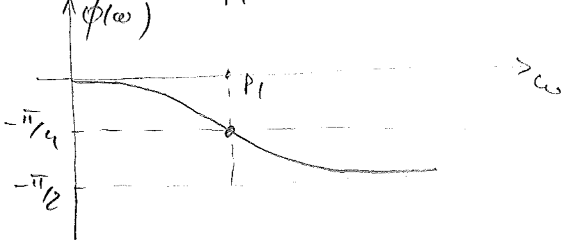
(c) $\boxed{H(s) = \dfrac{1}{s + p_1}}$ $\Rightarrow$ $H(j\omega) = \dfrac{1}{j\omega + p_1}$ $\Rightarrow$ $G(\omega) = -20 \cdot \log\sqrt{\omega^2 + p_1^2} =$

$p_1 > 0.$

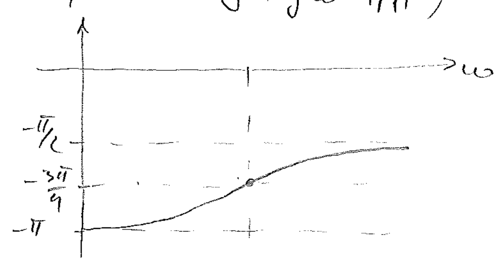
$$= \begin{cases} -20 \log p_1, & \omega \ll p_1 \\ -20 \log \omega, & \omega \gg p_1 \end{cases}$$

$\phi(\omega) = -\operatorname{arctg} \dfrac{\omega}{p_1}$



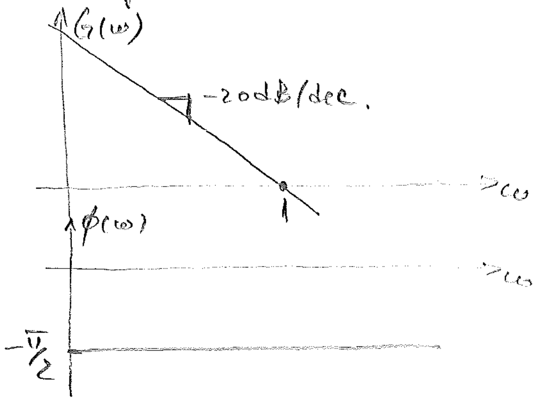


If. $\boxed{p_1 < 0}$ $\Rightarrow$ Gain is same, phase $\phi(\omega) = \arg\left(\dfrac{1}{j\omega - |p_1|}\right) = -\arg(j\omega - |p_1|)$



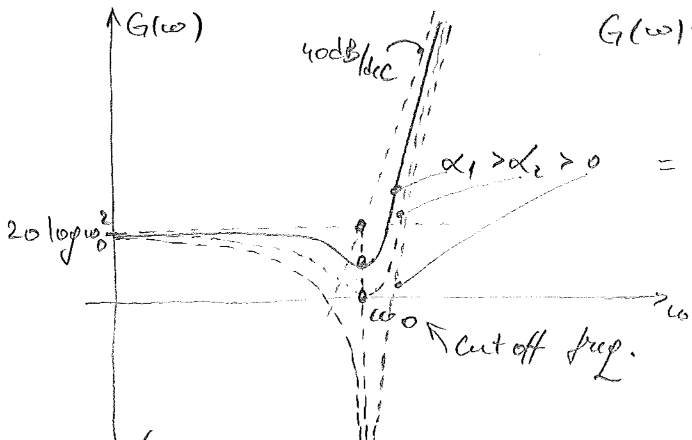
(d) integrator $H(s)=\frac{1}{s}$ $\Rightarrow$ $H(j\omega)=\frac{1}{j\omega}=-j\frac{1}{\omega}$

$$G(\omega)=-20\cdot\log(\omega)$$



(e) $\boxed{\text{Quadratic terms} \equiv \text{Notch filters}}$

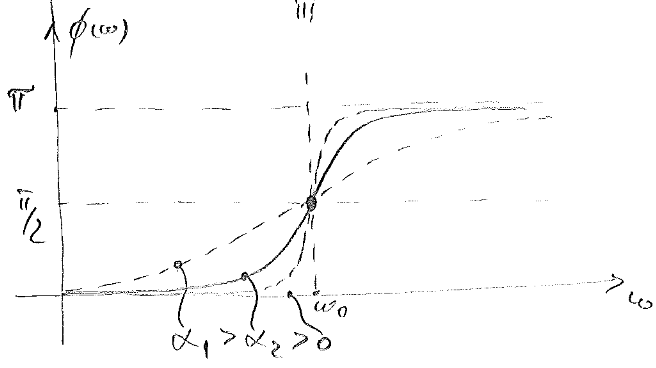
$H(s)=s^2+2\alpha s+\omega_0^2$ $\Rightarrow$ $H(j\omega)=-\omega^2+2j\alpha\omega+\omega_0^2 = (\omega_0^2-\omega^2)+j2\alpha\omega$, $\alpha>0$



$$G(\omega)=|H(j\omega)|_{dB}=20\cdot\log\sqrt{(\omega_0^2-\omega^2)^2+4\alpha^2\omega^2}$$

$$=\begin{cases}20\cdot\log\omega_0^2, & \omega\ll\omega_0^2\\ 20\cdot\log\omega^2, & \omega\gg\omega_0^2.\end{cases}$$

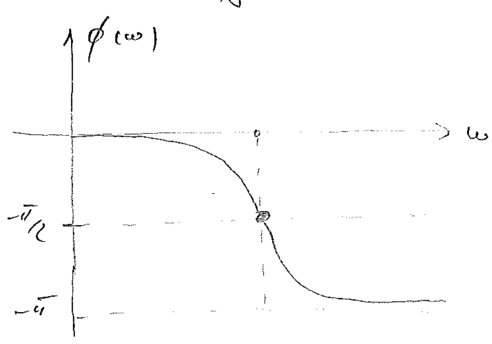
$$G(\omega_0)=20\cdot\log(2\alpha\omega_0)$$



$$\phi(\omega)=\arg(H(j\omega))=$$

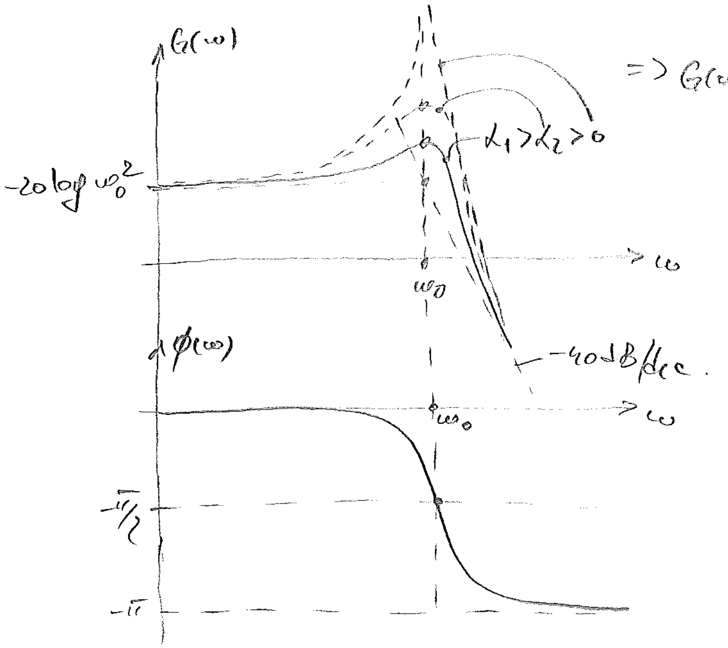
$$=\begin{cases}\operatorname{arctg}\frac{2\alpha\omega}{\omega_0^2-\omega^2}, & \omega\ll\omega_0\\ \pi/2, & \omega=\omega_0\\ \underbrace{\pi-\operatorname{arctg}\frac{2\alpha\omega}{\omega^2-\omega_0^2}}_{\text{written like this for simplicity and with Re, Im picture used}}, & \omega\gg\omega_0\end{cases}$$

If. $\alpha<0$ $\Rightarrow$ $H(s)=s^2-2\alpha s+\omega_0^2$ point remains the same but the phase:



(f) $H(s) = \dfrac{1}{s^2 + 2\alpha s + \omega_0^2}$ $\Rightarrow$ $H(j\omega) = \dfrac{1}{2j\alpha\omega + (\omega_0^2 - \omega^2)}$

$\Rightarrow G(\omega) = -20 \log \sqrt{(\omega_0^2 - \omega^2)^2 + 4\alpha^2\omega^2}$



$|H(0)| = \dfrac{1}{\omega_0^2}$

$|H(\omega_0)| = \dfrac{1}{2\alpha\omega_0}$

$\Rightarrow \dfrac{|H(\omega_0)|}{|H(0)|} = \boxed{\dfrac{\omega_0}{2\alpha} \triangleq Q}$

Quality factor

$\alpha = 0 \Rightarrow$ Oscillator!

Homework: if $\alpha < 0 \Rightarrow$ discuss the gain and phase!

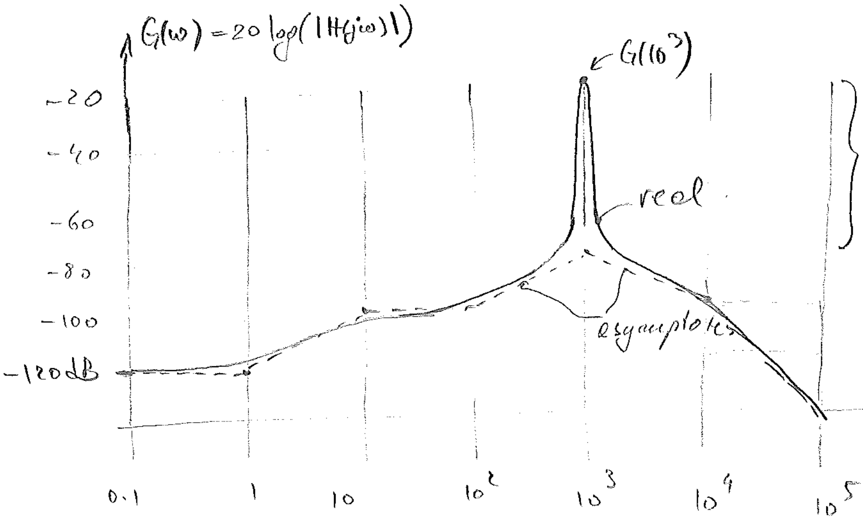
Example of Bode diagrams and "polar diagram" (hodograph)

(a) $H(s) = \dfrac{10^3 \cdot (s+1)(s+10^2)}{(s+10)(s^2+s+10^6)(s+10^4)}$ Bode characteristic will be plotted between 0.1 and 10^5!

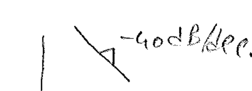
We'll start the Gain plot at:

$G(\omega)\big|_{\omega=0} \cong 20 \cdot \log \dfrac{10^3 \cdot 1 \cdot 10^2}{10 \cdot 10^6 \cdot 10^4} = 20 \cdot \log(10^{-6}) = -120\text{ dB}$.

$\phi(\omega)\big|_{\omega=0} = 0$

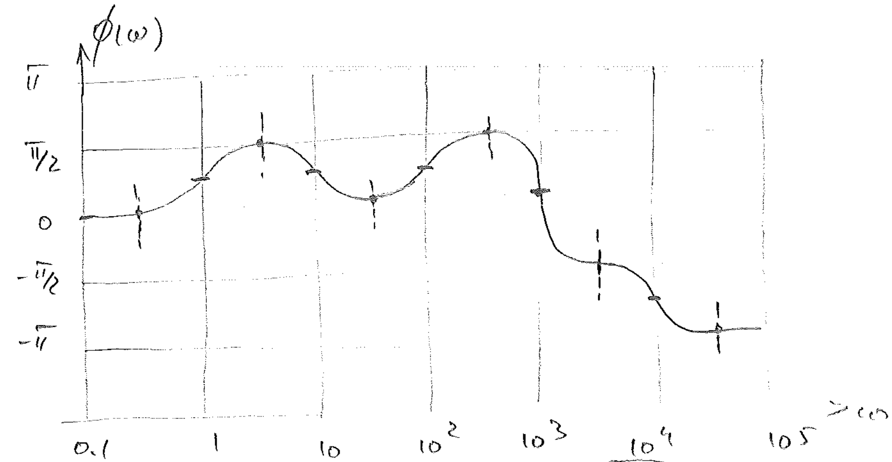


A zero will mean "going up" and a pole will mean "going down" with 20dB/dec. (for simple zeroes and poles!)

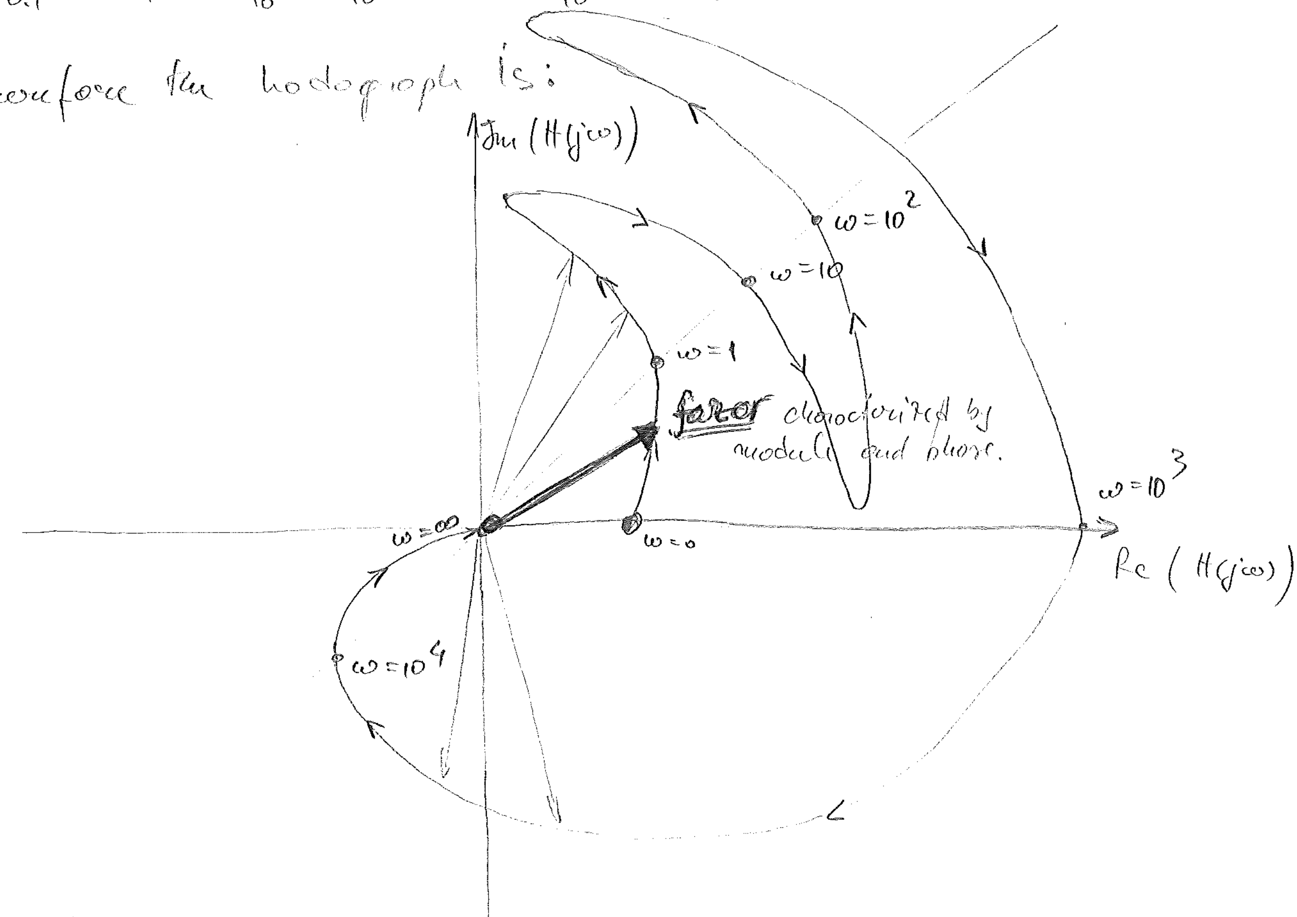


At HF, this transfer function will behave as $\dfrac{1}{s^2}$, $\phi(\omega) = -\pi$

$G(10^3) \cong 20\log \dfrac{10^3 \cdot 10^3 \cdot 10^3}{10^3 \cdot 10^3 \cdot 10^4} = -20\text{dB} \Rightarrow \left| Q = \dfrac{\omega_0}{2\alpha} = 10^3 \right| = 60\text{dB}$



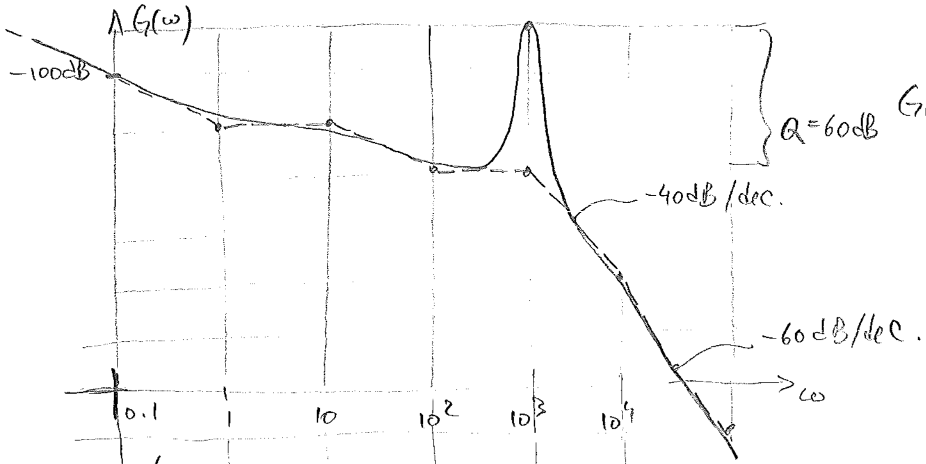
Therefore the hodograph is:



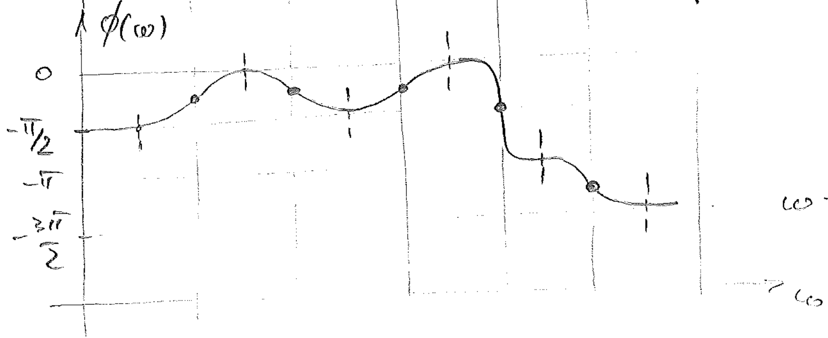
(b) $H(s)= \dfrac{10^3(s+1)(s+10^2)}{s(s+10)(s^2+s+10^6)(s+10^4)}$

- At low frequencies $\omega \to 0$
  $\begin{cases} G(\omega) \simeq \frac{1}{s} \\ \phi(\omega) = -\frac{\pi}{2} \end{cases}$ -20dB/dec. integrator.

- At high frequencies $\omega \to \infty$
  $\begin{cases} G(\omega) \simeq \frac{1}{s^3} \\ \phi(\omega) = -\frac{3\pi}{2}. \end{cases}$ -60dB/dec.

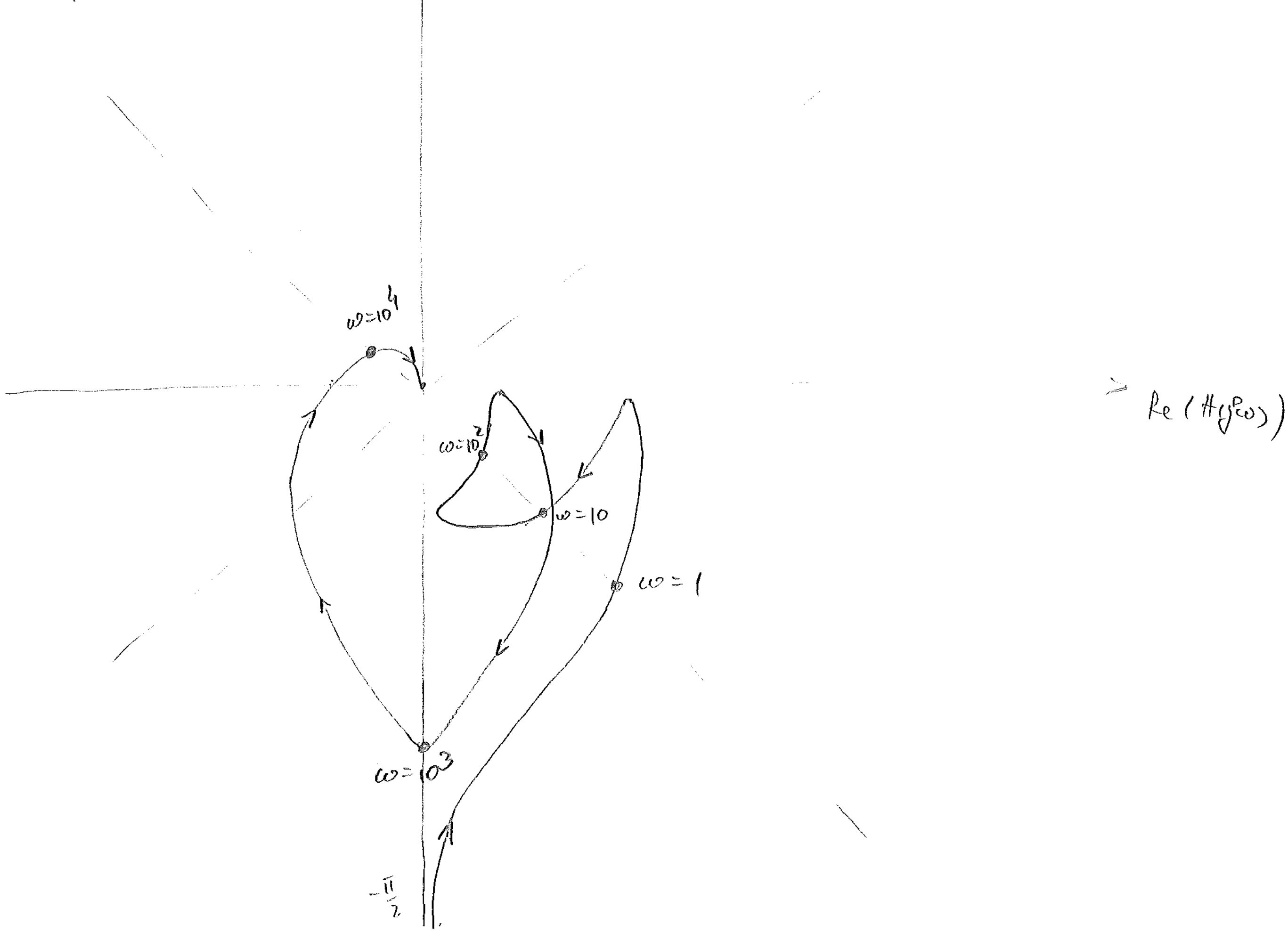


$$G(0.1) \cong 20 \cdot \log \frac{\cancel{10^3} \cdot 1 \cdot 10^2}{10^{-1} \cdot \cancel{10} \cdot 10^6 \cdot \cancel{10^4}} = -100\,\text{dB}.$$



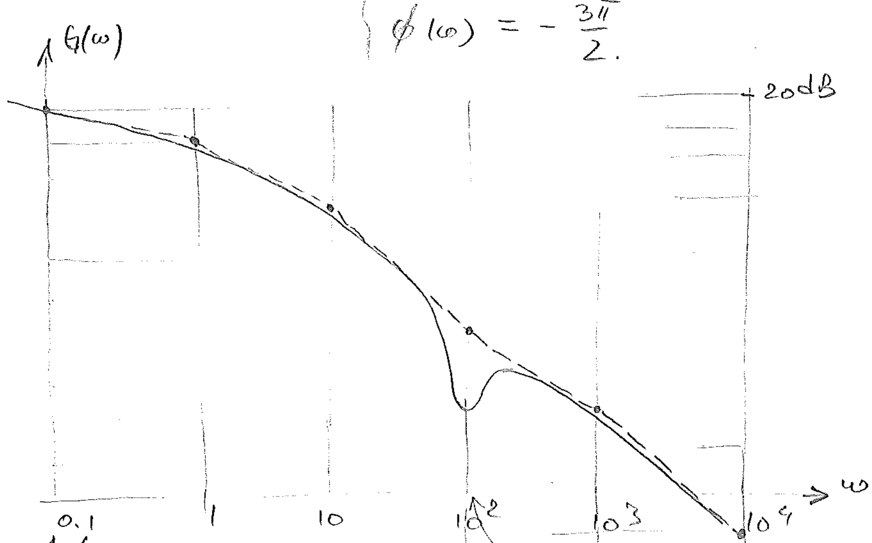
$$\omega \to \infty \Rightarrow \phi(\omega)\Big|_{\omega \to \infty} = \frac{\pi}{2} + \frac{\pi}{2} - \frac{\pi}{2} - \pi - \frac{\pi}{2} - \frac{\pi}{2} = -\frac{3\pi}{2}$$

Hodograph:
of $H(j\omega)$



(c) $H(s) = \dfrac{-100\cdot(s^2 - s + 10^4)}{s(s+1)(s-10)(s+10^2)(-s+10^3)}$

- At $\omega \to 0$: $G(\omega) \simeq +\frac{1}{s}$; $\phi(\omega) = -\frac{\pi}{2}$ because we have $\frac{-}{-s} = \frac{1}{s}$. These "minuses" matter! This is an integrator!
- At $\omega \to \infty$: $G(\omega) \simeq \frac{1}{s^3}$; $\phi(\omega) = -\frac{3\pi}{2}$.

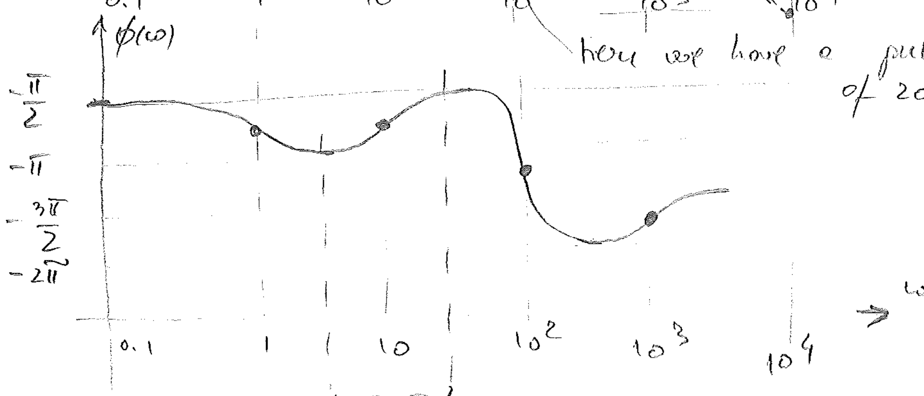


$G(0.1) = 20\cdot\log\frac{(-100)\cdot 10^4}{10^{-1}\cdot 1\cdot(-10)\cdot 10^2\cdot 10^3} = 20dB$

$G(10^2) \cong 20\cdot\log\frac{(-100)\cdot\cancel{10^4}}{\cancel{10^2\cdot 10^2}\cdot 10^2\cdot 10\cdot 10^3} =$

$= 100dB$

here we have a pull-up of 40dB and a pull-down of 20dB => pull-up of 20dB.



phase goes up due to (s-10) which has "-10"!