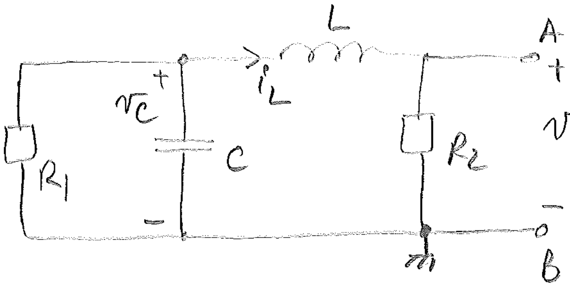
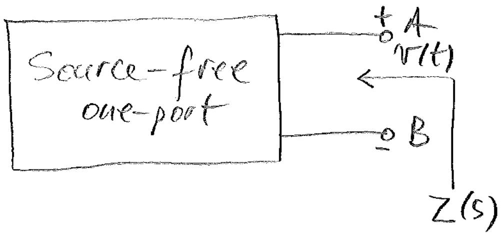
Example 1

Given source free circuit with initial conditions: $v_C(0^-) = 10\ V$ and $i_L(0^-) = 0$, find $v(t \geq 0) = ?$.

$R_1 = 2k$
$C = 5\ nF$
$L = 1\ mH$
$R_2 = 0.5\ k$



Solution



The poles of $Z(s)$ determine the functional form of the open-circuit voltage $v(t)$!

That is because $Z(s) = \frac{V(s)}{I(s)}$ and thus we "look" at $Z(s)$ same way as we looked at $H(s)$ in previous two lectures:

$$V(s) = Z(s) \cdot I(s)$$

when zero at the limit, $Z(s)$ is infinity, which corresponds to the poles of it!

- In our case:

$$Z(s) = R_2 \parallel \left[sL + \left(R_1 \parallel \frac{1}{sC}\right)\right]$$

$$Z(s) = \frac{R_2(R_1 LC s^2 + Ls + R_1)}{R_1 LC s^2 + (R_1 R_2 C + L)s + (R_1 + R_2)}$$

- Let's try the asymptotic checks:

$Z(0) = R_1 \parallel R_2$ confirmed!

$Z(\infty) = R_2$ confirmed!

$$Z(s) = 500 \cdot \frac{s^2 + 10^5 s + 20\times 10^{10}}{s^2 + 6\times 10^5 s + 25\times 10^{10}} \; [\Omega]$$

- Conjugate pole-pair: $P_{1,2} = (-3 \pm j4)\cdot 10^5$ complex Np/s

- From previous lecture, we know:

$$\boxed{v(t) = 2\cdot A \times e^{-3\times 10^5 t} \times \cos(4\times 10^5 t + \theta)} \quad (1)$$

- we now wish to find $\boxed{A}$ and $\boxed{\theta}$ on the basis of initial conditions.

- On one hand observe:

$$\boxed{v = R_2 \cdot i_L}$$

1st initial condition

$$v(0) = R_2 \cdot i_L(0) = R_2 \cdot 0 = 0$$

- On other hand, equation (1):

$$v(0) = 2\cdot A \cdot e^0 \cdot \cos(0+\theta) = 2A\cos\theta$$

$$\Rightarrow \boxed{2A\cos\theta = 0} \quad (2)$$

- Because we seek a solution with $A \neq 0$, we must have:

$$\cos\theta = 0 \quad \Rightarrow \quad \theta = 90^\circ$$

- Hence: $$\boxed{v(t) = 2A\, e^{-3\times 10^5 t} \cdot \sin(4\times 10^5 t)} \quad (3)$$

- For inductance:

$$v_C - v = L\frac{di_L}{dt}$$

$$\frac{v_C - v}{L} = \frac{d}{dt}\left(\frac{v}{R_2}\right) \quad \Rightarrow \quad \boxed{\frac{R_2}{L}(v_C - v) = \frac{dv}{dt}} \quad (4)$$

So, second initial condition:

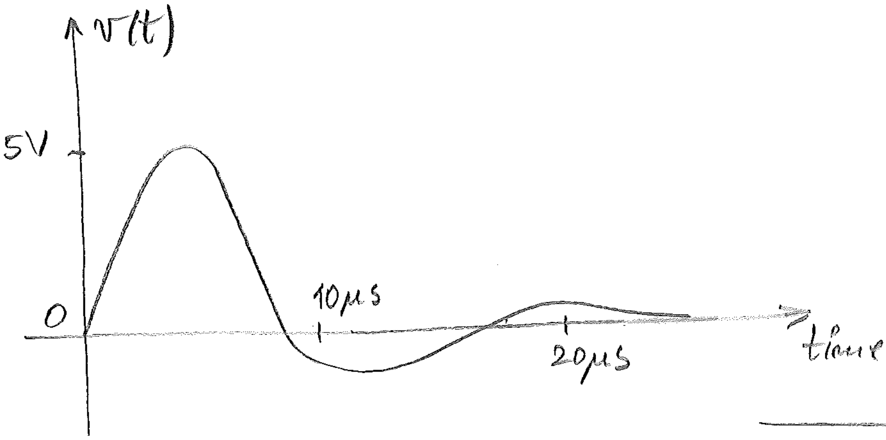
$$\frac{dv(0)}{dt} = \frac{R_2}{L}\left[v_C(0) - v(0)\right] = \frac{500}{10^{-3}}(10-0) = 5\times 10^6$$

$\Rightarrow$

But using equation (3): $$\frac{dv(0)}{dt} = 2A\left(-3\times 10^5 \cdot e^0 \cdot \sin 0 + 4\times 10^5 e^0 \cdot \cos 0\right) = 8\times 10^5 \times A$$

$\Rightarrow 8\times10^5 \times A = 5\times10^6 \Rightarrow \boxed{A = 6.25}\,[V]$ (5)

So, finally: $\boxed{v(t) = 12.5 \times e^{-3\times10^5 t} \times \sin(4\times10^5 t)}\,[V]$



IMPORTANT DISCUSSION

- This example is done without using or having any knowledge abut Laplace transforms. Because of that we had to do extra work to find constants $\boxed{A}$ and $\boxed{\theta}$! This extra work may become tedious and time consuming in general.
- That is why Laplace transform-based analysis should be preferred because Laplace transform takes initial conditions into account automatically!
- For more info on the subtle differences between these two approaches, see the "STORY OF $\boxed{S}$" lecture notes at the end of the semester!