Example 1'

Find the inverse Laplace transform of:

$$F(s) = \frac{8(s+2)}{(s+1)^3(s+3)}$$

Poles: $p_1 = -1$ Np/s multiplicity $r=3$

$p_2 = -3$ Np/s simple

Thus: $$F(s) = \frac{A_{1,3}}{(s+1)^3} + \frac{A_{1,2}}{(s+1)^2} + \frac{A_{1,1}}{s+1} + \frac{B_1}{s+3}$$

$$A_{1,3} = (s+1)^3 \cdot F(s)\Big|_{s=-1} = \frac{8(s+2)}{s+3}\Big|_{s=-1} = 4$$

$$A_{1,2} = \frac{d}{ds}\left[(s+1)^3 F(s)\right]\Big|_{s=-1} = \frac{d}{ds}\left(\frac{8(s+2)}{s+3}\right)\Big|_{s=-1} = \frac{8}{(s+3)^2}\Big|_{s=-1} = 2$$

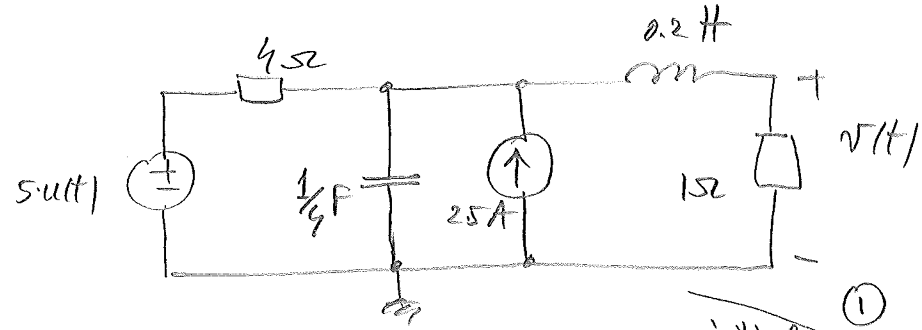
$$A_{1,1} = \frac{1}{2!}\frac{d^2}{ds^2}\left[(s+1)^3 F(s)\right]\Big|_{s=-1} = \frac{1}{2}\frac{d}{ds}\left(\frac{8}{(s+3)^2}\right)\Big|_{s=-1} = \frac{1}{2}\cdot\frac{-16}{(s+3)^3}\Big|_{s=-1} = -1$$

$$B_1 = (s+3)F(s)\Big|_{s=-3} = \frac{8(s+2)}{(s+1)^3}\Big|_{s=-3} = 1$$

$$F(s) = \frac{4}{(s+1)^3} + \frac{2}{(s+1)^2} - \frac{1}{s+1} + \frac{1}{s+3}$$

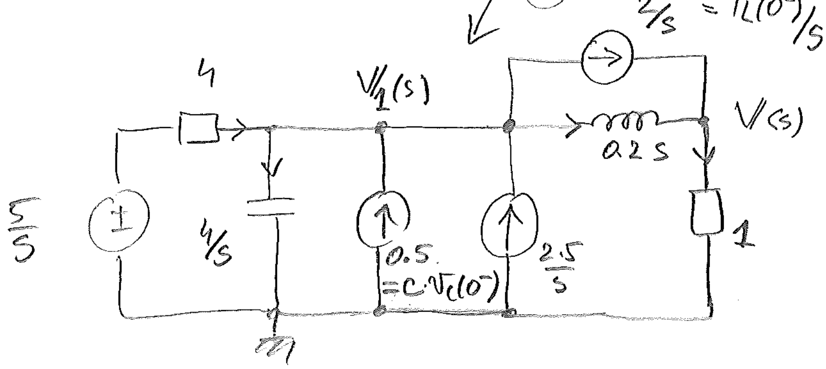
$$\boxed{f(t) = \mathcal{L}^{-1}\{F(s)\} = \left[(2t^2 + 2t - 1)e^{-t} + e^{-3t}\right]\cdot u(t)}$$

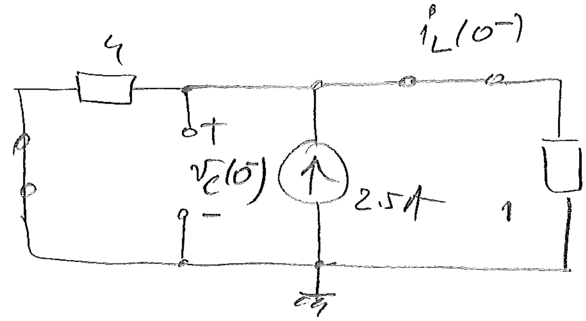
$\boxed{\text{Example 2}}$ Find $v(t)$



① initial conditions

s-domain circuit ②





$$v_C(0^-) = (4 \parallel 1) \times 2.5 = 2\,V$$

$$i_L(0^-) = \frac{4}{4+1} \times 2.5 = 2\,A$$

③ Solve circuit in s-domain:

Apply KCL at nodes V_1 and V:

$$\begin{cases} \dfrac{5/s - V_1(s)}{4} + 0.5 + \dfrac{2.5}{s} = \dfrac{V_1(s)}{4/s} + \dfrac{2}{s} + \dfrac{V_1(s) - V(s)}{0.2\,s} \\[2ex] \dfrac{2}{s} + \dfrac{V_1(s) - V(s)}{0.2\,s} = \dfrac{V(s)}{1} \end{cases}$$

2 eq.
2 unknowns

$$\Rightarrow V(s) = \frac{2s^2 + 12s + 75}{s(s^2 + 6s + 25)} = \frac{3}{s} + \frac{0.625\angle 143.13^\circ}{s + 3 - j4} + \frac{0.625\angle -143.13^\circ}{s + 3 + j4}$$

Taking the inverse Laplace transform:

$$v(t>0) = 3 + 1.25 \cdot e^{-3t} \cdot \cos(4t + 143.13^\circ)\ [V]$$