

ECE-311 (ECE, NDSU)  
Lab 8 – Simulation and experiment  
**Transfer functions**

**1. Objective**

To study transfer functions.

**2. Introduction**

The transfer function of a circuit is defined in the s domain as the ratio of the output (response)  $Y(s)$  of the circuit to an excitation or forcing function  $X(s)$ . All the initial conditions in the circuit are set to zero while computing the transfer function. The transfer function, denoted by  $H(s)$ , is then expressed as:

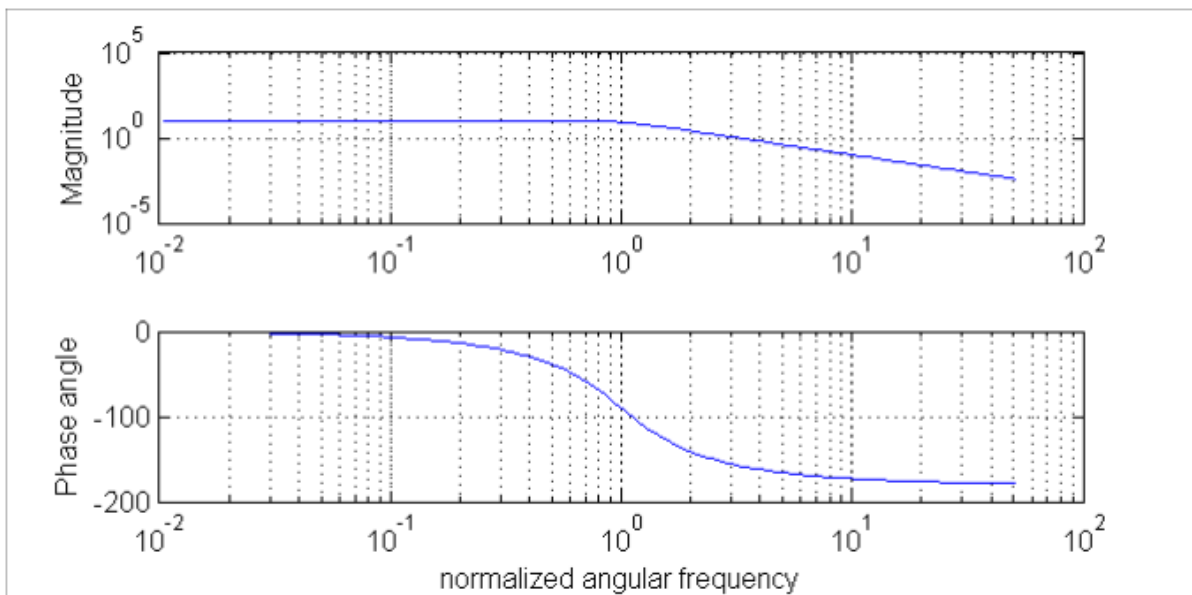
$$H(s) = Y(s) / X(s)$$

The circuit output (response) can be written as:

$$Y(s) = H(s)X(s)$$

which says that the Laplace transform of the output is equal to the product of the transfer function and Laplace transform of the input function.

The transfer function is a complex quantity with a magnitude and phase that are functions of frequency. A plot of the magnitude and the phase of a transfer function are shown in Figure 1.



**Figure 1: Frequency response plot**

Keep in mind that the transfer function applies to a single source. If more than one source exists in the circuit, a transfer function for each source needs to be determined first and the total response can be found using superposition. Also, it is important to note that a single circuit can have many transfer functions depending upon the output (response) of interest.

### 3. Preparation

For all circuits,  $R = 1 \text{ k}\Omega$ ,  $C = 0.1 \text{ }\mu\text{F}$ .

#### (1) Single-stage circuits

For circuits in Figure 2,

- Derive the transfer function.
- Find the frequency where the output voltage is  $45^\circ$  out of phase with the input voltage. Find the amplitude of the output voltage at this frequency.
- Find the frequency where the amplitude of the output voltage is  $1/\sqrt{2}$  that of the input voltage. Find the phase difference between the input and output voltages.
- Are the frequencies found in steps b) and c) the same? Please explain.

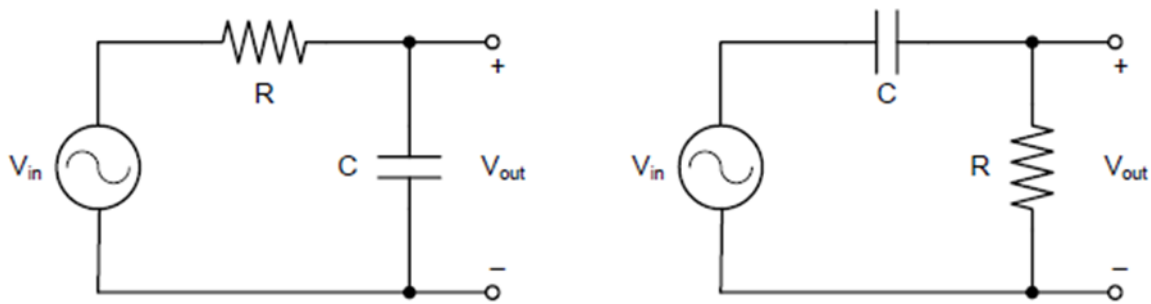


Figure 2: Circuits

#### (2) Two-stage RC ladder network

For circuits in Figure 3,

- Derive the transfer function.
- Find the frequency where the output voltage is  $90^\circ$  out of phase with the input voltage. Find the amplitude of the output voltage at this frequency.
- Find the frequency where the amplitude of the output voltage is  $1/3$  that of the input voltage. Find the phase difference between the input and output voltages.
- Are the frequencies found in steps b) and c) the same? Please explain.

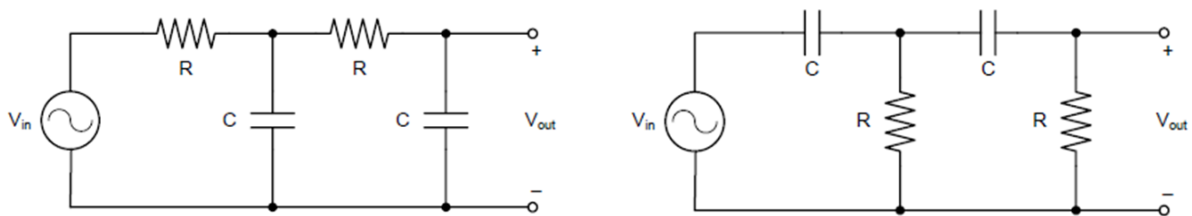


Figure 3: Circuits

### (3) Simulation

In LTSpice, build and simulate the circuits in Figures 2 and 3.

- (1) For circuits in Figure 2, find the frequency where the output voltage is  $45^\circ$  out of phase with the input voltage and find the amplitude of the output voltage at this frequency.
- (2) Find the frequency where the amplitude of the output voltage is  $1/\sqrt{2}$  that of the input voltage and find the phase difference between the input and output voltages.
- (3) For circuits in Figure 3, find the frequency where the output voltage is  $90^\circ$  out of phase with the input voltage. Find the amplitude of the output voltage at this frequency.
- (4) Find the frequency where the amplitude of the output voltage is  $1/3$  that of the input voltage and find the phase difference between the input and output voltages.

### (4) Experiment

- (1) Build circuits in Figures 2 and 3. Connect Ch1 to input and Ch2 to output so that both the input and the output are displayed on the oscilloscope.
- (2) Set the input voltage to  $\pm 5$  and frequency to that calculated in the corresponding step in Preparation. Measure the phase difference between the input and output. Adjust the frequency of the input signal appropriately so that the phase difference equals the values given below and measure the output voltage at that frequency for each case.
  - a)  $45^\circ$  for circuits in Figure 2
  - b)  $90^\circ$  for circuits in Figure 3

*Note: The way to measure the phase difference is as follows. First measure the time difference  $\Delta t$  between the two peaks of  $V_{in}$  and  $V_{out}$ . Then use the equation  $\phi = 2\pi f \times \Delta t$  (rad) to calculate  $\phi$ . Pay attention to the unit. Convert  $\phi$  to degrees if needed.*

- (3) Set the input voltage frequency to the one calculated in the corresponding step in Preparation. Measure the output voltage. Adjust the frequency of the input signal appropriately so that the output voltage equals the values given below and measure the phase difference between the input and output at this frequency for each case.
  - a)  $1/\sqrt{2}$  that of the input voltage for circuits in Figure 2
  - b)  $1/3$  that of the input voltage for circuits in Figure 3
- (4) Compare the results in steps 2 and 3 with those from Preparation and Simulation.