Chapter 11

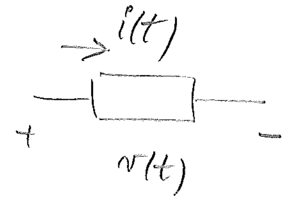
1 AC power

We'll apply phasor techniques to the study of $\boxed{\text{ac power}}$.

Definitions

ⓐ Instantaneous power - delivered to a device is given by the product of the instantaneous voltage across the device and the instantaneous current through it.

$$\boxed{p(t) \triangleq v(t) \cdot i(t)} \quad (1)$$



ⓑ average power

- Obtained for a time interval t_1 to t_2 by integrating the instantaneous power $p(t)$ and by dividing by the length of the interval:

$$\boxed{P \triangleq \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} p(t)\, dt} \quad (2)$$

- In the particular case that $p(t)$ is periodic with period T, the average power can be computed as:

$$\boxed{P = \frac{1}{T} \int_{t_x}^{t_x + T} p(t)\, dt} \quad (2')$$ where t_x is arbitrary.

Sinusoidal steady state power ("ac power")

(forcing function is a sinusoidal)

$v(t) = V_m \cos(\omega t + \theta)$

$i(t) = I_m \cos(\omega t + \phi)$

$p(t) = v(t) \cdot i(t) = V_m \cdot I_m \cdot \cos(\omega t + \theta) \cdot \cos(\omega t + \phi)$

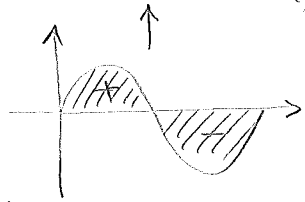
Use trig. identity $\cos a \cdot \cos b = \frac{1}{2}[\cos(a+b) + \cos(a-b)]$ to get:

$$\boxed{p(t) = \frac{1}{2} V_m I_m \cos(\theta - \phi) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta + \phi)} \quad (3)$$

time independent! time dependent!

Average power:

$$P = \frac{1}{T}\int_{t_x}^{t_x+T} \frac{1}{2} V_m I_m \cos(\theta-\phi)\,dt + 0 = \frac{1}{T}(\cancel{t_x} + T - \cancel{t_x})\,\frac{1}{2} V_m I_m \cos(\theta - \phi)$$

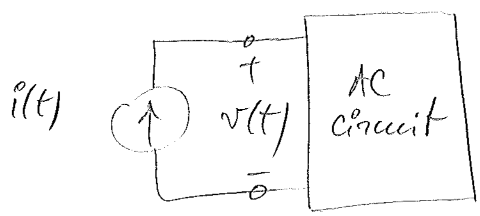


$$(4) \quad \boxed{P = \frac{1}{2} V_m I_m \cos(\theta - \phi)} = \left(\frac{1}{\sqrt{2}} V_m\right)\cdot\left(\frac{1}{\sqrt{2}} I_m\right)\cdot\cos(\theta-\phi) = \boxed{V_{rms}\cdot I_{rms}\cdot\cos(\theta-\phi)}$$

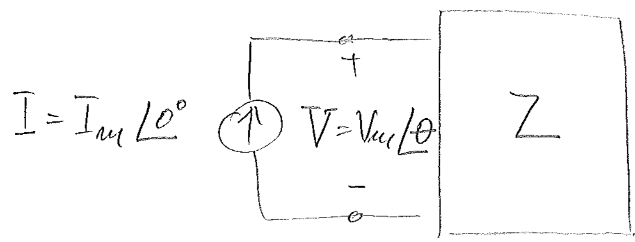
[W] watts

rms ≡ root-mean-square.

Example general



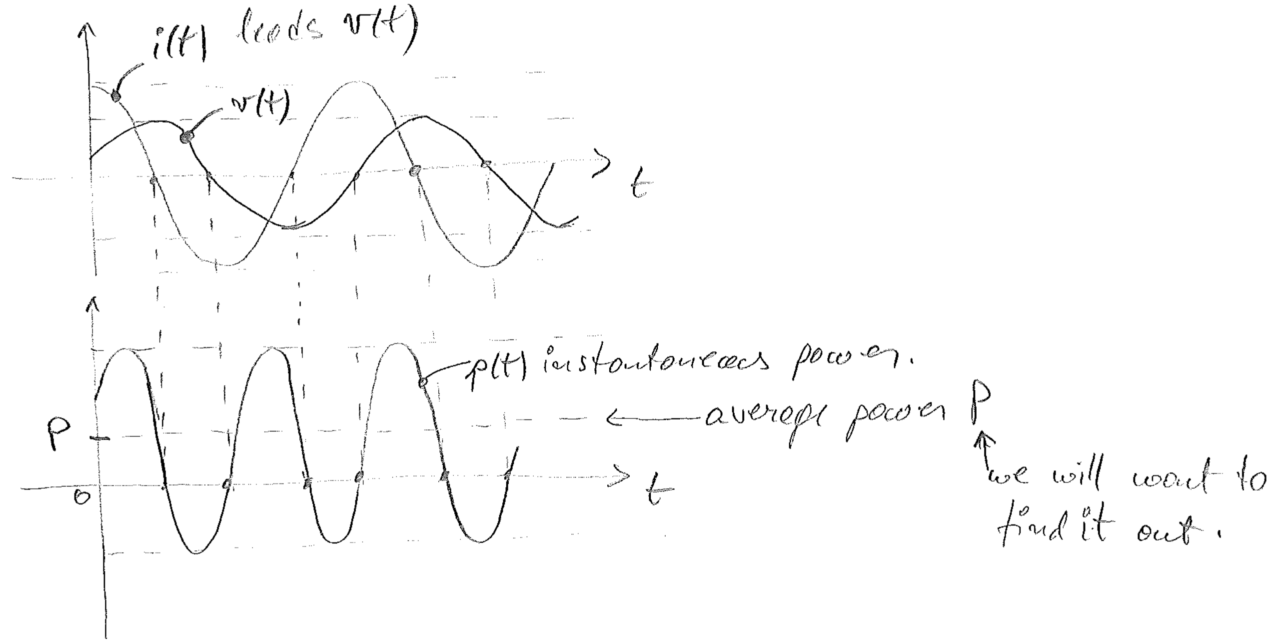
time domain. has resistive and reactive elements.



frequency-domain representation

$i(t) = I_m \cos\omega t$; $v(t) = V_m \cos(\omega t + \theta)$

$\phi = 0°$



We identify three important cases:

Case 1: $\boxed{\theta - \phi = 0^\circ}$ $\equiv$ v and i are in phase with each other as in a purely resistive load!

(3) $\Rightarrow$ $\boxed{p(t) = \frac{1}{2} V_m I_m \left[1 + \cos 2\omega t\right]}$ (5)

Note: it alternates between 0 and $V_m I_m$ and so always positive!

In this case 1, expression (4) reaches its maximum denoted $\boxed{S = \frac{1}{2} V_m I_m = V_{rms} \cdot I_{rms}}$ (6) and called the "apparent power".

Even though S has the same dimension as P, S is expressed in VA (volt amperes) to distinguish it from P.

Case 2: $\boxed{\theta - \phi = \pm 90^\circ}$: condition arises when circuit is purely reactive.

Because $\cos(\pm 90^\circ) = 0$:

(3) $\Rightarrow$ $\boxed{p(t) = \frac{1}{2} V_m I_m \cdot \cos(2\omega t \pm 90^\circ)}$ (7)

whose average is zero! which confirms that purely reactive loads dissipate <u>no</u> power: the energy absorbed during a positive alternation of $p(t)$ is returned to the source during the subsequent negative alternation!

Case 3: circuit is partly resistive, partly reactive

$\Rightarrow$ $0 \leq \cos(\theta - \phi) \leq 1$

$\Rightarrow$ $\boxed{0 \leq P \leq S}$ (8)

Definitions: The term $\cos(\theta - \phi)$ is called "power factor" (pf)

$$\boxed{pf = \cos(\theta - \phi)} \quad (9)$$

called "pf angle"

OBS: - Because $\cos(\theta-\phi) = \cos[-(\theta-\phi)]$ the power factor is the same whether current lags or leads the voltage!

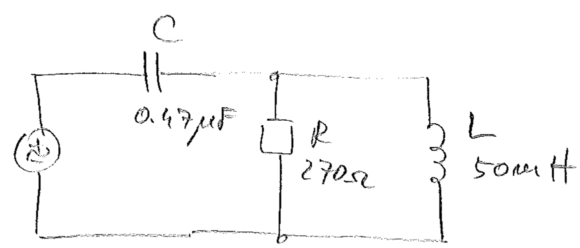
- Power factor pf is given by:

$$\boxed{pf = \frac{\text{average power}}{\text{apparent power}}} = \frac{\frac{1}{2} V_m I_m \cos(\theta-\phi)}{\frac{1}{2} V_m I_m} = \cos(\theta - \phi)$$

Example:

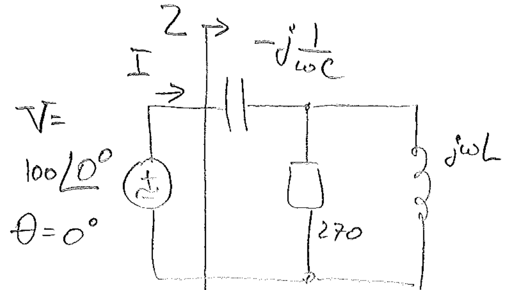
Find apparent power as well as avg. power delivered by the source $v(t)$:

$v(t) = 100 \cos(2\pi 10^3 t)$



time-domain

$\Rightarrow$



frequency-domain

where $\omega = 2\pi 10^3$

We need to find V_m, I_m, and $\theta - \phi$

$V_m = 100\,V$, $\theta = 0°$

$I = \frac{V}{Z}$ where $Z = Z_C + Z_R \| Z_L$

$$= -j\frac{1}{\omega C} + \frac{j\omega L R}{R + j\omega L} = \frac{-j1}{2\pi 10^3 \times 0.47 \times 10^{-6}} + \frac{j 2\pi 10^3 \times 0.05 \times 270}{270 + j2\pi 10^3 \times 0.05}$$

$$= 155.5 - j205.2 = 257.3 \angle -52.88° \ \Omega \quad \text{(capacitive load)}$$

Then: $I = \frac{V}{Z} = \frac{100\angle 0°}{257.3\angle -52.88°} = 0.3866 \angle 52.88° \ A$

Apparent power: $S = \frac{1}{2} \times 100 \times 0.3866 = 19.43 \ VA$

Power factor: $pf = \cos(0° - 52.88°) = \cos(52.88°) = 0.6035$

Average power: $P = 19.43 \times 0.6035 = 11.72 \ W$