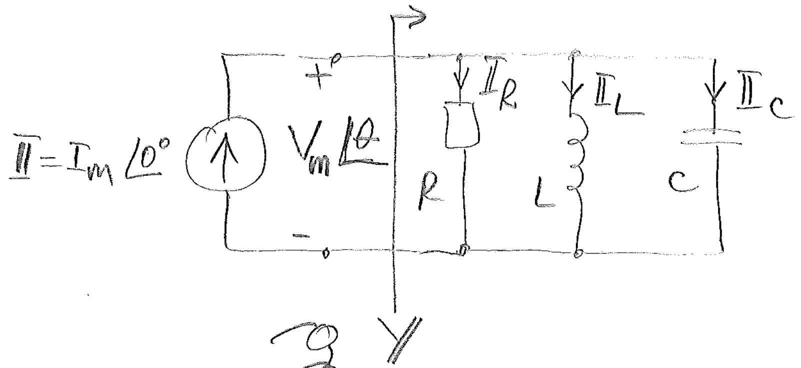
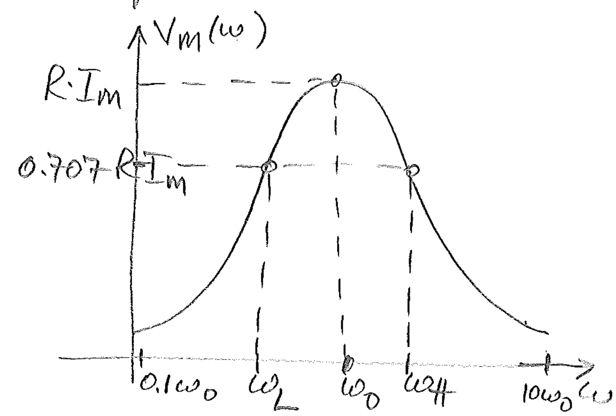
2 Parallel resonance

- capacitance & inductance connected in parallel.





$$\boxed{\mathbb{Y} = G + j\left(\omega C - \frac{1}{\omega L}\right)} \quad (1)$$

Doing a similar analysis as for the series resonance we can derive:

$$\boxed{\mathbb{Y} = G\left[1 + jQ\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right]} \quad (2) \quad \text{where} \quad \boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$$ is

the "resonance frequency"

$$\boxed{Q = R\sqrt{\frac{C}{L}}} \quad (3)$$ is the "quality factor" of the parallel RLC circuit!

$$\boxed{\mathbb{V} = Z \cdot \mathbb{I} = \frac{1}{\mathbb{Y}} \cdot \mathbb{I} = \frac{I_m}{G\left[1 + jQ\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right]}} \quad (4) \Rightarrow$$

$$\Rightarrow \begin{cases} V_m(\omega) = R I_m \cdot \dfrac{1}{\sqrt{1 + Q^2\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)^2}} & (5) \\ \measuredangle \mathbb{V} = \theta(\omega) = -\tan^{-1}\left[Q\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right] & (6) \end{cases}$$

← Has peak value: $V_{m(max)} = R \cdot I_m$

(1) $\omega < \omega_0 \Rightarrow L$ dominates $\Rightarrow$ parallel RLC is inductive

(2) $\omega > \omega_0 \Rightarrow C$ dominates $\Rightarrow$ -"- capacitive.

(3) $\boxed{\omega = \omega_0} \Rightarrow \omega C = \frac{1}{\omega L} \Rightarrow Y$ is purely conductive : $Y = G + j0$

Resonance

$\Rightarrow$ L & C combination effectively behave as an open circuit!

- V_m is maximized at resonance, and average power dissipated by R:

$$P_{max} = \frac{1}{2} R I_m^2 = R I_{rms}^2$$

- Frequencies at which P is down to half its maximum value are the "half power frequencies" ω_L, ω_H, and the "half power bandwidth" $BW = \omega_H - \omega_L$ is such that:

$$\boxed{Q = \frac{\omega_0}{BW}} \quad (7)$$

$$(7) + (3) \Rightarrow \boxed{BW = \frac{1}{RC}} \quad (8)$$

NOTE: In a parallel RLC circuit ω_0 is set by L, C and bandwidth BW is set by R, C!

- Following the same line as in series RLC we can derive that in a parallel RLC circuit, we have:

$$\begin{cases} \mathbb{I}_R = \frac{V_m}{R} \angle\theta \\ \mathbb{I}_C = \omega C V_m \angle\theta + 90^\circ \\ \mathbb{I}_L = \frac{1}{\omega L} V_m \angle\theta - 90^\circ \end{cases}$$

and at resonance:

$$\begin{cases} \mathbb{V} = R \cdot I_m \angle 0^\circ \\ \mathbb{I}_R = I_m \angle 0^\circ \\ \mathbb{I}_C = Q I_m \angle +90^\circ \\ \mathbb{I}_L = Q I_m \angle -90^\circ \end{cases}$$

Note: If $Q > 1$ $\Rightarrow$ current peak amplitudes $> I_m$
this is called "resonant current rise" !

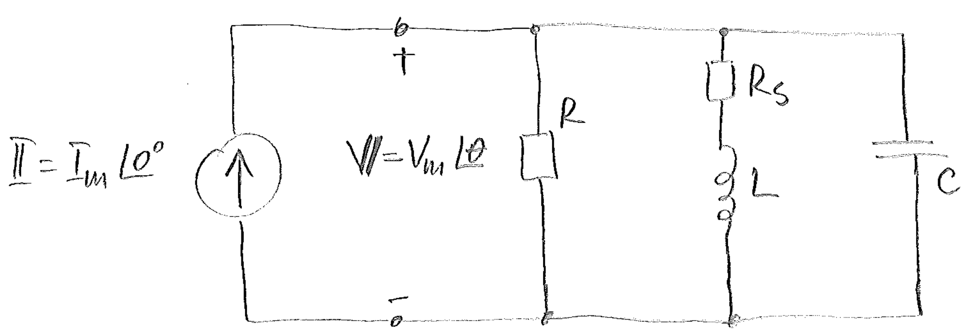
- "Maximum stored energy" at resonance: $w_L(t) + w_C(t)$
- "Power dissipated per cycle" at resonance: $w_{R(cycle)}$

$$\left.\begin{aligned} w_L(t) + w_C(t) &= \frac{Q}{\omega_0} R \cdot I_{rms}^2 \\ w_{R(cycle)} &= \frac{2\pi}{\omega_0} R\, I_{rms}^2 \end{aligned}\right| \Rightarrow$$

$$\Rightarrow \boxed{Q = 2\pi \cdot \frac{\text{maximum stored energy}}{\text{energy dissipated per cycle}}} \quad (9)$$

3 Practical Resonant Circuits

- A class of circuits called $\boxed{\text{tuned circuits}}$ such as tuned oscillators and tuned amplifiers in radio communications!
- L is the least ideal $\Rightarrow$ modeled in series with a "stray" resistance.



A practical tank circuit.

$$Y = \frac{1}{R} + j\omega C + \frac{1}{R_s + j\omega L}$$

$$Y = \frac{1}{R} + \frac{R_s}{R_s^2 + (\omega L)^2} + j\omega\left(C - \frac{L}{R_s^2 + (\omega L)^2}\right)$$

- We are interested in $\omega_{res} \neq 0$, which makes the circuit resonant, (makes Y purely real): it is the frequency that makes:

$$C - \frac{L}{R_s^2 + (\omega_{res} L)^2} = 0$$

$$\Rightarrow \boxed{\omega_{res} = \sqrt{\frac{1}{LC} - \frac{R_s^2}{L^2}}} \quad (10)$$

- In the limit $R_s \to 0 \Rightarrow \omega_{res} \to \frac{1}{\sqrt{LC}}$

- But, the effect of $R_s \neq 0$ is that $\omega_{res} < \frac{1}{\sqrt{LC}}$ a <u>downshift</u> in the resonance frequency!

- An appropriate figure of merit for a practical inductor is the "<u>quality factor of the inductor</u>" defined:

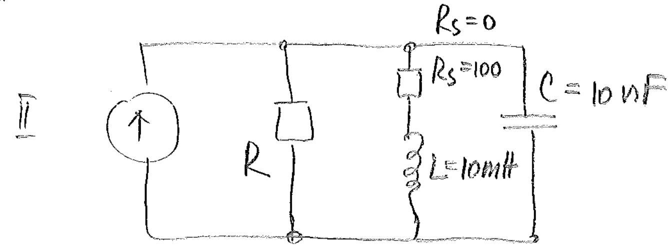
$$\boxed{Q_{coil} \triangleq \frac{|Z_L(\omega_{res})|}{R_s} = \frac{\omega_{res} L}{R_s}} \quad (11)$$

- when $\omega_{res} \to \frac{1}{\sqrt{LC}}$ we can approximate $R_s = \omega_{res} L / Q_{coil} \cong \frac{1}{\sqrt{LC}} \cdot \frac{L}{Q_{coil}}$

So:

$$\boxed{\omega_{res} = \frac{1}{\sqrt{LC}} \sqrt{1 - \frac{1}{Q_{coil}^2}}} \quad (12)$$

Example 1:



Case 1: $R_s=0$
Case 2: $R_s=100\,\Omega$

Find $\omega_{res}=?$
$Q_{coil}=?$

Case 1: $\boxed{R_s=0}$

Use eq. (11): $Q_{coil}=\dfrac{\omega_{res}L}{R_s}=\infty$

Use eq. (12): $\omega_{res}=\dfrac{1}{\sqrt{LC}}=\dfrac{1}{\sqrt{10^{-2}\times10^{-8}}}=10^5\ \dfrac{rad}{s}$

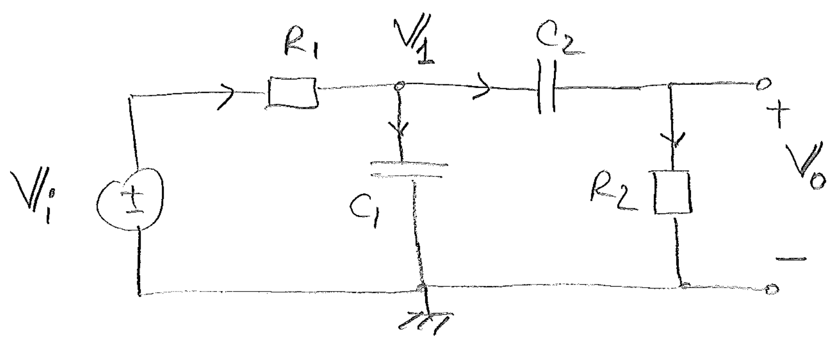
Case 2: $\boxed{R_s=100\,\Omega}$

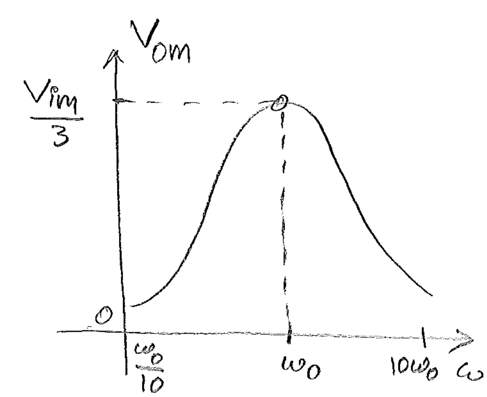
$$Q_{coil}=\frac{\omega_{res}L}{R_s}\cong\frac{10^5\times10^{-2}}{100}=10$$

$$\omega_{res}=\frac{1}{\sqrt{LC}}\times\sqrt{1-\frac{1}{Q_{coil}^2}}=10^5\sqrt{1-\frac{1}{10^2}}=99{,}499\ \frac{rad}{s}$$ indicating

a 0.5% frequency downshift!

Example 2: $\boxed{\text{A passive inductorless Band-Pass circuit}}$





(1) $$\frac{V_i-V_1}{R_1}=\frac{V_1}{\frac{1}{j\omega C_1}}+\frac{V_1-V_o}{\frac{1}{j\omega C_2}}$$

(2) $$\frac{V_1 - V_0}{\frac{1}{j\omega C_2}} = \frac{V_0}{R_2}$$

(1)+(2) $\Rightarrow$ $$\boxed{V_0 = \frac{j\omega R_2 C_2}{1 + j\omega(R_1C_1 + R_2C_2 + R_1C_2) - \omega^2 R_1R_2C_1C_2} \cdot V_i} \quad (3)$$

can be manipulated into <u>standard</u> band-pass form:

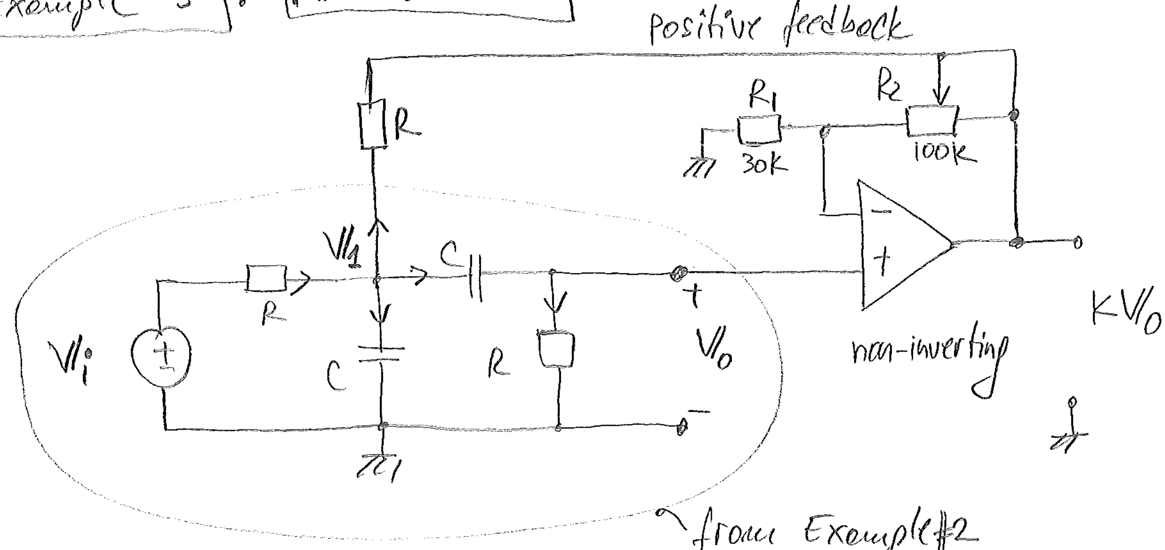
(4) $$\boxed{V_0 = \frac{A}{1 + jQ\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)} \cdot V_i}$$

where: $$\begin{cases} A = \dfrac{1}{1 + \frac{R_1}{R_2}\left(1 + \frac{C_1}{C_2}\right)} \\[2ex] \omega_0 = \dfrac{1}{\sqrt{R_1R_2C_1C_2}} \\[2ex] Q = \dfrac{\sqrt{R_1R_2C_1C_2}}{R_1C_1 + R_2C_2 + R_1C_2} \end{cases}$$

<u>Note</u>:

- Because $A \leq 1$ $\Rightarrow$ no matter values of $R_1 R_2 C_1 C_2$ we always have: $\boxed{V_{om\,(max)} \leq V_{im}}$
- Our circuit is incapable of resonance signal rise!
- Let $\begin{cases} R_1 = R_2 = R \\ C_1 = C_2 = C \end{cases}$ $\Rightarrow$ $\begin{cases} A = \frac{1}{3} \\ \omega_0 = \frac{1}{RC} \\ Q = \frac{1}{3} \end{cases}$

Example 3 : KRC circuit



- V_o is amplified by $K = 1 + \frac{R_2}{R_1}$

  and and then re-injected via an additional R into circuit itself!

- Due to the positive feedback action by OpAmp, it is possible to bolster both A & Q.

$$\left.\begin{cases} (1) & \dfrac{V_i - V_1}{R} - \dfrac{V_1}{\frac{1}{j\omega C}} + \dfrac{V_1 - V_0}{\frac{1}{j\omega C}} + \dfrac{V_1 - KV_o}{R} \\ (2) & \dfrac{V_1 - V_o}{\frac{1}{j\omega C}} = \dfrac{V_o}{R} \end{cases}\right| \Rightarrow$$

$$\Rightarrow \boxed{V_o = \frac{j\omega RC}{2 - (\omega RC)^2 + (4-K)j\omega RC} \cdot V_i} \quad (3)$$

which when put into the band-pass form (eq.4 in Example 2) we get:

$$\boxed{A = \frac{1}{4-K}} \quad \boxed{\omega_0 = \frac{\sqrt{2}}{RC}} \quad \boxed{Q = \frac{\sqrt{2}}{4-K}}$$

still set by RC

controlled by K : vary to achieve wide range of A, K

In the limit $K \to 4 \Rightarrow A = \infty,\ Q = \infty \Rightarrow$ $\boxed{\text{oscillation}}$!!!